Model of a self-fulfilling debt crisis

- Motivation: Mexican crisis
  - Crisis occurred with “sound” fundamentals.
    - why now, and not earlier?
    - if in Mexico, why not elsewhere?
- Model:
  - Crisis occurrence depends upon extrinsic uncertainty.
  - Crises zone: conditions for crisis depend on fundamentals:
    - debt vs. output
    - term structure of debt.
  - In crisis zone probability of a crisis is arbitrary.
  - Examine optimal government policy if a crisis can occur in equilibrium:
    - motivates fleeing zone by lowering debt.
  - Examine role of debt maturity in preventing a crisis.

Prior literature

- Diamond and Dybvig (1993)
- Calvo (1988)
  - multiple equilibria with different default levels
  - investors expectations of default change interest rate
  - change in interest rate induces different default levels
  - (our model shows that government cannot peg interest rate)
- Alesina, Prati, and Tabellini (1989)
  - a simple model of lending crisis
  - with 2 period as opposed to 1 period debt possibility of crisis reduced
  - (our model shows that once crisis has started there is nothing government can do)
- Chari and P. Kehoe (1996)
  - information cascade story for crises
  - opposite information assumption: no communication

Model

- infinite horizon, discrete time model with three types of actors and one good in each period
- actors:
  - **Government:**
    - cannot commit to its policies or repayment of debts
    - sequentially chooses spending and borrowing levels
    - only source of revenue is a flat rate income tax
    - borrows by issuing pure discount bonds
    - benevolent concern for consumers

$$g_t + z_t B_t \leq \theta a_t f(k_t) + q_t B_{t+1}$$

- **(International) Bankers:**
  - large number of risk neutral lenders with discount factor β
  - price of government's one period bonds is q
  - q depends on amount of debt issued

$$x_t + q_t b_{t+1} \leq \bar{x} + z_t b_t$$

- **Consumers:** (risk neutral in consumption for simplicity) choose $\{c_t, k_{t+1}\}$ and have

$$E \sum_{t=0}^{\infty} \beta^t [c_t + v(g_t)]$$

$$s.t.\ c_t + k_{t+1} \leq (1-\theta) a_t f(k_t)$$

- **Default penalty** a_t falls from 1 to $\alpha < 1$ forever, and government is excluded from credit market
  - Bulow and Rogoff (1989a): saving mechanisms undercuts reputation motivation.
  - They argue for direct penalties enforcing repayment.
  - Cole and P. Kehoe show spillovers can motivate.
  - Both provide explanations for a_t.

- **Exogenous sunspot variable:** ζ_t is i.i.d. and uniformly distributed on $[0,1]$

- **Timing within a period:**

1. ζ_t is realized, and the aggregate state is $s_t = (B_t, K_t, a_{t-1}, \zeta_t)$;

2. the government chooses B_{t+1};

3. each banker chooses b_{t+1}, which along with z_t determines x_t;

4. the government chooses z_t and g_t;

5. each consumer chooses k_{t+1} and c_t.

Recursive equilibrium

- aggregate state is $s_t = (B_t, K_t, a_{t-1}, \zeta_t)$
- a collection of value functions and policy functions:
  - for consumers, $V_c(k, s, B', g, z)$ and $c(k, s, B', g, z)$, $k'(k, s, B', g, z)$
  - for bankers, $V_b(b, s, B')$
  - for the government, $V_g(s)$ and $B'(s)$, and $g(s', B', q)$, $z(s', B', q)$
- an equation of motion for the aggregate capital stock $K'(s, B', g, z)$.
  - consistency of consumer's behavior K' :

$$K'(s, B', g, z) = k'(K, s, B', g, z).$$

- **Banker's problem:** price function

$$q(s, B') = \beta E z(s', B', q(s', B'(s')))$$

- **Consumer's problem:**

$$V_c(k, s, B', g, z) = \max_{c, k'} \; c + v(g) + \beta E V_c(k', s', B'', g', z')$$

subject to

$$c + k' \leq (1 - \theta) a(s, z) f(k)$$

$$c, k' \geq 0$$

Use the government's policy functions, $B'(\bullet), z(\bullet), g(\bullet),$ along with $q(\bullet)$ and $K'(\bullet)$ to determine s', B', g' and z'.

- **Government problems:**
  - **problem one** pick B'

$$V_g(s) = \max_{B'} \; c(K, s, B', g, z) + v(g) + \beta E V_g(s')$$

Use the government's policy functions, $z(\bullet), g(\bullet),$ along with $q(\bullet)$ and $K'(\bullet)$ to determine $g, z,$ and s'.

  - **problem two** pick z and g

$$\max_{g,z} \; c(K, s, B', g, z) + v(g) + \beta E V_g(s')$$

subject to

$$g + zB \leq \theta a(s,z) f(K) + qB'$$

$$z = 0 \text{ or } z = 1$$

$$g \geq 0.$$

Use $K'(\bullet)$ to determine s'.

Agenda

- zero probability of crisis equilibrium
  - conditions for no-lending continuation equilibrium
- positive probability of crisis equilibrium
  - changes behavior before crisis
  - changes interest rate
- maturity of debt
  - little role if no sunspot
  - important role if sunspot
  - nothing government can do once crisis has started
- examine Mexican crisis.

What happens in default?

- Productivity falls from $a_t = 1$ to $a_t = \alpha < 1$.
- Government loses all access to credit markets after a default.
- Equilibrium price of government debt is $q = 0$.
- **Consumers:** invest k^d and eat the remainder:

$$k^d : (1-\theta)\alpha\beta f'(k^d) = 1$$

$$c^d(k) = (1-\theta)\alpha f(k) - k^d$$

- **Bankers:** buy none of the government debt since they believe that $z = 0$.
- **Government:** eats everything it raises in revenue, issues no new claims and sets $z = 0$; its post-default payoff is given by

$$c^d(K) + v(\theta\alpha f(K)) +$$
$$\beta[c^d(k^d) + v(\theta\alpha f(k^d))]/(1-\beta).$$

No crisis equilibrium with lending

- State is (B, K, a_{-1}) (ignore ζ).
- Equilibrium price function:
  - $q(s, B') = \beta$ if government has no incentive to default
  - $q(s, B') = 0$ if it does.
- **Consumers:**
  - if prior default, follow default continuation equilibrium
  - if default next period, set $k' = k^d$ and $c = (1-\theta)f(k) - k^d$
  - otherwise invest k^n and eat $c^n(k)$

$$k^n : (1-\theta)\beta f'(k^n) = 1$$

$$c^n(k) = (1-\theta)f(k) - k^n.$$

- **Bankers:**
  - buy any amount of debt at the price β if they believe government will not default
  - buy none if they believe that the government will default.

- **Government:**
  - payoff from defaulting today:

$$V_g^d(s,B',q) = c^d(K) + v(\theta\alpha f(K) + qB') \\ + \beta[c^d(k^d) + v(\theta\alpha f(k^d))]/(1-\beta)$$

  - payoff from not defaulting today, given $a_{-1} = 1$ and others do not believe will default:

$$V_g^n(s,B',q) = c^n(K) + v(\theta f(K) - B + qB') \\ + \beta V_g(s')$$

  - optimal default rule: choose the maximum of two payoffs,

$$z(s,B',q) = \begin{cases} 1 & \text{if } V_g^n(s,B',\beta) \geq V_g^d(s,B',\beta) \\ 0 & \text{otherwise} \end{cases}$$

- **Equilibrium:**
  - if initial debt low enough, get commitment outcome and payoff

$$V_g^n(s, B, \beta)$$

    - government follows stationary policy: $B' = B$
  - if initial debt too high for stationary policy, then government runs down debt to reduce incentive to default and then goes stationary
    - recursively construct the no defaults sets of states and the government's payoff from defaulting and show that this occurs in no more than two recursions
  - if too high for this, then no lending/default only equilibrium with payoff

$$V_g^d(s, 0, 0).$$

No-lending equilibrium

- belief that the government will default can be self-fulfilling if it induces the government to default
  - **no-lending continuation condition**

$$V_g^d(s,0,0) > V_g^n(s,0,0)$$

    - payoff in no-lending continuation equilibrium:

$$V_g^d(s,0,0)$$

    - in continuation equilibrium $z(s) = 0$ and $q(s,B') = 0$
    - government sets $B'(s) = 0,$ and consumers set $k' = k^d$.

Crisis equilibrium?

- lending equilibrium **participation constraint**:

$$V_g^n(s, B', \beta) \geq V_g^d(s, B', \beta)$$

  - define $\bar{B}$: largest B such that there exists a $B' \geq 0$ for which participation constraint is satisfied

- **no-lending continuation condition:**

$$V_g^d(s, B', 0) = V_g^d(s, 0, 0) > V_g^n(s, 0, 0) = V_g^n(s, B', 0)$$

  - define $\bar{b}(K)$:

$$V_g^d((\bar{b}(K), K, 1), 0, 0) = V_g^n((\bar{b}(K), K, 1), 0, 0)$$

  (notice that $\bar{b}'(K) > 0$)

- **CRISIS ZONE** exists if $\bar{b}(k^n) < \bar{B}$.

Self-fulfilling crisis equilibrium

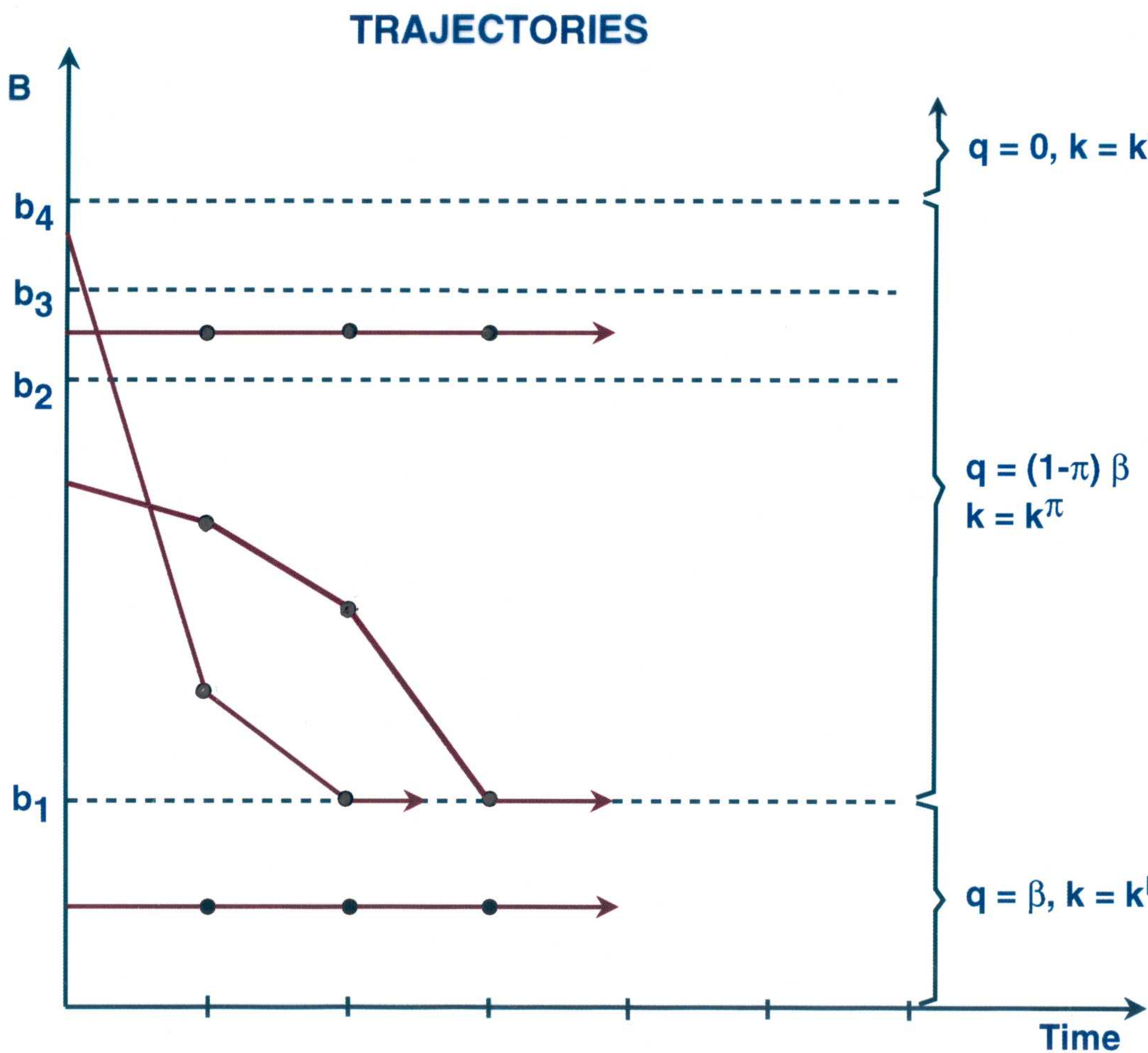
- equilibrium description:
  - if $\zeta < \pi$ and $B > \bar{b}(K)$, then a crisis occurs
  - if $\zeta \geq \pi$ or $B \leq \bar{b}(K)$, then a crisis cannot occur today
- equilibrium price of government debt:
  - β if a crisis cannot occur next period
  - $\beta(1-\pi)$ if it can occur
  - 0 if the government does not weakly prefer to repay
- policy function of the consumer:
  - $k' = k^n$ if the probability a default next period is zero
  - $k' = k^\pi$ if there could be a crisis next period

$$k^\pi : [(1-\pi)+\pi\alpha](1-\theta)\beta f'(k^\pi) = 1$$

  - $k' = k^d$ if default has either already occurred or is believed will occur next period.

- **Equilibrium outcomes:**
  - If debt below $\bar{b}(K)$ then
    - government not in crisis zone
    - optimal policy is stationary g and B.
  - If debt slightly above $\bar{b}(K)$ then
    - government is in crisis zone
    - interest rate on debt discretely higher
    - capital stock discretely lower
    - optimal to run debt down to $\bar{b}(K)$ in one step
    - capital stock jumps up and interest rate jumps down when leave the crisis zone.
  - Yet higher debt,
    - government in crisis zone
    - optimal to run down debt in several steps
    - if sufficiently high and π sufficiently small may go stationary.

- Debt higher still,
  - participation constraint binds
  - jump debt down to secure new borrowing.
- Debt too high, default only outcome.


TRAJECTORIES
B
b4
b3
b2
b1
q = 0, k = k^d
q = (1-π) β
k = k^π
q = β, k = k^n
Time

- **Three possible payoffs to the government:**
  - if a crisis cannot occur because $B' \leq \bar{b}(k^n)$, then the payoff to government is $V_g^n(s, B', \beta)$
  - if the government prefers to default, then its payoff is $V_g^d(s, 0, 0)$
  - if a crisis can occur,
    - let $V_g^T(s)$ denote the payoff to the government if it reduces its debt level in T periods to $\bar{b}(k^n)$, at which point its payoff is $V_g^n(\bar{b}(k^n), k^n, 1, \bar{b}(k^n), \beta)$
    - note that the capital level next period is k^π, and it will continue at this level until T periods hence when it rises to k^n
    - g is constant between now and period $T-1$ (may need to adjust in initial period to satisfy participation constraint)
    - as $T \to \infty$, $V_g^T(s)$ converges to the payoff from a policy in which the government's debt always exceeds $\bar{b}(k^n)$, $V_g^\infty(s)$.
    - exists a best T if we include ∞.

Maturity of the debt and debt crises

- present value of the debt is B and maximum maturity of the debt is N
- assume that we are in a no crisis equilibrium and hence $q = \beta$
- policy of maintaining a flat maturity structure: the value of the payments coming due in each period is constant
  - if B_N is the amount coming due, then

    $$B_N = B/(1 + \beta + \ldots + \beta^{N-1}) = \frac{1 - \beta}{1 - \beta^N} B$$

    to maintain this debt structure the government issues B_N units of N period discount bonds
  - its net payments are $B_N(1 - \beta^N)$
- participation constraint converges to no-lending continuation condition because

  $$\lim_{N \to \infty} \beta^N B_N = 0$$

- without possibility of crisis there is little role for maturity, but with possibility of crisis there is a big role.

1994-1995 Mexican Crisis

- in 1994
  - political crisis in Mexico
  - international reserves fell sharply in March and April, then stabilized
  - Mexican central bank sterilized
  - large fraction of public debt converted to *tesobonos* and maturity shrank
  - November another run on reserves occurred
  - December-January markets refused to roll over debt coming due.
- puzzle: fundamentals sound even after devaluation
  - debt/GDP had been falling
  - maturity structure of debt did shorten.

- interpretation of events using model
  - with dollar-indexed debt, default discrete event with discrete penalty
  - shortened maturity put Mexico in the CRISIS ZONE
  - political turmoil helped to stir the caldron
  - crisis only ended with offer of Clinton’s 31 January loan package.
- results of model
  - models with debt roll over have crises equilibria
  - crisis can be avoided only by keeping debt down and maturity long.
- given the model, surprise is that we do not see more crises.

Debt/GDP for Selected Countries
(Percent)

	1990	1991	1992	1993	1994
Mexico	55.2	45.8	35.1	35.0	37.4
Belgium	130.7	132.6	134.4	141.3	140.1
France	40.4	41.1	45.6	52.9	56.8
Germany	43.4	42.7	47.3	51.8	54.6
Greece	77.7	81.7	88.6	117.1	119.8
Italy	100.5	103.9	111.4	120.2	122.6
Spain	48.7	49.9	53.0	59.4	63.5

Sources: International Monetary Fund (1995c), Organization for Economic Co-Operation and Development (1995).

Calendar of Maturing Debt 1995
Tesobonos and Cetes

(millions USD)

	Cetes	Tesobonos	Cetes plus Tesobonos
1st quarter	3,015.00	9,873.94	12,888.94
2nd quarter	1,563.47	6,429.26	7,992.72
3rd quarter	1,042.66	8,425.70	9,468.36
4th quarter	943.13	3,927.83	4,870.97

Tesobonos Auctions

Date	Yield (percent)	Amount Sold (million dollars)	Amount Offered (millions dollars)
6 Dec 1994	8.39	420	420
13 Dec 1994	8.23	375	375
20 Dec 1994	8.61	416	600
27 Dec 1994	10.23	28	600
3 Jan 1995	12.31	52	500
10 Jan 1995	19.63	63	400
17 Jan 1995	19.75	400	300
24 Jan 1995	21.40	50	50
31 Jan 1995	24.98	155	150

Source: International Monetary Fund (1995a).

Numerical Example

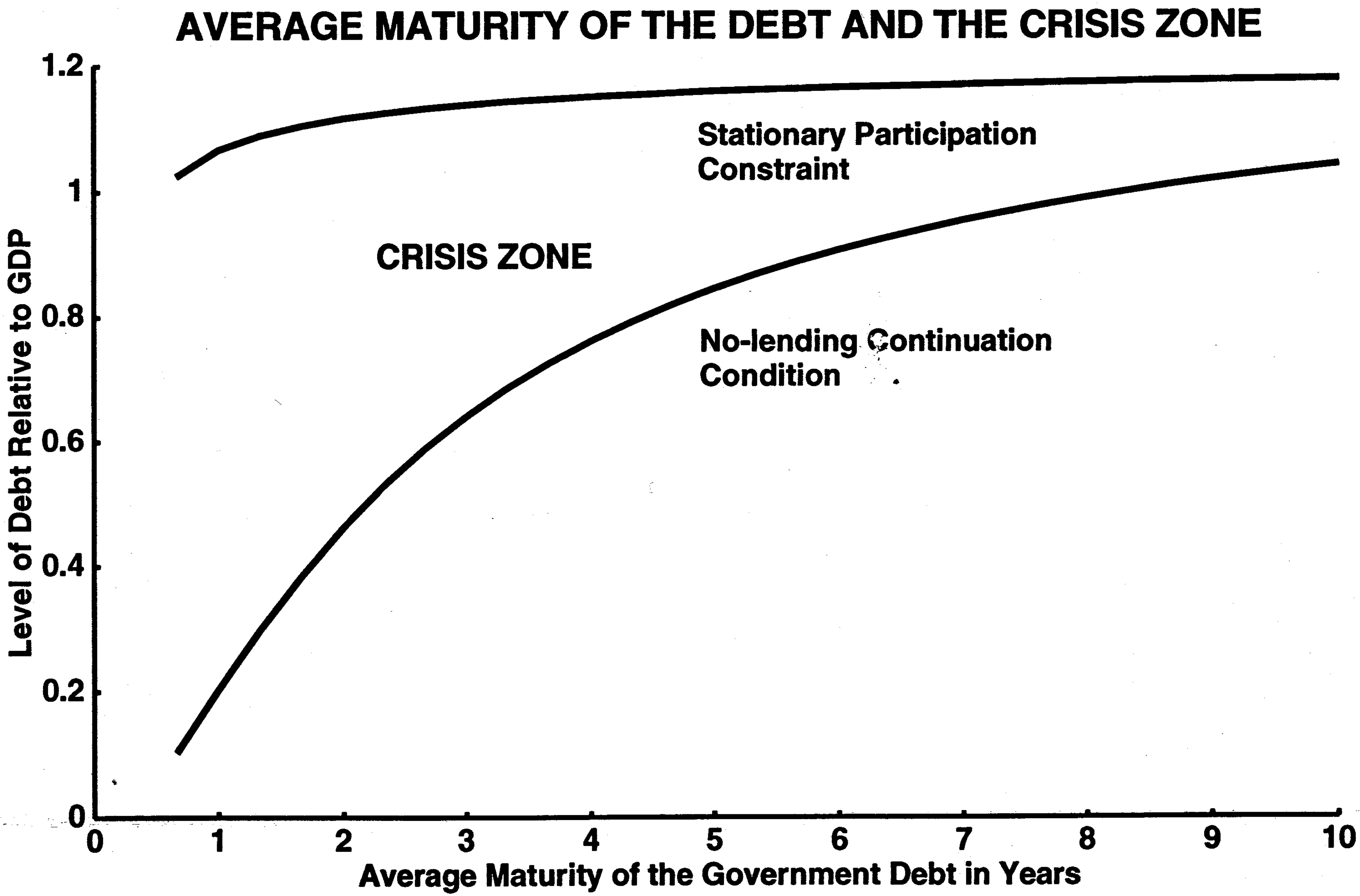
- Period: 2/3 year
- Utility: $E\sum_{t=0}^{\infty} 0.97^t(c_t + \log(g_t))$
- Possibility of default: $\pi = 0.02$
  - $\beta = 0.97$ implies yearly discount factor 0.955, which implies a yearly yield of 0.047 on risk free bonds
  - $\pi = 0.02$ implies a yearly yield of 0.079 on Mexican government bonds
- Feasibility constraint:

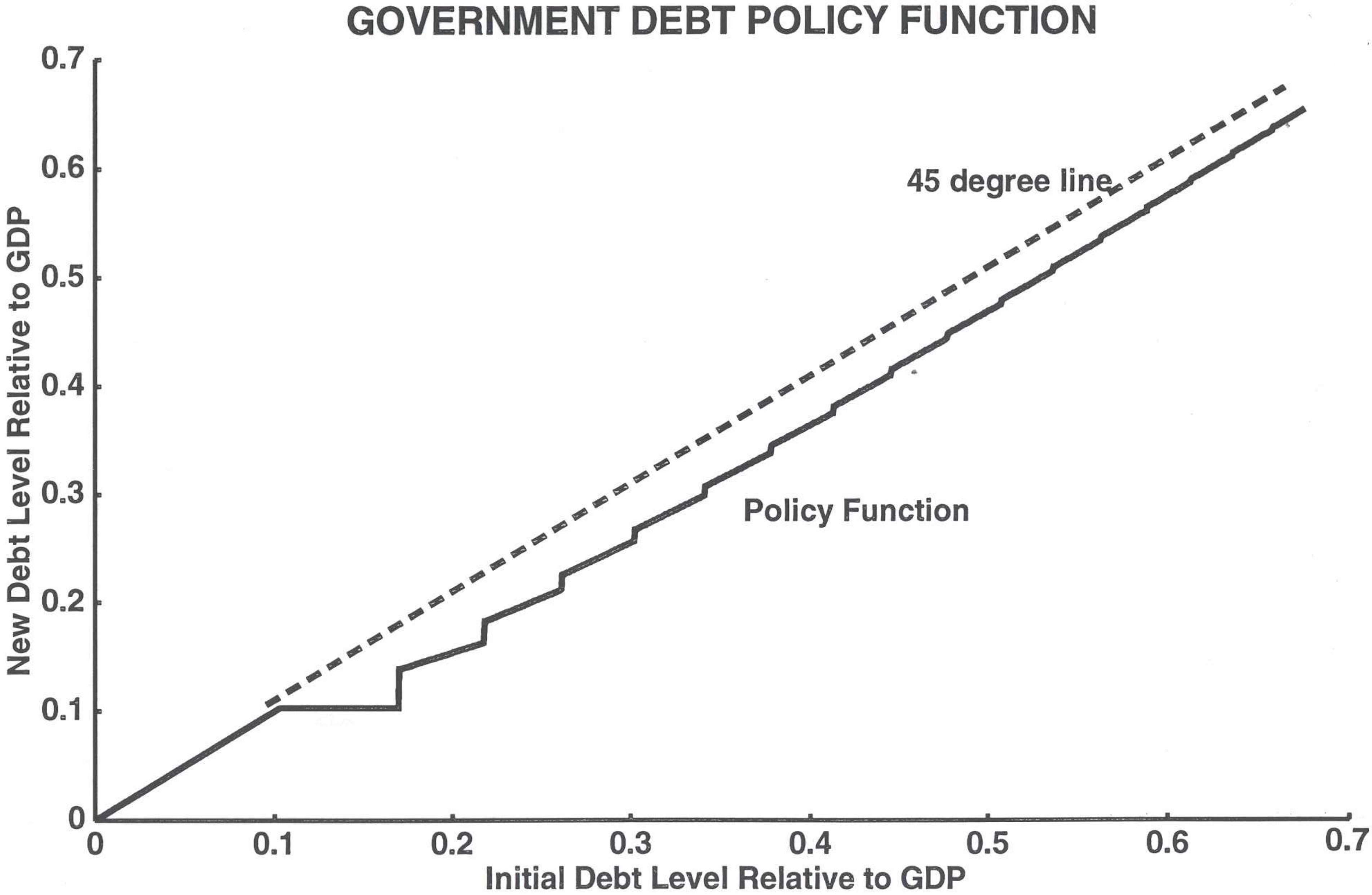
$$c + g + k' - 0.95k + zB \leq 2k^{0.4} + qB'$$

- $\delta = 0.05$ corresponds to a yearly discount rate of 0.074
- Capital Stock:

$$(1-\theta)[(0.98 + 0.02\alpha)0.8(k^{\pi})^{-0.6} - 0.05] = 0.97^{-1} - 1$$

- Tax rate: $\theta = 0.20$
- Default Penalty: 0.05 ($\alpha = 0.95$)
  - $k^{\pi} = 39.04$
  - GDP= $(3/2)2(k^{\pi})^{0.4} = 12.99$
  - capital/output ratio = 3.00
  - investment/GDP ratio = 0.23
  - tax revenues/GDP ratio = 0.15
- Initial debt: $B_0 = 2.67$
  - debt/GDP = 0.20


AVERAGE MATURITY OF THE DEBT AND THE CRISIS ZONE
Stationary Participation Constraint
CRISIS ZONE
No-lending Continuation Condition
Level of Debt Relative to GDP
Average Maturity of the Government Debt in Years
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7
8
9
10


GOVERNMENT DEBT POLICY FUNCTION
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
45 degree line
Policy Function
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Initial Debt Level Relative to GDP
New Debt Level Relative to GDP

GOVERNMENT DEBT POLICY FUNCTION WITH GOVERNMENT DISCOUNT RATE 0.93