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TAREA: GROWTH ACCOUNTING AND STRUCTURAL CHANGE

Questions 1, 2, and 3 require you to find annual time series data on constant price GDP, current price GDP, current price investment, hours worked, and working age population for the some country not the United States. Instead of constant price GDP, you may need to use a chained weighted quantity index. If you cannot find data on hours worked, use data on employment and assume that all workers worked a fixed amount, like 40 hours per week.

1. Construct a real investment series by calculating

$$I_t = \frac{\tilde{I}_t}{P_t},$$

where $\tilde{I}_t$ is nominal (current price) investment and P_t is the GDP deflator

$$P_t = \frac{\tilde{Y}_t}{Y_t},$$

where $\tilde{Y}_t$ is nominal (current price) GDP and Y_t is real (constant price) GDP or the chain weighted quantity index for GDP.

a) Use the data for real investment to construct a series for the capital stock following the rule

$$K_{t+1} = (1-\delta)K_t + I_t$$
$$K_{T_0} = \bar{K}_{T_0}.$$

where T_0 is the first year for which you have data on output and investment. Choose $\bar{K}_{T_0}$ so that

$$K_{T_0+1} / K_{T_0} = (K_{T_0+10} / K_{T_0})^{1/10}.$$

If you have sufficient data, calibrate the depreciation rate δ. Otherwise, use $\delta = 0.05$.

b) Repeat part a, but choose $\bar{K}_{T_0}$ so that

$$K_{T_0} / Y_{T_0} = \left(\sum_{t=T_0}^{T_0+9} K_t / Y_t\right)/10.$$

c) Compare the two series constructed in parts a and b.

2. Suppose that the aggregate production function for the country that you are studying takes the form

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}.$$

If you have sufficient data, calibrate the capital share α. Otherwise, use $\alpha = 0.35$.

a) Perform a growth accounting exercise for this economy. That is, decompose the growth and fluctuation in real GDP per working-age person into three factors, one of which depends on total factor productivity, one of which depends on the capital/output ratio, and the third of which depends on hours worked per working-age person:

$$\frac{Y_t}{N_t} = A_t^{\frac{1}{1-\alpha}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \frac{L_t}{N_t}.$$

b) Discuss what happens during different time periods.

3. Consider an economy in which the equilibrium solves the optimal growth problem

$$\begin{aligned} \max & \textstyle\sum_{t=0}^{\infty} \beta^t \left[\gamma \log C_t + (1-\gamma) \log(N_t \bar{h} - L_t) \right] \\ \text{s.t.} \quad & C_t + K_{t+1} - (1-\delta) K_t \le (g^{1-\alpha})^t A_0 K_t^{\alpha} L_t^{1-\alpha} \\ & C_t, K_t \ge 0 \\ & K_0 = \bar{K}_0 \\ & N_t = \eta^t N_0. \end{aligned}$$

a) Define a balanced growth path for this economy. Write down conditions that characterize this balanced growth path. Verify that the balanced growth path exhibits characteristics consistent with Kaldor's stylized facts on economic growth.

b) Calibrate the parameters of this economy — β, γ, g, A_0, and η and, if you have sufficient data, α and δ — so that that the behavior of this economy matches that in the data. Do the data for this country look like those of a balanced growth path? Discuss.

4. Consider a closed economy in which there are three goods: primaries, manufactures, and services. Each good is produced using labor with the production function

$$y_t^i = \theta_t^i \ell_t^i$$

for $i = a, m, s$, $t = 0, 1, \ldots$. There is a continuum of consumer/workers in the economy of measure $\bar{\ell}$. Each is endowed with one unit of labor and has the utility function

$$\textstyle\sum_{t=0}^{\infty} \beta^t u(c_t^a, c_t^m, c_t^s).$$

a) Define a competitive equilibrium for this economy.

b) Suppose that $\theta_t^i = \theta_0^i g_i^t$ for $i = a, m, s$, $t = 0, 1, \ldots$. Suppose that

$$g_m > g_a > 1,\ g_m > g_s > 1 .$$

Also suppose that

$$u(c_t^a, c_t^m, c_t^s) = \alpha_a \log(c_t^a + \bar{c}^a) + \alpha_m \log(c_t^m + \bar{c}^m) + \alpha_s \log(c_t^s + \bar{c}^s)$$

where $\bar{c}^a < 0$, $\bar{c}^m = 0$, $\bar{c}^s \geq 0$. Provide an interpretation of this utility function. Assume that the parameters are such that, in equilibrium $c_t^s > 0$ for all $t = 0, 1, \ldots$. Calculate expressions for c_t^i, $p_t^i c_t^i / w_t$, and real GDP,

$$y_t = p_0^a y_t^a + p_0^m y_t^m + p_0^s y_t^s .$$

c) Write a computer program or set up an Excel file to calculate the equilibrium in part b. Set

$$\begin{gathered}
\theta_0^a = \theta_0^m = \theta_0^s = 1 \\
g_a = 1.02,\ g_m = 1.04,\ g_s = 1.01 \\
\alpha_a = 0.05,\ \alpha_m = 0.30,\ \alpha_s = 0.65 \\
\bar{c}^a = -0.50,\ \bar{c}^m = 0,\ \bar{c}^s = 0.60 \\
\bar{\ell} = 100 .
\end{gathered}$$

Calculate the equilibrium for $t = 0, 1, \ldots, 200$. Graph the sectoral composition of output,

$$\left(\frac{p_t^a y_t^a}{p_t^a y_t^a + p_t^m y_t^m + p_t^s y_t^s}, \frac{p_t^m y_t^m}{p_t^a y_t^a + p_t^m y_t^m + p_t^s y_t^s}, \frac{p_t^s y_t^s}{p_t^a y_t^a + p_t^m y_t^m + p_t^s y_t^s} \right).$$

Graph the growth rate of real GDP. Discuss your results.

d) Suppose now that

$$u(c_t^a, c_t^m, c_t^s) = \frac{1}{\rho}\left[\alpha_a (c_t^a + \bar{c}^a)^\rho + \alpha_m (c_t^m + \bar{c}^m)^\rho + \alpha_s (c_t^s + \bar{c}^s)^\rho - 1 \right].$$

Calculate the utility function in the limit as ρ tends to 0. Repeat the analysis of part b. Explain how the case where $\rho < 0$ is very different from the case where $1 > \rho > 0$.

e) Repeat the exercise in part c setting $\rho = -1$ and

$$\alpha_a = 0.004504\,,\ \alpha_m = 0.882883\,,\ \alpha_s = 0.112613$$
$$\bar{c}^a = -0.6\,,\ \bar{c}^m = 0\,,\ \bar{c}^s = 0\,.$$

f) Repeat the exercise in part c setting $\rho = 0.5$ and

$$\alpha_a = 0.152968\,,\ \alpha_m = 0.216329\,,\ \alpha_s = 0.630703$$
$$\bar{c}^a = -0.6\,,\ \bar{c}^m = 0\,,\ \bar{c}^s = 0\,.$$