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PROBLEM SET #1

1. Consider an economy in which there are two countries and a continuum of goods in indexed $z \in [0,1]$. Goods are produced using labor:

$$y_j(z) = \ell_j(z) / a_j(z).$$

where

$$a_1(z) = e^{\alpha z}$$

$$a_2(z) = e^{\alpha(1-z)}.$$

Here $y_j(z)$ is the production of good z in country j and $\ell_j(z)$ is the input of labor. The stand-in consumer in each country has the utility function

$$\int_0^1 \log c_j(z)\, dz.$$

This consumer is endowed with $\bar{\ell}_j$ units of labor where $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}$.

a) Define an equilibrium of the economy. Calculate expressions for all of the equilibrium prices and quantities. Draw a graph that illustrates the pattern of specialization in production and trade.

b) Suppose now the each country faces iceberg transportation costs of τ to import the goods from the other country. Repeat the analysis of part a.

c) Suppose finally that the two countries engage in a tariff war in which each country imposes an *ad valorem* tariff τ on imports from the other country. Repeat the analysis of part a.

2. Consider an economy in which there are two types of goods, agriculture and manufactured goods. Agricultural goods are homogeneous and are produced using labor according to the constant returns to scale production function

$$y_0 = \ell_0.$$

Manufactured goods are differentiated by firm. The production function for firm *j* is

$$y_j = (1/b)\max[\ell_j - f, 0].$$

Here f is the fixed cost, in terms of labor, necessary to operate the firm and b is the unit labor requirement. Suppose that there is a representative consumer with preferences

$$\log c_0 + (1/\rho)\log \sum_{j=1}^{n} c_j^{\rho} ,$$

where $1 \geq \rho > 0$. There is an endowment of $\bar{\ell}$ units of labor.

a) Define a monopolistically competitive equilibrium for this economy in which firms follow Cournot pricing rules and there is free entry and exit.

b) Suppose that $b = 2$, $f = 4$, $\rho = 1/2$, and $\bar{\ell} = 36$. Calculate the autarky equilibrium.

c) Suppose now that $\bar{\ell} = 180$. Calculate the equilibrium.

d) Interpret the equilibrium in part c as a trading equilibrium among two countries, one with $\bar{\ell}^1 = 36$ and the second with $\bar{\ell}^2 = 144$. Assume that production of the homogeneous good is distributed proportionally across the two countries. What impact does trade have on the number of manufacturing firms in each country? The average output of firms? The total number of products available? Consumer utility and real income? Illustrate the efficiency gains using an average cost curve diagram.

e) Suppose that consumers have the utility function

$$\log c_0 + (1/\rho)\log \int_0^n c(j)^{\rho}\, dj.$$

Here there is a continuum $[0, n]$ of differentiated goods. (Hint: You need to be very careful in taking derivatives when solving the firms' profit maximization problems. In particular, the answers change drastically.)

f) Compare the gains in real income in parts e with those in part d.