Roll-Over Crises in Sovereign Debt Markets

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Simple dynamic model in which the government of a small, open economy can experience a financial crisis during which it defaults on its sovereign debt.

Whether or not a crisis occurs can depend on the expectations of international investors and not only on fundamental factors like changes in national income or changes in the international interest rate. If investors expect a crisis to occur, they do not purchase new bonds.

Unlike in models in which crises depend only on changes in fundamentals, government can eliminate the possibility of a roll-over crisis by lengthening the maturity of its debt.

Based on Cole-Kehoe (1996, 2000), Conesa-Kehoe (2017).

3 types of agents:

- Government (or successive governments)
- International investors, continuum [0,1]
- Consumers, continuum [0,1]

Investors: risk neutral, deep pockets.

Consumers: utility $E\sum_{t=0}^{\infty}\beta^t u(c_t, g_t)$, passive.

State of the economy $s = (B, z_{-1}, \zeta)$:

- B is outstanding debt.
- z_{-1} is indicator of past default: $z_{-1} = 1$ no, $z_{-1} = 0$ yes.
- ζ is realization of sunspot variable.

Total output is $y(z) = Z^{1-z}\bar{y}$ where $1 > Z > 0$.

$(1 - Z)$ is output loss from past default.

Sunspot

Coordinating device for investors' expectations (Cass and Shell, 1981)

$\zeta \in U[0,1]$, uniform on interval [0,1]

$\zeta \geq 1-\pi$, investors panic and expect default

π, $0 < \pi < 1$, exogenously given

We construct a Markov equilibrium with two debt thresholds $\bar{b}$, $\bar{B}(\pi)$:

- If $B' \leq \bar{b}$, no default next period, $q = \beta$;
- If $\bar{b} < B' \leq \bar{B}(\pi)$, default with probability π next period, $q = \beta(1-\pi)$;
- If $\bar{B}(\pi) < B'$.

Problem of government:

$$V(s) = \max \; u(c,g) + \beta EV(s')$$

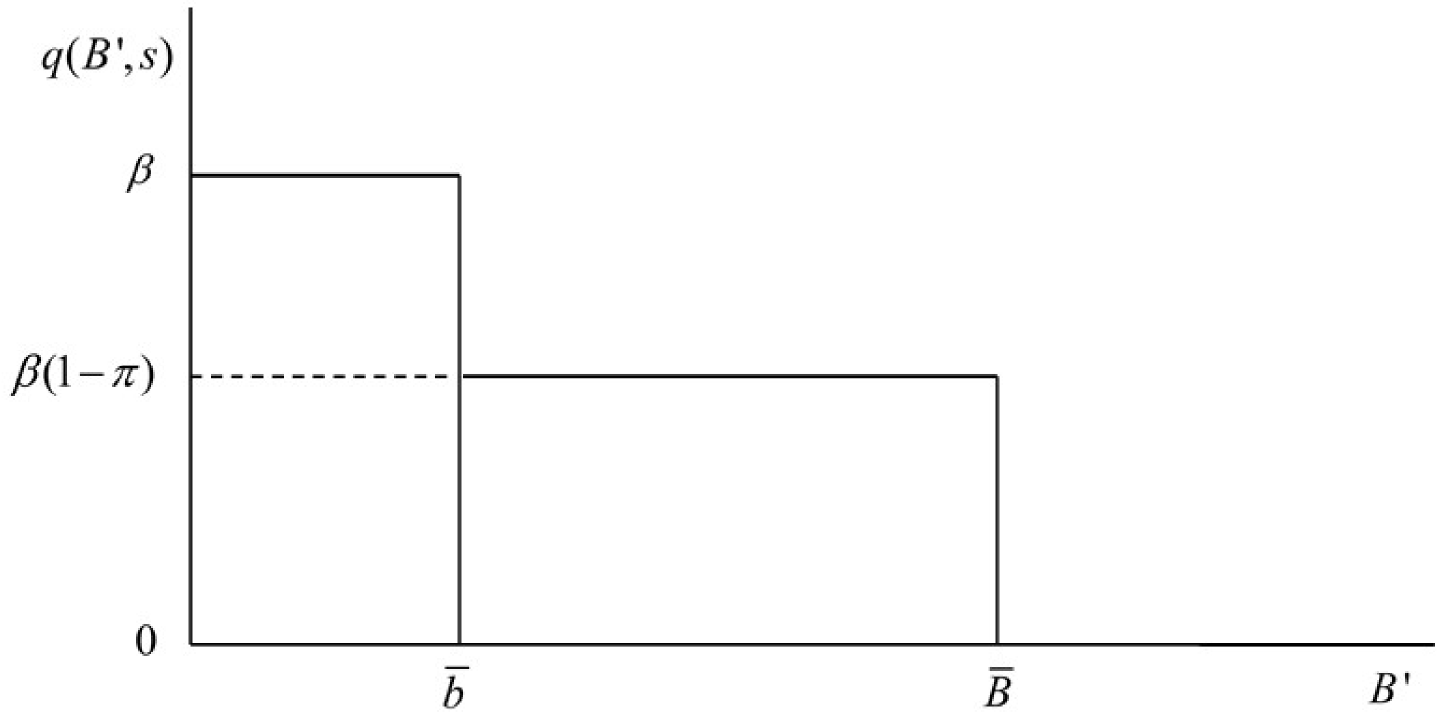
$$\text{s.t.} \;\; c = (1-\theta)y(z)$$

$$g + zB = \theta y(z) + q(B',s)B'$$

$$z \in \{0,1\}$$

$$z = 0 \text{ if } z_{-1} = 0.$$

$$q(B',(B,1,\zeta)) = \begin{cases} \beta & \text{if } B' \le \overline{b} \\ \beta(1-\pi) & \text{if } \overline{b} < B' \le \overline{B}(\pi) \\ 0 & \text{if } \overline{B}(\pi) < B' \end{cases}$$


$q(B',s)$
β
$\beta(1-\pi)$
0
$\bar{b}$
$\bar{B}$
B'

Timing

ζ_t is realized. State is $s_t = (B_t, z_{t-1}, \zeta_t)$. Government offers debt B_{t+1} at public auction,

↓

Investors decide whether or not to buy B_{t+1}, q_t is determined

↓

Government decides whether to repay B_t, z_t, determines (y_t, c_t, g_t)

Definition: An equilibrium is value function $V(s)$, policy functions, $B'(s), z(B',s,q)$, and $g(B',s,q)$, and price function $q(B',s)$ such that:

1. Beginning of period: given price function $q(B',s)$ and policy functions $z(B',s,q)$ and $g(B',s,q)$, value function $B'(s)$ is B' that solves

$$V(s) = \max \ u(c,g) + \beta EV(s')$$

$$\text{s.t.} \quad c = (1-\theta) y(z(B',s,q(B',s))$$

$$g(B',s,q(B',s)) + z(B',s,q(B',s))B = \theta y(z) + q(B',s)B'.$$

2. Bond market clears:

$$q(B',s) = \beta Ez(B'(s),s',q(B'(s'),s')).$$

3. End of period: given value function $V(s') = V(B', z, \zeta')$, new debt issued $B' = B'(s)$, and price function $q = q(B'(s), s)$, policy functions $z(B', s, q)$ and $g(B', s, q)$ are z and g that solve

$$\max \; u(c, g) + \beta EV(B', a', z, \zeta')$$

$$\text{s.t. } c = (1-\theta) y(a, z)$$

$$g + zB = \theta y(a, z) + qB'$$

$$z \in \{0,1\}, \; z = 0 \text{ if } z_{-1} = 0.$$

To calculate equilibrium, we need to calculate thresholds $\bar{b}$ and $\bar{B}(\pi)$.

Calculation of $\overline{b}$

Suppose $s = (B, z_{-1}, \zeta) = (B, 0, \zeta)$, then

$$V(B,0,\zeta) = \frac{u((1-\theta)Z\overline{y}, \theta Z\overline{y})}{1-\beta}.$$

Consequently, $\overline{b}$ is the solution to

$$u((1-\theta)\overline{y}, \theta\overline{y} - \overline{b}) + \frac{\beta u((1-\theta)\overline{y}, \theta\overline{y})}{1-\beta} = \frac{u((1-\theta)Z\overline{y}, \theta Z\overline{y})}{1-\beta}.$$

To calculate $\overline{B}(\pi)$, we need to know government's optimal policy $B'(s)$ in crisis zone, $\overline{b} < B' \leq \overline{B}(\pi)$.

Assume $u(c, g)$ is additively separable, continuously differentiable, and strictly concave in g. Then

$$q_t u_g(g_t) = \beta(1-\pi) u_g(g_{t+1}),$$

which implies that it is optimal to keep g_t constant in crisis zone.

Increasing $B'(s)$ is not optimal because $B'(s) > \overline{B}(\pi)$ eventually.

Keep $B'(s)$ constant or reduce it to $\overline{b}$ in T steps.

Easy to show it is not optimal to reduce $B'(s)$ to less than $\overline{b}$.

Reduce B_t to $\overline{b}$ in T periods, $T = 1, 2, ..., \infty$. Constant expenditure g_t throughout the sequence is

$$g_t = g^T(B_0).$$

To calculate $g^T(B_0)$, we use the government's budget constraints:

$$g^T(B_0) + B_0 = \theta\overline{y} + \beta(1-\pi)B_1$$

$$g^T(B_0) + B_1 = \theta\overline{y} + \beta(1-\pi)B_2$$

$$\vdots$$

$$g^T(B_0) + B_{T-2} = \theta\overline{y} + \beta(1-\pi)B_{T-1}$$

$$g^T(B_0) + B_{T-1} = \theta\overline{y} + \beta\overline{b}.$$

Multiplying each equation by $(\beta(1-\pi))^t$ and adding, we obtain

$$\sum_{t=0}^{T-1} (\beta(1-\pi))^t g^T(B_0) + B_0 = \sum_{t=0}^{T-1} (\beta(1-\pi))^t \theta\overline{y} + (\beta(1-\pi))^{T-1}\beta\overline{b}$$

$$\frac{1-(\beta(1-\pi))^T}{1-\beta(1-\pi)} g^T(B_0) + B_0 = \frac{1-(\beta(1-\pi))^T}{1-\beta(1-\pi)} \theta\overline{y} + (\beta(1-\pi))^{T-1}\beta\overline{b}$$

$$g^T(B_0) = \theta\overline{y} - \frac{1-\beta(1-\pi)}{1-(\beta(1-\pi))^T}\left(B_0 - (\beta(1-\pi))^{T-1}\beta\overline{b}\right).$$

Observe that

$$g^\infty(B_0) = \lim_{T\to\infty} g^T(B_0) = \theta\overline{y} - (1-\beta(1-\pi))B_0$$

We compute $V^T(B_0)$ as

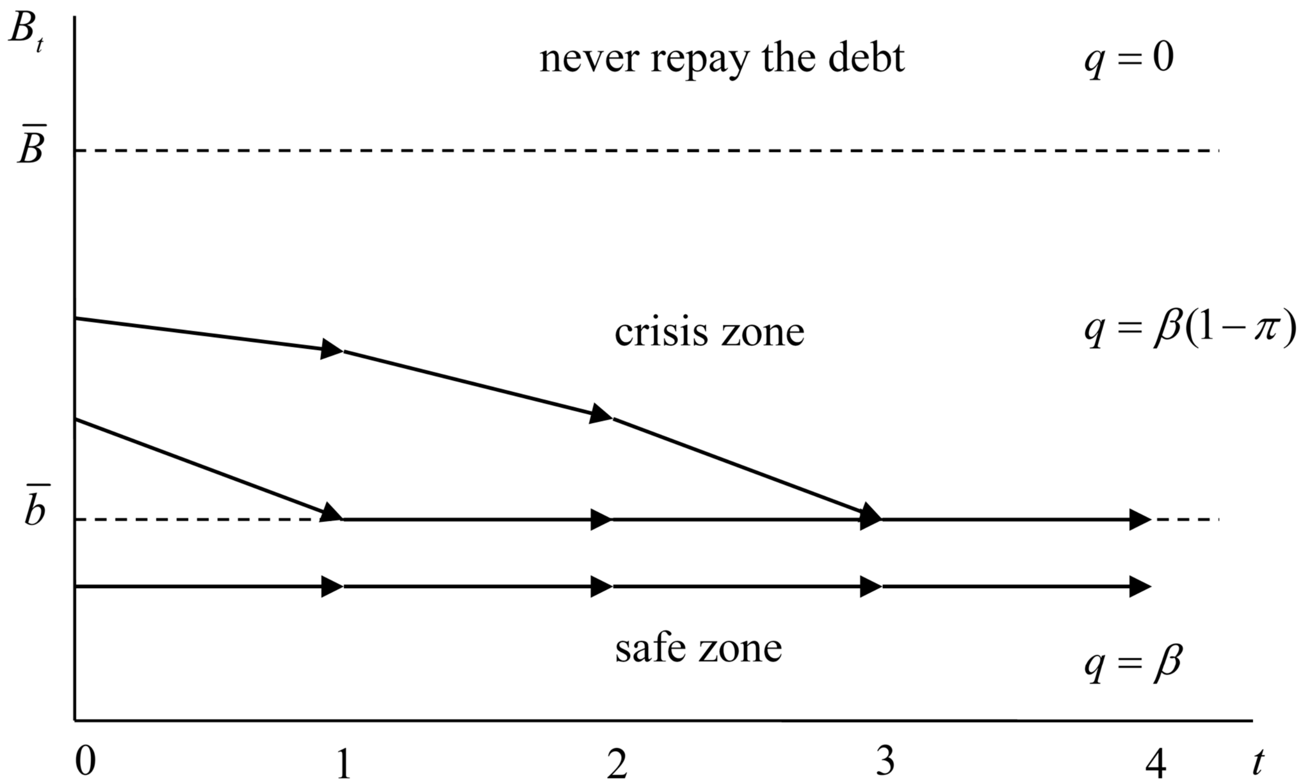
$$V^T(B_0) = (1+\beta(1-\pi)+(\beta(1-\pi))^2+\ldots+(\beta(1-\pi))^{T-1})u((1-\theta)\overline{y}, g^T(B_0))$$
$$+(1+\beta(1-\pi)+(\beta(1-\pi))^2+\ldots+(\beta(1-\pi))^{T-2})\frac{\beta\pi u((1-\theta)Z\overline{y},\theta Z\overline{y})}{1-\beta}$$
$$+(\beta(1-\pi))^{T-2}\frac{\beta u((1-\theta)\overline{y},\theta\overline{y}-(1-\beta)\overline{b}(1))}{1-\beta},$$

$$V^T(B_0) = \frac{1-(\beta(1-\pi))^T}{1+\beta(1-\pi)}u((1-\theta)\overline{y}, g^T(B_0))$$
$$+\frac{1-(\beta(1-\pi))^{T-1}}{1+\beta(1-\pi)}\frac{\beta\pi u((1-\theta)Z\overline{y},\theta Z\overline{y})}{1-\beta} \quad .$$
$$+(\beta(1-\pi))^{T-2}\frac{\beta u((1-\theta)\overline{y},\theta\overline{y}-(1-\beta)\overline{b})}{1-\beta}$$

To calculate $\overline{B}(\pi)$, we solve

$$V(\overline{B}(\pi)) = \max\left[V^1(\overline{B}(\pi)), V^2(\overline{B}(\pi)), \ldots, V^\infty(\overline{B}(\pi))\right]$$
$$= u((1-\theta)Z\overline{y}, \theta Z\overline{y} + \beta(1-\pi)\overline{B}(\pi)) + \beta\frac{u((1-\theta)Z\overline{y}, \theta Z\overline{y})}{1-\beta}\cdot$$

$$V(B,1,\zeta) = \begin{cases} \dfrac{u((1-\theta)\overline{y}, \theta\overline{y} - (1-\beta)B)}{1-\beta} & \text{if } B \le \overline{b} \\ \max\left[V^1(B), V^2(B), \ldots, V^\infty(B)\right] & \text{if } \overline{b} < B \le \overline{B}(\pi),\ \zeta \le 1-\pi \\ \dfrac{u((1-\theta)Z\overline{y}, \theta Z\overline{y})}{1-\beta} & \text{if } \overline{b} < B \le \overline{B}(\pi),\ \zeta > 1-\pi \\ \dfrac{u((1-\theta)Z\overline{y}, \theta Z\overline{y})}{1-\beta} & \text{if } \overline{B}(\pi) < B \end{cases}$$


B_t
never repay the debt
$q = 0$
$\overline{B}$
crisis zone
$q = \beta(1-\pi)$
$\overline{b}$
safe zone
$q = \beta$
0
1
2
3
4
t

Long maturity bonds

Hatchondo and Martinez (2009) and Chaterjee and Eyigungor (2012)

Government solves

$$V(s) = \max \ u(c, g) + \beta EV(s')$$

$$\text{s.a.} \quad c = (1-\theta) y(z)$$

$$g + z\delta B = \theta y(z) + q(B', s)\left(B' - (1-\delta)B\right)$$

$$z \in \{0,1\}$$

$$z = 0 \text{ if } z_{-1} = 0.$$

δ, $0 \leq \delta \leq 1$, is fraction of total debt due each period. Average maturity of debt is $1/\delta$ periods.

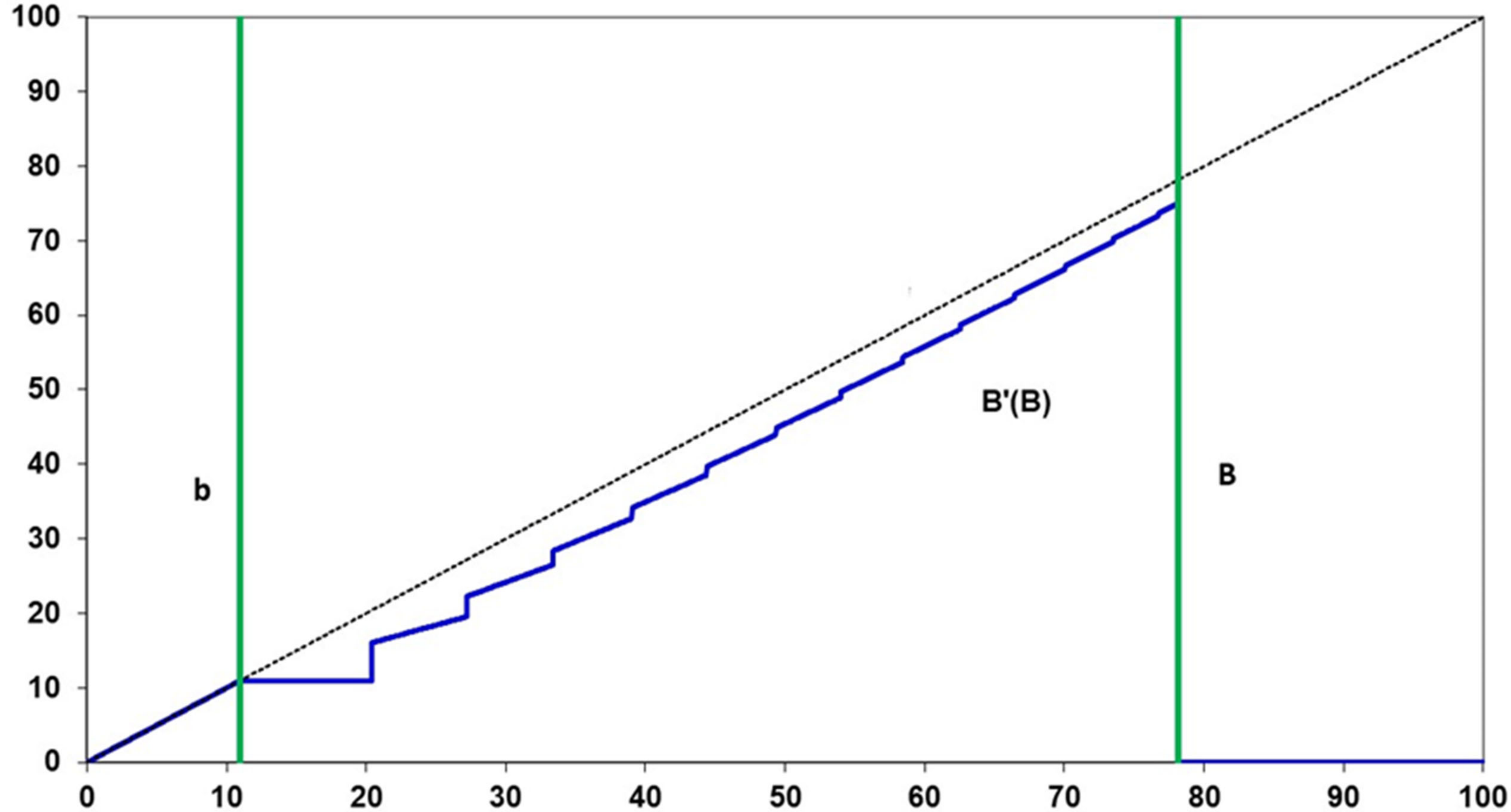
Model becomes more complex because bond prices depend on the policy function for debt accumulation of the government, $B'(B)$:

$$q(B',\pi) = \begin{cases} \beta\left[\delta + (1-\delta) q(B'(B'))\right] & \text{if } B' \le \overline{b} \\ \beta(1-\pi)\left[\delta + (1-\delta) q(B'(B'))\right] & \text{if } \overline{b} < B' \le \overline{B} \\ 0 & \text{if } \overline{B}(\pi) < B' \end{cases}$$

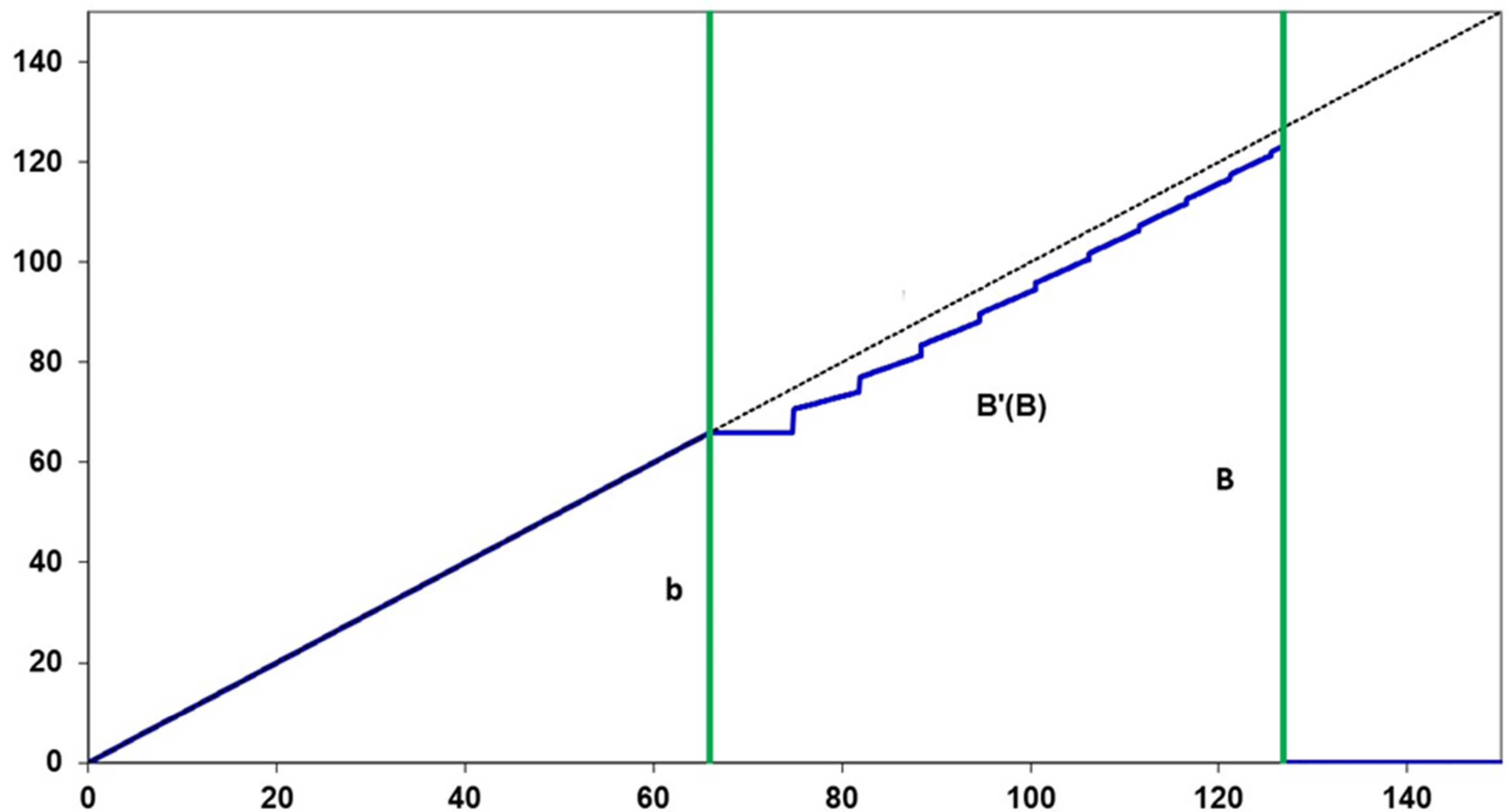
Conesa and Kehoe's calibration to Spanish economy in 2010:

parameter	value	target
$\bar{y}$	100.0	normalization
Z	0.95	default cost of 5% of output (1996)
β	0.98	risk-free interest rate of 2%
π	0.04	interest rate in the crisis zone of 6%
γ	0.20	arbitrary value
θ	0.36	fiscal revenue as fraction of output
$\bar{g}$	25.0	mínimum public spending

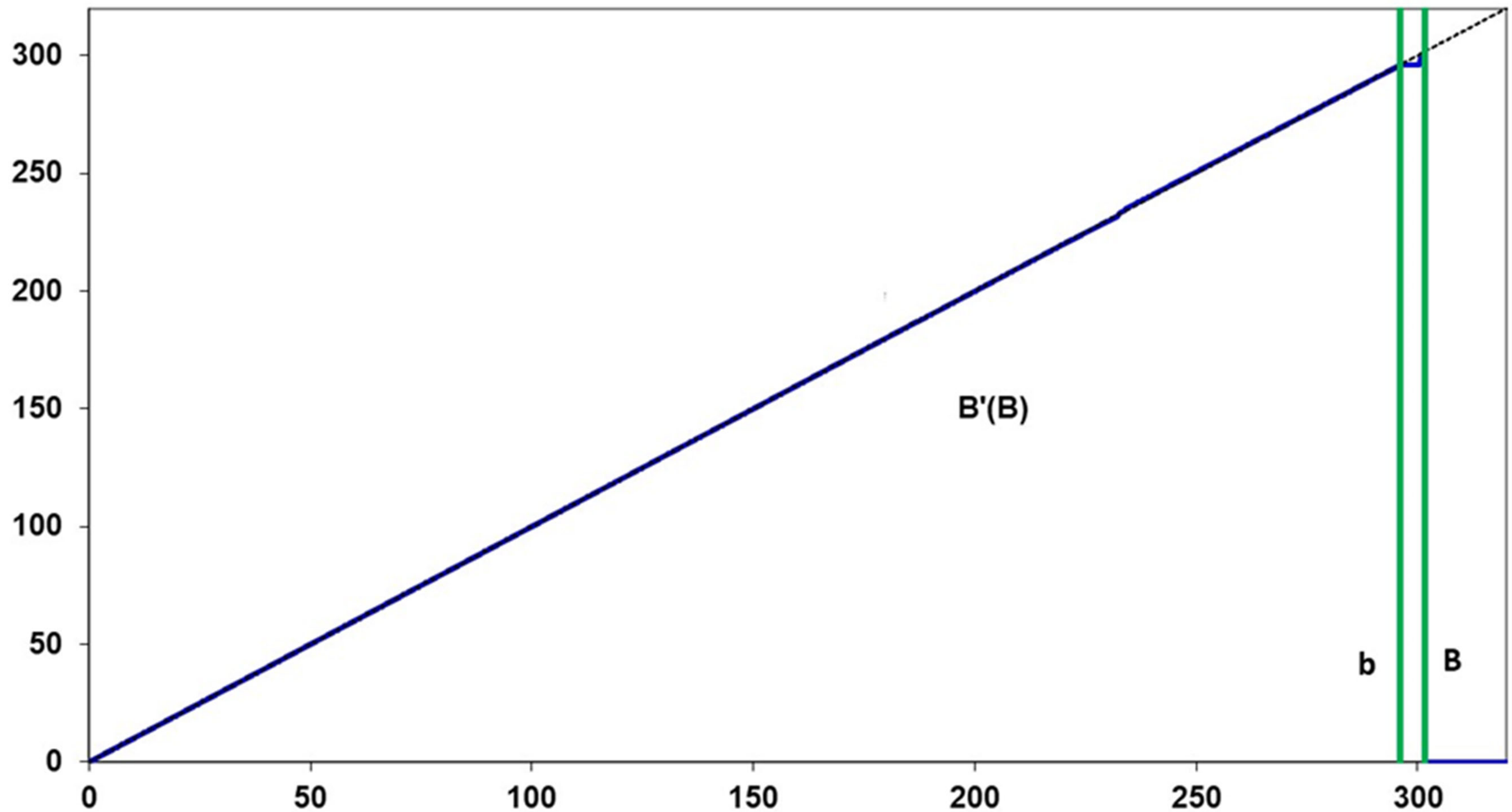
$$u(c,g) = \log(c) + \gamma \log(g - \bar{g})$$



$$\delta = 1$$



$$\delta = 1/6$$



$$\delta = 1/30$$

Impatient government, $\beta_g = 0.90$