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INTERNATIONAL TRADE AND PAYMENTS THEORY **FALL 2016**

PROBLEM SET #3

1. Consider an economy where the consumers have Dixit-Stiglitz utility functions and solve the problem

$$\max\ (1-\alpha)\log c_0 + (\alpha/\rho)\log\int_0^m c(z)^\rho dz$$
$$\text{s.t. } p_0 c_0 + \int_0^m p(z)c(z)dz = w\overline{\ell} + \pi$$
$$c(z) \geq 0 .$$

Here π are profits of the firms, which are owned by the consumers.

a) Suppose that the producer of good z takes the prices $p(z')$, for $z' \neq z$, as given. Suppose too that this producer has the production function

$$y(z) = \max\left[x(z)\left(\ell(z) - f\right), 0\right].$$

Solve the firm's profit maximization problem to derive an optimal pricing rule.

b) Suppose that there is a measure μ of potential firms. Firm productivities are distributed on the interval $x \geq 1$ according to the Pareto distribution with distribution function

$$F(x) = 1 - x^{-\gamma} .$$

Define an equilibrium for this economy.

c) Characterize the equilibrium of this economy in two different cases: In the first, μ is so small that all firms make nonnegative profits. In the second, μ is large enough so that there is a cutoff level of productivity $\overline{x} > 1$ such that all firms with productivity $x(z) \geq \overline{x}$ produce and earn nonnegative profits but no firm with productivity $x(z) < \overline{x}$ finds it profitable to produce. Find the relation between m and μ and $\overline{x}$.

d) Suppose that $\alpha = 0.5$, $\rho = 0.5$, $\overline{\ell} = 40$, $\mu = 10$, $\gamma = 4$, and $f = 2$. Calculate the equilibrium of this economy.

2. Consider a world with two countries like that in question 1 that engage in free trade. Each country i, $i = 1,2$, has a population of $\overline{\ell}_i$ and a measure of potential firms of μ_i. Firms' productivities are again distributed according to the Pareto distribution,

$F(x) = 1 - x^{-\gamma}$. A firm in country i faces a fixed cost of exporting to country j, $j \neq i$, of f_e where $f_e > f_d = f$ and an iceberg transportation cost of $\tau_i^j - 1 = \tau - 1 \geq 0$, $j \neq i$.

a) Define an equilibrium for this economy.

b) Explain the different cases for cutoff productivity levels in the equilibrium of the economy with trade. Suppose that the two countries are symmetric in the sense that $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}$ and $\mu_1 = \mu_2 = \mu$. Suppose too that μ is large enough so that not all firms can earn nonnegative profits in equilibrium. Characterize the equilibrium of this economy in the case where there are two cutoff levels, $\bar{x}_d$ and $\bar{x}_d$, where $\bar{x}_e > \bar{x}_d > 1$. Firms with $x(z) \geq \bar{x}_e$ produce for both the domestic and the export market; firms with $\bar{x}_e > x(z) \geq \bar{x}_d$ produce only for the domestic market, and firms with $\bar{x}_d > x(z)$ do not produce.

c) Suppose that $\alpha = 0.5$, $\rho = 0.5$, $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell} = 40$, $\mu_1 = \mu_2 = \mu = 10$, $\gamma = 4$, $f_d = 2$, $f_e = 3$, and $\tau_2^1 = \tau_1^2 = \tau = 1.2$. Calculate the equilibrium of this economy.

d) Suppose now that a free trade agreement sets $\tau_2^1 = \tau_1^2 = \tau = 1$. Recalculate the equilibrium in part c.

e) Suppose now that a different reform sets $f_e = 2$. Again recalculate the equilibrium in part b. Contrast the results with those in parts c and d. What sort of reform can lower f_e?