MACROECONOMIC THEORY **T. J. KEHOE**
ECON 8105 **FALL 2018**

MIDTERM EXAMINATION

Answer ***two*** of the following three questions.

1. Consider an economy with two infinitely lived consumers. There is one good in each period. Consumer i, $i = 1, 2$, has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t^i$$

Here β, $0 < \beta < 1$, is the common discount factor. Each of the consumers is endowed with a sequence of goods:

$$(w_0^1, w_1^1, w_2^1, w_3^1, ...) = (3, 1, 3, 1, ...)$$
$$(w_0^2, w_1^2, w_2^2, w_3^2, ...) = (1, 4, 1, 4, ...).$$

There is no production or storage.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Carefully state a proposition or propositions that establish the essential equivalence of the equilibrium concept in part a with that in part b. Be sure to specify the relationships between the objects in the Arrow-Debreu equilibrium and those in the sequential markets equilibrium.

(d) Calculate the Arrow-Debreu equilibrium for this economy. (This equilibrium is unique, but you do not have to prove this fact.) Use this answer and the answer to part c to calculate the sequential markets equilibrium.

(e) Define a Pareto efficient allocation for this economy. Prove that the allocations in parts a and b are Pareto efficient.

2. Consider an overlapping generations economy in which the representative consumer born in period $t, t = 1, 2, \ldots,$ has the utility function over consumption of the single good in periods t and $t+1$

$$u(c_t^t, c_{t+1}^t) = \log c_t^t + \log c_{t+1}^t$$

and endowments $(w_t^t, w_{t+1}^t) = (w_1, w_2)$. Suppose that the representative consumer in the initial old generation has the utility function

$$u^0(c_1^0) = \log c_1^0$$

and endowment $w_1^0 = w_2$ of the good in period 1 and endowment m of fiat money.

There is no production or storage.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Suppose that $m = 0$. Calculate both the Arrow-Debreu equilibrium and the sequential markets equilibrium.

(d) Define a Pareto efficient allocation. Suppose that $w_2 > w_1$. Is the equilibrium allocation in part c Pareto efficient? Explain carefully why or why not.

(e) Relax now the assumption that the good is not storable. Suppose instead that 1 unit of the good in period t, $t = 1, 2, \ldots$, can be transformed into $\theta > 0$ units of the good in period $t+1$. Define a sequential markets equilibrium for this economy.

3. Consider an economy in which there are both a continuum of measure one of infinitely lived consumers and a continuum of measure one of two-period lived overlapping generations consumers born every period. There is a single good in every period $t, t = 1, 2, \ldots$. The representative infinitely lived consumer is born in period 1, has the utility function

$$\sum_{t=1}^{\infty} \beta^{t-1} \log c_t^1 ,$$

and is endowed with a sequence of goods

$$(w_1^1, w_2^1, w_3^1, w_4^1 \ldots) = (3,1,3,1,\ldots) .$$

The representative overlapping generations consumer born in every period $t, t = 1, 2, \ldots,$ has the utility function

$$\log c_t^{2t} + \log c_{t+1}^{2t}$$

and endowment $(w_t^{2t}, w_{t+1}^{2t}) = (3,1)$. The representative initial old generations consumer has the utility function

$$\log c_1^{20}$$

and endowment $w_1^{20} = 1$ of the good in period 1. There is no fiat money, and there is no production or storage.

Notice that the demographic structure of the economy is such that in every period $t, t = 1, 2, \ldots,$ the active economic agents are a continuum of measure one infinitely lived consumers, a continuum of measure one of old overlapping generations consumers, and a continuum of measure one of young overlapping generations consumers.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Define a Pareto efficient allocation. Is the equilibrium allocation in part a Pareto efficient? Explain carefully why or why not.