TOPICS IN INTERNAIONAL TRADE **T. J. KEHOE**
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EXERCISES: COMPARATIVE ADVANTAGE

1. Consider an economy in which there are two countries and a continuum of goods in indexed $z \in [0,1]$. Goods are produced using labor:

$$y_j(z) = \ell_j(z) / a_j(z) .$$

where

$$a_1(z) = e^{\alpha z}$$

$$a_2(z) = e^{\alpha(1-z)} .$$

Here $y_j(z)$ is the production of good z in country j and $\ell_j(z)$ is the input of labor. The stand-in consumer in each country has the utility function

$$\int_0^1 \log c_j(z)\, dz .$$

This consumer is endowed with $\overline{\ell}_j$ units of labor where $\overline{\ell}_1 = \overline{\ell}_2 = \overline{\ell}$.

a) Define an equilibrium of the economy. Calculate expressions for all of the equilibrium prices and quantities. Draw a graph that illustrates the pattern of specialization in production and trade.

b) Suppose now the each country faces iceberg transportation costs of τ to import the goods from the other country. Repeat the analysis of part a.

c) Suppose finally that the two countries engage in a tariff war in which each country imposes an *ad valorem* tariff τ on imports from the other country. Repeat the analysis of part a.

d) For the model in part c, calculate gross domestic product, exports, and the real income index

$$v_j = \exp \int_0^1 \log c_j(z)\, dz$$

as functions of τ. Suppose that in the base period $\tau = 0$ and calculate real GDP — that is, GDP in base period prices — as well as GDP in current prices. Now suppose that in the base period $\tau = \tau_0$ and calculate real GDP.

2. Consider an economy in which the representative consumer has the utility function

$$u(c_1, c_2) = a_1 \log c_1 + a_2 \log c_2, \ i = 1,2 .$$

where $a_i > 0$ and $a_1 + a_2 = 1$. This consumer is endowed with capital and labor in the amounts $(\bar{k}, \bar{\ell})$. There are two goods, which are produced with the production technologies

$$y_j = \theta_j k_j^{\alpha_j} \ell_j^{1-\alpha_j}, \ j = 1,2 ,$$

where $1 > \alpha_1 > \alpha_2 > 0$.

a) Define an autarky equilibrium for this economy. Calculate this equilibrium.

b) Now suppose the country opens to trade. Suppose that the country is so small that it does not affect the world prices p_1, p_2. Define an equilibrium for this small open economy.

c) Find conditions on p_1, p_2, $\bar{k}$, and $\bar{\ell}$ such that the country produces positive amounts of both goods in the trade equilibrium.

d) Use the answer to part c to calculate a cone of diversification that depends on (p_1, p_2) — a set of $(\bar{k}, \bar{\ell})$ — such that small open economies with their endowments in this cone produce both goods.

e) State and prove a version of the Stolper-Samuelson theorem. [Hint: The proofs of this theorem and those in part e and in question 3, parts c and d are simple for this particular model. Please do not copy a general proof out of a book. Do the proof for this specific model. You should provide careful statements of the theorems. In particular, you should be careful about the restrictions on parameters needed for the theorem to hold.]

f) State and prove a version of the Rybczynski theorem.

3. Now consider a two-country version of question 2. The representative consumer in each country has the utility function

$$u(c_1^i, c_2^i) = a_1 \log c_1^i + a_2 \log c_2^i, \ i = 1,2 .$$

This consumer is endowed with capital and labor in the amounts $(\bar{k}^i, \bar{\ell}^i)$ where $\bar{k}^1 / \bar{\ell}^1 > \bar{k}^2 / \bar{\ell}^2$. The production technologies in the two countries are identical.

$$y_j^i = \theta_j (k_j^i)^{\alpha_j} (\ell_j^i)^{1-\alpha_j}, \; i, j = 1,2.$$

a) Define a free trade equilibrium. Explain the four possible patterns of specialization that are possible. [Hint: You need to distinguish among three conceptually different cases: one in which both countries produce both goods, another in which one country produces both goods and the other produces only one good, and the last in which each country produces only one good.]

b) Under what conditions on $(\overline{k}^1, \overline{\ell}^1)$ and $(\overline{k}^2, \overline{\ell}^2)$ do both countries produce both goods? Calculate a cone of diversification that depends on $(\overline{k}^1 + \overline{k}^2, \overline{\ell}^1 + \overline{\ell}^2)$ for the world economy such that, if both countries have endowments of labor and capital in the cone, they both produce positive amounts of both goods. [Be careful: This concept of the cone of diversification differs from that in question 1.]

c) Calculate the equilibrium in the case where both country produce both goods and for the case where country 1 specializes in the production of good 1 but country 2 produces both goods.

d) State and prove a version of the factor price equalization theorem for this particular world economy.

e) State and prove a version of the Heckscher-Ohlin theorem. Does this theorem hold when the endowments of one or both countries are outside the cone of diversification?