ECON 8401 **T. J. KEHOE**
INTERNATIONAL TRADE AND PAYMENTS THEORY **FALL 2013**

PROBLEM SET #2

1. Consider an economy in which there are two types of goods, agriculture and manufactured goods. Agricultural goods are homogeneous and are produced using labor according to the constant returns to scale production function

$$y_0 = \ell_0.$$

Manufactured goods are differentiated by firm. The production function for firm *j* is

$$y_j = (1/b)\max[\ell_j - f, 0].$$

Here f is the fixed cost, in terms of labor, necessary to operate the firm and b is the unit labor requirement. Suppose that there is a representative consumer with preferences

$$\log c_0 + (1/\rho)\log \sum_{j=1}^{n} c_j^{\rho},$$

where $1 \geq \rho > 0$. There is an endowment of $\bar{\ell}$ units of labor.

a) Define a monopolistically competitive equilibrium for this economy in which firms follow Cournot pricing rules and there is free entry and exit.

b) Suppose that $b = 2$, $f = 4$, $\rho = 1/2$, and $\bar{\ell} = 36$. Calculate the autarky equilibrium.

c) Suppose now that $\bar{\ell} = 180$. Calculate the equilibrium.

d) Interpret the equilibrium in part c as a trading equilibrium among two countries, one with $\bar{\ell}^1 = 36$ and the second with $\bar{\ell}^2 = 144$. Assume that production of the homogeneous good is distributed proportionally across the two countries. What impact does trade have on the number of manufacturing firms in each country? The average output of firms? The total number of products available? Consumer utility and real income? Illustrate the efficiency gains using an average cost curve diagram.

2. Repeat the analysis of question 1 for two variants of the model. Compare the gains from trade in these two alternative models with those in the model in question 1.

a) Suppose that there are again a finite number of differentiated goods but that firms are now Bertrand competitors, rather than Cournot competitors.

b) Suppose that consumers have the utility function

$$\log c_0 + (1/\rho)\log \int_0^n c(j)^\rho \, dj.$$

Here there is a continuum $[0, n]$ of differentiated goods. (Hint: You need to be very careful in taking derivatives when solving the firms' profit maximization problems. In particular, the answers change drastically.)

c) Compare the gains in real income in parts a and b with each other and with those in question 1, part d.

3. Consider an economy in which the consumption space is the set of functions $c: R_+ \times R_+ \to R_+$. In $c(x,t)$ the index x denotes the type of good and the index t denotes the date at which it is consumed. An individual consumer has preferences given by the functional

$$u(c) = \int_0^\infty e^{-\rho t}\left[\int_0^\infty \log(c(x,t)+1)dx\right]dt.$$

Goods are produced using a single factor of production, labor:

$$y(x,t) = \ell(x,t)/a(x,t).$$

Each consumer has an endowment of labor equal to 1, and the total number of consumers is fixed at $\bar{\ell}$. The unit labor requirement $a(x,t)$ is bounded from below, $a(x,t) > \bar{a}(x)$, where

$$\bar{a}(x) = e^{-x}.$$

At $t = 0$ there is a $z(0) > 0$ such that $a(x,0) = e^{-x}$ for all $x < z(0)$ and that $a(x,0) = e^{x-2z(0)}$ for all $x \geq z(0)$. There is learning by doing of the form

$$\frac{\dot{a}(x,t)}{a(x,t)} = \begin{cases} -\int_0^\infty b(v,t)\ell(v,t)dv & \text{if } a(x,t) > \bar{a}(x) \\ 0 & \text{if } a(x,t) = \bar{a}(x) \end{cases}.$$

Here $\dot{a}(x,t)$ denotes the partial derivative of $a(x,t)$ with respect to t and $b(v,t) = b > 0$ if $a(v,t) > \bar{a}(v)$ and $b(v,t) = 0$ if $a(v,t) = \bar{a}(v)$. There is no borrowing or lending, and there is no storage.

a) Provide a motivation for the production technology described above.

b) Define an equilibrium for this economy. Characterize the equilibrium as much as possible.

c) Consider now a two country world in which the two countries are identical except in their endowments of labor and their initial technology levels. In particular, $z^1(0) > z^2(0)$. Define an equilibrium for this economy.

d) Describe the environment of a static Ricardian model whose equilibrium has the same values of prices and quantities as $p(x,0)$, $w^1(0)$, $w^2(0)$, $y^1(x,0)$, $y^2(x,0)$, $c^1(x,0)$, $c^2(x,0)$ in the economy of part c. Illustrate and explain the (five) different possibilities for patterns of production and consumption in this model. (To make things easy assume that $z^1(0)$ and $z^2(0)$ are sufficiently large so that good $x = 0$ is not produced in equilibrium.)

e) Describe the dynamics of the model, explaining the crucial role played by the sizes of the two countries, $\overline{\ell}^1$ and $\overline{\ell}^2$.