MACROECONOMIC THEORY   **T. J. KEHOE**
ECON 8105   **FALL 2020**
MIDTERM EXAMINATION

Answer **two** of the following three questions..

1. Consider an economy with two infinitely lived consumers. There is one good in each period. Consumer i, $i = 1, 2$, has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t^i .$$

Here β, $0 < \beta < 1$, is the common discount factor. Each of the consumers is endowed with a sequence of goods:

$$(w_0^1, w_1^1, w_2^1, w_3^1, ...) = (2, 1, 2, 1, ...)$$
$$(w_0^2, w_1^2, w_2^2, w_3^2, ...) = (1, 4, 1, 4, ...).$$

There is no production or storage.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Calculate the Arrow-Debreu equilibrium for this economy. (This equilibrium is unique, but you do not have to prove this fact.)

(d) Suppose now that there is a production technology that transforms labor and capital into output that can be consumed or saved as capital:

$$y_t = \theta k_t^\alpha \ell_t^{1-\alpha},$$

where $\theta > 0$ and $1 > \alpha > 0$. Capital depreciates at the rate δ, $1 > \delta > 0$, every period. The consumers' endowments of labor are

$$(\bar{\ell}_0^1, \bar{\ell}_1^1, \bar{\ell}_2^1, \bar{\ell}_3^1, ...) = (2, 1, 2, 1, ...)$$
$$(\bar{\ell}_0^2, \bar{\ell}_1^2, \bar{\ell}_2^2, \bar{\ell}_3^2, ...) = (1, 4, 1, 4, ...).$$

Their endowments of capital in period 0 are $\bar{k}_0^i > 0$, $i = 1, 2$. Define a sequential markets equilibrium for this economy.

2. Consider an overlapping generations economy in which the representative consumer born in period t, $t = 1, 2, \ldots$, has the utility function over consumption of the single good in periods t and $t+1$

$$u\left(c_t^t, c_{t+1}^t\right) = \log c_t^t + \log c_{t+1}^t$$

and endowments $\left(w_t^t, w_{t+1}^t\right) = \left(w_1, w_2\right)$. Suppose that the representative consumer in the initial old generation has the utility function

$$u^0\left(c_1^0\right) = \log c_1^0$$

and endowment $w_1^0 = w_2$ of the good in period 1 and endowment m of fiat money.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Suppose that $m = 0$. Calculate both the Arrow-Debreu equilibrium and the sequential markets equilibrium.

(d) Still supposing that $m = 0$, relax now the assumption that the good is not storable. Suppose instead that 1 unit of the good in period t, $t = 0, 1, \ldots$, can be transformed into $\theta > 0$ units of the good in period $t+1$. Define a sequential markets equilibrium for this economy. Provide a condition on $\left(w_1, w_2\right)$ that guarantees that the storage technology will be used in equilibrium.

3. Consider an economy with a representative consumer with the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$

where $0 < \beta < 1$. This consumer has an endowment of $\bar{\ell} = 1$ units of labor in each period and $\bar{k}_0$ units of capital in period 0. Feasible allocation/production plans satisfy

$$c_t + k_{t+1} \le \theta k_t^{\alpha} \ell_t^{1-\alpha}.$$

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium.

(b) Describe a sequential markets structure for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium.

(c) Carefully state a proposition or propositions that establish the essential equivalence of the equilibrium concept in part a with that in part b. Be sure to specify the relationships between the objects in the Arrow-Debreu equilibrium and those in the sequential markets equilibrium.

(d) Suppose now that the representative consumer faces the choice of selling his labor services or consuming them as leisure. The consumer's utility function is

$$\sum_{t=0}^{\infty} \beta^t \left(\gamma \log c_t + (1-\gamma) \log x_t \right)$$

where $x_t = 1 - \ell_t$ is leisure. Define an Arrow-Debreu equilibrium for this economy.

(e) Define sequential market equilibrium for the economy in part d.