MACROECONOMIC THEORY **T. J. KEHOE**
ECON 8105 **FALL 2023**

FINAL EXAMINATION

Answer ***two*** of the following four questions.

1. Consider an overlapping generations economy in which there is one good in each period and each generation, except the initial one, lives for two periods. The representative consumer in generation t, $t = 1,2,\ldots$, has the utility function

$$\log c_t^t + \log c_{t+1}^t$$

and the endowment $(w_t^t, w_{t+1}^t) = (3,1)$. The representative consumer in generation 0 lives only in period 1, has the utility function $\log c_1^0$, and has the endowment $w_1^0 = 1$. Goods are not storable, and there is no fiat money.

a) Define an Arrow-Debreu equilibrium for this economy. Calculate the unique Arrow-Debreu equilibrium.

b) Define a sequential markets equilibrium for this economy. Calculate the unique sequential markets equilibrium.

c) Define a Pareto efficient allocation for this economy. Is the equilibrium allocation in part a Pareto efficient? Explain carefully why or why not.

d) Suppose now that consumers live for three periods, that the representative consumer in generation t, $t = 1,2,\ldots$, has the utility function

$$\log c_t^t + \log c_{t+1}^t + \log c_{t+2}^t$$

and the endowment $(w_t^t, w_{t+1}^t, w_{t+2}^t) = (2,3,1)$. There is also a generation -1 that lives only in period 1, whose representative consumer has the utility function $\log c_1^{-1}$ and who has the endowment $w_1^{-1} = 1$. In addition, there is a generation 0, whose representative consumer lives in periods 1 and 2 and who has the utility function $\log c_1^0 + \log c_2^0$ and endowment $(w_1^0, w_2^0) = (3,1)$. There is no fiat money. Define a sequential markets equilibrium for this economy.

e) Relax now the assumption that goods are not storable. Suppose instead that 1 unit of the good in period t, $t = 1,2,\ldots$, can be transformed into $\theta > 0$ units of the good in period $t+1$. Define a sequential markets equilibrium for this economy with storage in which consumers live for three periods.

2. Consider the optimal growth problem

$$\max \sum_{t=0}^{\infty} \beta^t \log c_t$$
$$\text{s.t.} \quad c_t + k_{t+1} - (1-\delta)k_t \le \theta k_t^{\alpha}$$
$$c_t, k_t \ge 0$$
$$k_0 = \bar{k}_0 .$$

Here $1 > \beta > 0$, $1 > \delta > 0$, $1 > \alpha > 0$, $\theta > 0$.

a) Write down the Euler conditions and the transversality condition for this problem. Calculate the nontrivial steady state values of c and k. (The trivial steady state is $\hat{c} = \hat{k} = 0$.)

b) Let $C(K)$ be the set of bounded continuous functions on $K \subset R_+$. Define a contraction mapping $T : C(K) \to C(K)$. State Blackwell's sufficient conditions for the mapping T to be a contraction mapping.

c) Let

$$T(V)(k) = \max \log c + \beta V(k')$$
$$\text{s.t.} \quad c + k' - (1-\delta)k \le \theta k^{\alpha}$$
$$c, k' \ge 0 .$$

Explain how you can choose the set $K \subset R_+$ so that $T(V)$ is bounded above without restricting the set of solutions to the optimal growth problem when $V = T(V)$. Ignore the fact that $T(V)$ is not bounded below. Prove that T satisfies Blackwell's sufficient conditions. Explain why it is useful that T is a contraction.

d) Suppose for the moment that $\delta = 1$. Guess that the value function has the form $a_0 + a_1 \log k$. Use the method of undetermined coefficients — often called guess and verify by economists — to calculate the value function $V(k) = T(V)(k)$ and the policy function $k' = g(k)$.

e) Define a sequential markets equilibrium for the economy for which the optimal growth problem in part a is the social planner's problem.

f) Suppose that you have solved the dynamic programming problem with the Bellman equation given by $V = T(V)$ in part c. Explain how you can use this solution to calculate the equilibrium in part e.

3. Consider an economy with two types of consumers. There are equal measures of each type. The representative consumer of type i, $i = 1,2$, has the utility function

$$\sum_{t=0}^{\infty} \beta^t \left[\theta_i \log c_t^i + (1-\theta_i) \log(\bar{h}_t^i - \ell_t^i) \right],$$

where $0 < \beta < 1$ and $0 < \theta_i < 1$, $i = 1,2$. This consumer has an endowment of $\bar{k}_0^i$ units of capital in period 0. Labor endowments are

$$(\bar{h}_0^1, \bar{h}_1^1, \bar{h}_2^1, \bar{h}_3^1, \ldots) = (2,\ 1,\ 2,\ 1, \ldots)$$

and

$$(\bar{h}_0^2, \bar{h}_1^2, \bar{h}_2^2, \bar{h}_3^2, \ldots) = (1,\ 3,\ 1,\ 3, \ldots).$$

Feasible allocation/production plans satisfy

$$c_t + k_{t+1} - (1-\delta)k_t \leq A k_t^{\alpha} \ell_t^{1-\alpha},$$

where variables without superscripts denote aggregates.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium.

(b) Describe a sequential markets structure for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium.

(c) Carefully state a proposition or propositions that establish the essential equivalence of the equilibrium concept in part a with that in part b. Be sure to specify the relationships between the objects in the Arrow-Debreu equilibrium and those in the sequential markets equilibrium.

(d) Define a Pareto efficient allocation and production plan. Prove that the equilibrium allocation and production plan in part a is Pareto efficient.

(e) Does the equilibrium allocation and production plan in part a solve a social planner's problem? If so, explain why it does and write down the Bellman equation. If not, explain carefully why not.

4. Consider the problem faced by an unemployed worker searching for a job. Every period that the worker searches, she receives a job offer with the wage w drawn independently from the time invariant probability distribution $F(v) = \text{prob}(w \le v)$, $v \in [0, B]$, $B > 0$. After receiving the wage offer w the worker faces the choice (1) to accept it or (2) to reject it, receive unemployment benefit b, and search again next period. That is,

$$y_t = \begin{cases} w & \text{if job offer has been accepted} \\ b & \text{if searching} \end{cases}.$$

The worker solves

$$\max E \sum_{t=0}^{\infty} \beta^t y_t$$

where $1 > \beta > 0$. Once a job offer has been accepted, there are no fires or quits.

a) Formulate the worker's problem as a dynamic programming problem by writing down the Bellman equation.

b) Using the Bellman equation in part a, characterize the value function $V(w)$ in a graph and argue that the worker's problem reduces to determining a reservation wage $\bar{w}$ such that she accepts any wage offer $w \ge \bar{w}$ and rejects any wage offer $w < \bar{w}$.

c) Consider two economies with different unemployment benefits b_1 and b_2 but otherwise identical. Let $\bar{w}_1$ and $\bar{w}_2$ be the reservation wages in these two economies. Suppose that that $b_2 > b_1$. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.

d) Consider two economies with different wage distributions F_1 and F_2 but otherwise identical. Let $\bar{w}_1$ and $\bar{w}_2$ be the reservation wages in these two economies. Define a mean preserving spread. Suppose that F_2 is a mean preserving spread of F_1. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.