A chapter called "References" gives a section-by-section commentary, including specific literature citations, for each of the chapters and the appendices.

An alphabetical bibliography of 20 pages concludes the text and is followed by a seven-page index.

This book is an excellent, readable reference work. Its approximately 600 pages are densely packed. Each of Chapters 3–10 concludes with a set of nontrivial exercises. In the Preface, the authors describe how to select sections of the book for a variety of senior-level courses.

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Harmonic Analysis on Symmetric Spaces and Applications II. *By Audrey Terras.* Springer-Verlag, Berlin, 1988. xii + 385 pp. $45.00, paper. ISBN 0-387-96663-3 (US).

The aim of this book is to introduce the harmonic analysis of (noncompact) symmetric spaces and some of the many applications of this theory. This is accomplished by the restriction of the general theory to the specific (and important) special case where the space is $\mathscr{P}_n$ (the space of all positive n by n matrices, where positivity of a matrix is defined in terms of positivity for the corresponding quadratic form). The last chapter of the book explains how the results for $\mathscr{P}_n$ fit into the framework of harmonic analysis on $X = G/K$, and on X/Γ, where X is a noncompact symmetric space specified by a Lie group G with Cartan involution σ and fixed-point subgroup K for σ, and where Γ is a given discrete subgroup of G. The Iwasawa decomposition is worked out in the specialized setting with very interesting explicit formulas, and it is explained, in the last chapter, how this fits into the general theory for symmetric spaces involving the known decompositions and integration theory.

Another aim of the book is to develop this dual-track method (examples versus the general theory) for Harish-Chandra's spherical function formulas. With the spherical functions given, Helgason's Fourier transform is introduced, and is used to explain the inversion formula for the Fourier decomposition of X. Some of the differences between the distinct types of symmetric spaces are highlighted. A detailed and novel account is provided for the case of the quaternionic upper-half 3-space as an example of harmonic analysis on symmetric spaces of type IV. The discrete subgroup analysis for $\Gamma\backslash G/K$ is illustrated for the Siegel modular group $Sp(n, \mathbb{Z})$, and the modular group $GL(n, \mathscr{O}_K)$ over an algebraic number field, i.e., $\mathscr{O}_K$ is the ring of integers in a given algebraic number field K. This includes specific examples: the Picard group (with the Dedekind zeta function), the Hilbert modular group, and the general linear group over any number field. The specific results include a novel exposition of the Siegel cusp–ideal class correspondence, the Fourier expansion of Epstein's zeta function (for K), and a formula for the product of the class number and the regulator.

The present book is a continuation of Volume I of the same title, where the harmonic analysis of the Poincaré upper half-plane is developed with view to applications in number theory and a variety of other fields, within mathematics itself as well as in physics and engineering. Volume II, the present work, builds to some extent on concepts (such as automorphic forms, Dirichlet series, Hecke theory, the spectral resolution of the Laplacian, and the Selberg trace formula) introduced in Volume I.

I expect that the refreshing style of the book will appeal to students, and to researchers in neighboring fields who want to learn the subject. The large number of exercises included should be a great help in a course based on this book. Readers who want a treatment in full generality of harmonic analysis on symmetric spaces may supplement their reading with the recent book [1].

REFERENCE

[1] R. GANGOLI AND V. S. VARADARAJAN, *Harmonic Analysis of Spherical Functions on Real Reductive Groups*, Springer-Verlag, Berlin, New York, 1988.

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Microeconomic Theory. *By Yoshihiko Otani and Mohamed El-Hodiri.* Springer-Verlag, Berlin, 1987. xiv + 274 pp. $35.00, paper. ISBN 0-397-17994-1 (US).

The position of mathematics in economics has risen considerably since the days of Alfred Marshall. In 1906 this great English economist (who had distinguished himself as a mathematician while an undergraduate in Cambridge) wrote to a friend explaining his rules for using mathematics in economic research [3, p. 427]:

> (1) Use mathematics as a shorthand language, rather than as an engine of inquiry. (2) Keep to them till you have done. (3) Translate into English. (4) Then illustrate by examples that are important in real life. (5) Burn the mathematics. (6) If you can't succeed in 4 burn 3. This last I did often.

The textbook by Otani and El-Hodiri turns Marshall on his head: much of it is a translation of the material found in intermediate-level microeconomics textbooks into mathematically formal and rigorous language. It would surprise Marshall that the typical economics graduate student today would find this book far easier to follow than Marshall's *Principles of Economics* [2]. Perhaps even more surprising is the high level of mathematical sophistication in this book compared to that in a comparable textbook used by a previous generation of graduate students, say, Henderson and Quandt's *Microeconomic Theory: A Mathematical Approach* [1].

The book under review is organized into four parts. The first part deals with the derivation of consumer demand functions from consumer preferences and with the properties of individual and aggregate demand functions. Among topics not usually covered in microeconomics texts are the recoverability of consumer preferences from consumer demand functions, and the properties of the demand functions generated by consumers who over time maximize paths of consumption in a continuous-time, multicommodity world. The second part deals with production functions and with cost minimization and profit maximization by competitive firms. The third part considers market structures that offer alternatives to price competition: monopoly, oligopoly, and monopolistic competition. This is clearly the most original and innovative part of the book. Among the many topics covered are oligopoly models with free entry and many firms, and Hotelling-type models of product differentiation and monopolistic competition. The fourth part is a brief introduction to general equilibrium theory and welfare economics. In addition to these four parts, an appendix reviews the significant mathematical results used in the text.

Perhaps the easiest way to assess the relative strengths and weaknesses of this book is to compare it to Varian's *Microeconomic Analysis* [5], which is currently the most popular textbook for graduate microeconomics. Both employ a modern, mathematical approach to the subject, and there is substantial overlap. Yet the emphasis is clearly different. Otani and El-Hodiri put significantly more emphasis on market structure than does Varian: more than 37 percent (103 of 274 pages) of Otani and El-Hodiri's book is devoted to market structure, while less than 10 percent (28 of 284 pages) of Varian's book is. In particular, Varian gives oligopolies and monopolistic competition only cursory attention. One of my major disappointments with Otani and El-Hodiri's book is the little attention that the authors pay to welfare economics and general equilibrium: they devote 30 pages to this, a central topic in modern microeconomics, as compared to 73 pages in Varian's book. Another shortcoming of this book is that no attention is given to the economics of incomplete information: models of moral hazard, adverse selection, and signalling. Since Akerlof's market for lemons article [6], in which he discussed a used-car market where sellers know the qualities of their cars but buyers do not, this has become an important research field in microeconomics.

A major strength of this book, especially for students, is that the mathematics is worked out completely. As compared to Varian's book, there are very few steps between equations that the reader needs to fill in. In this way, the authors take the risk of seeming mechanical and tedious to the more sophisticated reader, but I am sure it is a risk that students will thank them for taking. Like Varian's book, this book contains a collection of problems and exercises at the end of each section. These may be the most valuable part of the book for many teachers and students.

Who then would be interested in this book? It could obviously be used as a textbook for an undergraduate intermediate microeconomics course with a heavy mathematical emphasis or for a graduate microeconomics course. The material on market structure makes this an ideal textbook for an instructor with a strong interest in industrial organization. In fact, the book would serve as an excellent introduction to Tirole's book on industrial organization theory [4]. It would also work well as a supplement to more conventional textbooks for teachers and students interested in a mathematical presentation of familiar topics. Since it translates many concepts often given in contorted English and mostly communicated by examples elsewhere into a precise mathematical language, the book will be of some value as a reference for many research economists.

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[3] A. C. PIGOU, *Memorials of Alfred Marshall*, Macmillan, London, 1925.
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[5] H. R. VARIAN, *Microeconomic Analysis*, Norton, New York, 1978.
[6] G. AKERLOFF. *The market for lemons: Qualitative uncertainty and the market mechanism*, Quart. J. Econom., 84 (1970), pp. 488–500.

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Singularities and Groups in Bifurcation Theory. Volume II. *By Martin Golubitsky, Ian Stewart, and David G. Schaeffer*. Springer-Verlag, New York, 1988. xvi + 533 pp. $69.50. ISBN 0-387-96652-8. Applied Mathematical Sciences, Vol. 69. Revised for Volume II.

My review of Volume I of this set can be found in an earlier issue of *SIAM Review* [1], where the general setting for the whole subject is given. Since writing that review, I taught a special topics course in dynamical systems to graduate students and I chose to present selections from three recently published books. To teach a course from a book is to delve into the proof of each lemma and comprehend the overall strategy, so it gives a very good measure of a book. One book was fair and one a total disaster, but Volume I of this set was excellent. When a detail was needed in a proof it was there, and readers were always well informed about where the authors were leading them. Volume I was the best-written advanced book that I have taught from in many years.

Briefly (see [1]), the general problem discussed is the nature of the solutions of the equations $F(x, \lambda) = 0$, where $F: \mathbb{R}^n \times \mathbb{R}^k \to \mathbb{R}^n$ is a smooth ($\mathbb{C}^\infty$) mapping. The variable $x \in \mathbb{R}^n$ is to be thought of as the state of some physical system, and $\lambda \in \mathbb{R}^k$ is a parameter. Thus the problem is to understand the zero set in $\mathbb{R}^n$, i.e., $\{x \in \mathbb{R}^n: F(x, \lambda) = 0\}$, as a function of the parameter $\lambda \in \mathbb{R}^k$. A bifurcation occurs when this zero set changes drastically as λ passes through some particular value. The analysis is totally local and finite-dimensional. The approach is that developed in singularity and catastrophe theory—in fact this book could be called "Applied Singularity Theory."

There is a natural notion of local change of variable for this problem, and two bifurcations are considered equivalent if there is a change of variable taking one to the other. These books discuss three basic questions. First is the "recognition" problem, i.e., how much of the Taylor expansion is needed to determine the type of bifurcation. Second is the "unfolding" problem, i.e., what is the general way in which a bifurcation changes under further perturbations. Third is the "classification" problem, i.e., how to build a dictionary of generic bifurcations.

Volume I deals almost exclusively with the case where x, λ, and F are all scalars and no symmetries are present. Volume II deals with the higher-dimensional case and attacks symmetries head on. A *symmetry* of the system of equations is a nonsingular, $n \times n$ matrix A such that $F(Ax, \lambda) \equiv AF(x, \lambda)$. The set of all such matrices, Γ, forms a closed subset of the general linear group and so is a Lie group, called the *symmetry group*. For some problems the correct concept is Γ-*invariant*, i.e., $F(Ax, \lambda) \equiv F(x, \lambda)$ for all $A \in \Gamma$. The main results of the book require Γ to be compact, but this includes discrete (finite) symmetries and rotational symmetries because finite groups and the orthogonal groups are compact.