ECON 8401 **T. J. KEHOE**
INTERNATIONAL TRADE AND PAYMENTS THEORY **FALL 2014**

PROBLEM SET #3

1. Consider an economy similar to that in part b, question 2 on problem set 2. Again, consumers have the utility function

$$\log c_0 + (1/\rho)\log \int_0^n c(j)^\rho \, dj.$$

Also the production function for the differentiated good remains

$$y(j) = (1/b)\max[\ell(j) - f, 0].$$

Suppose now, however, that production of the agricultural good is governed by the function

$$y_0 = \ell_0^\alpha t_0^{1-\alpha}.$$

Here t_0 denotes inputs of land. Suppose now that there are two such countries, one with endowments $(\bar{\ell}^1, \bar{t}^1)$ and the other with endowments $(\bar{\ell}^2, \bar{t}^2)$, but otherwise identical.

a) Define an autarky equilibrium. Calculate this autarky equilibrium.

b) Define a trade equilibrium.

c) Suppose that $\bar{\ell}^1 / \bar{t}^1 > \bar{\ell}^2 / \bar{t}^2$. Derive conditions on $(\bar{\ell}^1, \bar{t}^1)$ and $(\bar{\ell}^2, \bar{t}^2)$ for which factor price equalization holds. Explain carefully what patterns of specialization are possible and what pattern of trade you would expect to see.

d) Continuing to suppose that $\bar{\ell}^1 / \bar{t}^1 > \bar{\ell}^2 / \bar{t}^2$, explain what changes you would expect to see in prices, factor prices, number of firms, average output levels, and utility levels as these two countries, initially in autarky, open to trade.

2. Consider an economy with two goods that enter both consumption and investment. The utility function of the representative consumer is

$$\sum_{t=0}^{\infty} \beta^t \log(c_{1t}^{a_1} c_{2t}^{a_1}).$$

Here $0 < \beta < 1$, $a_1 \geq 0$, $a_2 \geq 0$, and $a_1 + a_2 = 1$. Investment goods are produced according to

$$k_{t+1} - (1-\delta)k_t = dx_{1t}^{a_1} x_{2t}^{a_1}.$$

Feasible consumption/investment plans satisfy the feasibility conditions

$$c_{1t} + x_{1t} = \phi_1(k_{1t}, \ell_{1t}) = k_{1t}$$

$$c_{2t} + x_{2t} = \phi_2(k_{2t}, \ell_{2t}) = \ell_{2t}$$

where

$$k_{1t} + k_{2t} = k_t$$

$$\ell_{1t} + \ell_{2t} = \ell_t.$$

The initial endowment of k_t is $\bar{k}_0$. ℓ_t is equal to 1. (In other words, all variables are expressed in per capita terms.)

a) Carefully define a competitive equilibrium for this economy.

b) Reduce the equilibrium conditions to two difference equations in k_t and c_t and a transversality condition. Here $c_t = dc_{1t}^{a_1} c_{2t}^{a_1}$ is aggregate consumption. [Here is one possible approach: Prove a version of the first and second welfare theorems for this economy. Show that the two-sector social planer's problem is equivalent to a one-sector social planner's problem. Derive the difference equations and transversality conditions from the one-sector social planner's problem.]

3. Suppose now that there is a world composed of n different countries, all with the same preferences and technologies, but with different initial endowments of capital per worker, $\bar{k}_0^i$ The countries also have different population sizes, L^i, which are constant over time. (In other words, there is a continuum of identical consumers/workers of measure L^i in country i.) Suppose that there is no international borrowing or lending and no and no international capital flows. Define an equilibrium for this world economy. Prove that in this equilibrium the variables $c_{jt} = \sum_{i=1}^{n} L^i c_{jt}^i \Big/ \sum_{j=1}^{n} L^i$, $k_t = \sum_{i=1}^{n} L^i k_t^i \Big/ \sum_{i=1}^{n} L^i$, p_{it}, r_t, and w_t satisfy the equilibrium conditions of the economy in part a when $\bar{k}_0 = \sum_{i=1}^{n} L^i \bar{k}_0^i \Big/ \sum_{i=1}^{n} L^i$.

a) State and prove versions of the factor price equalization theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and the Heckscher-Ohlin theorem for this particular world economy.

b) What effects do the assumptions of no international borrowing and lending and no international capital flows have on your analysis? Try to be precise about which variables are uniquely determined in the equilibrium with borrowing and lending and/or capital flows are which would not be uniquely determined.

c) Consider the case where $\delta = 1$. Set $z_0 = c_0 / \left(\beta r_0 k_0 \right)$ and $z_t = c_{t-1} / k_t$, $t = 1, 2, \ldots$. Transform the two difference equations in part b into two difference equations in k_t and z_t. Prove that

$$\frac{k_t^i - k_t}{k_t} = \frac{z_t}{z_{t-1}} \left(\frac{k_{t-1}^i - k_{t-1}}{k_{t-1}} \right) = \frac{z_t}{z_0} \left(\frac{\bar{k}_0^i - \bar{k}_0}{\bar{k}_0} \right).$$

d) Consider again the case where $\delta = 1$. Let $s_t = c_t / y_t$ where

$$y_t = p_{1t} k_t + p_{2t} = d k_t^{a_1} = r_t k_t + w_t .$$

Transform the two difference equations in part b into two difference equations in k_t and s_t. Prove that

$$\frac{y_t^i - y_t}{y_t} = \frac{s_t}{s_{t-1}} \left(\frac{y_{t-1}^i - y_{t-1}}{y_{t-1}} \right) = \frac{s_t}{s_0} \left(\frac{y_0^i - y_0}{y_0} \right)$$

where $y_t^i = p_{1t} y_{1t}^i + p_{2t} y_{2t}^i = r_t k_t^i + w_t$. Calculate an expression for s_t and discuss the significance of this result.