

PROBLEM SET #5

Consider a small open economy whose government borrows from international bankers. In every period, the value of output is

$$y(z) = Z^{1-z}\bar{y}$$

where  $1 > Z > 0$  is a constant and  $z = 0$  if the government defaults in that period or has defaulted in the past and  $\bar{y}$  is a constant. The government's tax revenue is  $\theta y(z)$  where the tax rate  $1 > \theta > 0$  is constant. The consumers in the economy consume  $c = (1 - \theta)y(z)$ . The government is benevolent and makes choices to maximize the expected discounted value of

$$u(c, g) = \log c + \gamma \log g$$

where  $\gamma > 0$  and  $1 > \beta > 0$  is the discount factor. At the beginning of every period, the state of the economy is  $s = (B, z_{-1}, \zeta)$  where  $B$  is the level of government debt;  $z_{-1} = 0$  if the government has defaulted in the past, and  $z_{-1} = 1$  if not, and  $\zeta \sim U[0, 1]$  is the realization of a sunspot variable. The government first offers  $B'$  to international bankers. The international bankers have the same discount factor  $\beta$  as the government. They are also risk neutral and have deep pockets. These international bankers buy the bonds at a competitive auction that determines a price for  $B'$ ,  $q(B', s)$ . The government finally chooses to default or not, which determines private consumption  $c$ . Government spending  $g$  is determined by the government's budget constraint

$$g + zB = \theta y(z) + q(B', s)B'.$$

If the government defaults, setting  $z = 0$ , then assume that  $z_{-1} = 0$  implies  $z = 0$  thereafter; that is, the economy suffers from the default penalty  $1 - Z$  forever. Furthermore,  $z_{-1} = 0$  implies  $q(B', s) = 0$ ; that is, the government is permanently excluded from credit markets.

- a) Define a recursive equilibrium.
- b) Assume that the bankers expect the government to default if  $\zeta > 1 - \pi$  and if such an expectation would be self-fulfilling, where  $1 > \pi \geq 0$  is an arbitrary constant. Find a level of debt  $\bar{b}$  such that, if  $B \leq \bar{b}$ , no default occurs in equilibrium, but that, if  $B > \bar{b}$ , default occurs in equilibrium.
- c) Suppose that  $B_0 > \bar{b}$ , and the government chooses to run down its debt to  $B_T \leq \bar{b}$  in  $T$  periods. Prove that it cannot be optimal to set  $B_T < \bar{b}$ . Prove that it is optimal for the

government to set  $g_t$  constant as long as  $B_t > \bar{b}$  and no crisis occurs. Find expressions for  $g_t$  and  $B_t$  that depend on  $B_0$  and  $T$ . Find an expression for the expected discounted value of the utility of running down the debt that starts at  $B_0$  to  $\bar{b}$  in  $T$  periods. Find the limit of these expressions when  $T = \infty$ .

d) Using the answers to part c, write down a formula that determines a value of debt  $\bar{B}(\pi)$  such that the government would choose to default if  $B > \bar{B}(\pi)$  even if international bankers do not expect a default.

e) Using the answers to parts a–d, construct a recursive equilibrium.

f) Use this model to interpret events of the Mexican financial crisis of December 1994 through January 1995.

g) Assume that  $Z = 0.9$ ,  $\bar{y} = 100$ ,  $\theta = 0.4$ ,  $\gamma = 0.5$ ,  $\beta = 0.95$ , and  $\pi = 0.05$ . Calculate  $\bar{b}$ . Calculate the expected discounted value of the utility of running down the debt that starts at  $B_0$  to  $\bar{b}$  in  $T$  periods for  $T = 1, 2, 3, 4, 5, 6, 7$ . Calculate  $\bar{B}(0.05)$ . Graph a policy function for government debt  $B'(B)$ . Graph a policy function for government spending  $g(B)$ .