MACROECONOMIC THEORY **T. J. KEHOE**
ECON 8105 **FALL 2018**

PROBLEM SET #2

1. Consider an overlapping generations economy in which the representative consumer in generation t, $t = 1, 2, \ldots,$ has preferences over the consumption of the single good in each of the two periods of her life given by the utility function

$$u(c_t^t, c_{t+1}^t) = \log\ c_t^t + \log\ c_{t+1}^t .$$

This consumer is endowed with quantities of labor $(\ell_t^t, \ell_{t+1}^t) = (\bar{\ell}_1, \bar{\ell}_2)$. In addition, there is a generation 0 who representative consumer lives only in period 1 and has the utility function

$$u^0(c_1^0) = \log\ c_1^0 ,$$

and the endowment of $\bar{\ell}_2$ units of labor and $\bar{k}_1$ units of capital in period 1. In addition, this consumer has an endowment of fiat money m, which can be positive, negative or zero.

The production function is

$$f(k_t, \ell_t) = \theta k_t^{\alpha} \ell_t^{1-\alpha} ,$$

and capital depreciates at the rate δ per period, $0 \le \delta \le 1$.

a) Define a sequential market equilibrium for this economy.

b) Define an Arrow-Debreu equilibrium for this economy. Assume that consumers own capital and rent it to the firms. State and prove two theorems that establish the equivalence between a sequential market equilibrium and an Arrow-Debreu equilibrium.

c) Reduce the equilibrium conditions to a second-order difference equation in k_t, that is, an equation in k_{t+1}, k_t, k_{t-1} that includes no other endogenous variables.

d) Suppose that $m = 0$. Reduce the equilibrium conditions to a first-order difference equation in k_t. (Hint: in this case you know that the savings of generation t in period t in the sequential market equilibrium must equal k_{t+1}.)

2. Consider an economy with a representative, infinitely lived consumer who has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$

where $0 < \beta < 1$. The consumer owns one unit of labor in each period and $\bar{k}_0$ units of capital in period 0. The depreciation rate on capital is δ.

a) Suppose that the consumer borrows b_{t+1} bonds in period *t* to be paid off in period $t+1$. The consumer's initial endowment of bonds is $\bar{b}_0 = 0$, the wage rate in period *t* is w_t, the rental rate on capital is r_t^k, and the interest rate on bonds is r_t^b. Write down the consumer's utility maximization problem in a sequential markets economy. Explain why you need to include a constraint to rule out Ponzi schemes.

b) Write down the Euler conditions and the transversality conditions for this problem.

Suppose that feasible consumption/investment plans satisfy

$$c_t + k_{t+1} - (1-\delta)k_t \le F(k_t, \ell_t)$$

where $F(k_t, \ell_t) = \theta k_t^{\alpha} \ell_t^{1-\alpha}$.

c) Define a sequential markets equilibrium with borrowing and lending for this economy. Prove that in equilibrium $r_t^k - \delta = r_t^b$ if $k_t > 0$.

d) Suppose that the consumer sells his endowment of capital to the firm in period 0. Thereafter, firms buy and sell capital from each other. Describe the production set for the Arrow-Debreu economy, the set of feasible $k_0, k_1, \ldots, \ell_0, \ell_1, \ldots, c_0, c_1, \ldots$.

e) Define the Arrow-Debreu equilibrium for this economy.

f) Suppose that the consumer can buy new capital in each period and rent capital services to the firm. Define the Arrow-Debreu equilibrium for this economy.

g) Carefully state theorems that relate the equilibrium allocations in parts c, e, and f. In particular, state two theorems that relate the equilibrium in part c to that in part f and two theorems that relate that in part e to that in part f.