Suppose that $B_0 > \bar{b}$ and the government decides to reduce B to $\bar{b}$ in T periods, $T = 1, 2, ..., \infty$. The first order conditions for the government's problem imply that

$$g_t = g^T(B_0).$$

The government's budget constraints are

$$\begin{aligned} g^T(B_0) + B_0 &= \theta\bar{y} + \beta(1-\pi)B_1 \\ g^T(B_0) + B_1 &= \theta\bar{y} + \beta(1-\pi)B_2 \\ &\vdots \\ g^T(B_0) + B_{T-2} &= \theta\bar{y} + \beta(1-\pi)B_{T-1} \\ g^T(B_0) + B_{T-1} &= \theta\bar{y} + \beta\bar{b}. \end{aligned}$$

Notice that in period $T-1$ the government is able to sell its new debt, $\bar{b}$, at the price β, rather than at the price $\beta(1-\pi)$, because there is no risk that it will default on $\bar{b}$. Multiplying each equation by $(\beta(1-\pi))^t$ and adding, we obtain

$$\sum_{t=0}^{T-1}(\beta(1-\pi))^t g^T(B_0) + B_0 = \sum_{t=0}^{T-1}(\beta(1-\pi))^t \theta\bar{y} + (\beta(1-\pi))^{T-1}\beta\bar{b}$$

$$\frac{1-(\beta(1-\pi))^T}{1-\beta(1-\pi)} g^T(B_0) + B_0 = \frac{1-(\beta(1-\pi))^T}{1-\beta(1-\pi)}\theta\bar{y} + (\beta(1-\pi))^{T-1}\beta\bar{b}$$

$$g^T(B_0) = \theta\bar{y} - \frac{1-\beta(1-\pi)}{1-(\beta(1-\pi))^T}\left(B_0 - (\beta(1-\pi))^{T-1}\beta\bar{b}\right).$$

Notice that

$$g^\infty(B_0) = \lim_{T\to\infty} g^T(B_0) = \theta\bar{y} - (1-\beta(1-\pi))B_0.$$