Numerical methods in economics. By KENNETH L. JUDD. Cambridge, MA: The MIT Press, 1998. Pp. xv, 633. $59.95. ISBM 0-262-10071-1.

Computers have changed the way we do economics. Twenty-five years ago a graduate student studying international economics, for example, would have spent most of the time working out the complexities of diagrams with lots of curves and often lots of quadrants. The role of a theorist was to prove "theorems" using such diagrams or to generalize these theorems to the multi-good or the multi-country case using lots of total differentials and linear algebra. An applied economist could then run a linear regression in which confirmation of the theory depended on a crucial coefficient having both the sign predicted by the theory and a large t statistic. International economists now build models whose equilibria can be computed. Those who do statistical analysis now estimate the parameters of structural models. The problems solved by the people in the models are far richer, especially in terms of dynamic and stochastic features. The change in the way we do international economics and teach it to graduate students is enormous, and it is larger still in fields like macroeconomics, industrial organization, and labor economics.

Ken Judd is worried that economists use the tools of numerical analysis in computing their models' equilibria or in estimating the models' coefficients without paying enough attention to how their computational methods work and without appreciating what is out there in the non-economics literature. Some economic researchers are at the frontier of applications of numerical analysis; others are spending large amounts of their own time and of journal space in crudely reinventing the wheel. To remedy this, he has written a book that every graduate student and researcher in economics should seriously consider for his or her bookshelf.

Following two introductory chapters, in which Judd makes his case that economists need to know more about numerical analysis, his book has four parts. The first part, which takes up more than half of the book, provides an introduction to numerical analysis for economists. The areas that are covered - linear equations, optimization, nonlinear equations, approximation methods, numerical differentiation and integration, and Monte Carlo methods - are ones that show up in standard numerical analysis textbooks. There are some applications of particular interest to economists: The chapter on optimization, for example, includes material on solving principal-agent problems and non-cooperative games; the chapter on nonlinear equations includes material on calculating equilibria of static exchange economies; and the chapter on numerical differentiation and integration includes material on solving portfolio choice problems. This part of the book has same sort of relation to a textbook on numerical analysis that much of the material in *Recursive Methods in Dynamic Economics* by Nancy Stokey and Robert Lucas with Edward Prescott (Harvard University Press, 1989) has with a textbook on measure theory and stochastic processes.

The second part of the book deals with numerical methods for solving functional equations. Applications include solving the differential equations in deterministic continuous-time growth models and solving the difference equations in their discrete-time counterparts. The principal application, however, is to dynamic programming for stochastic models. Once again, the obvious comparison is to the Stokey-Lucas-Prescott book. Judd's book does for the numerical analysis of dynamic programming what Stokey-Lucas-Prescott does for the mathematical theory.

The third part of the book deals with perturbation, or asymptotic, methods for solving the same sorts of functional equations as those dealt with in the second part. This sort of method starts with a particular case of a general problem that has a known solution and then uses Taylor series expansions to compute approximate solutions to nearby problems. As Judd explains, these methods are used with much success in physics. Applications in economics are fewer, although Judd himself, with coauthors, has made some notable contributions. Before reading the book, I would have thought that two obvious sets of applications in economics of these sorts of methods are asymptotic techniques in econometrics and linear-quadratic approximations in business cycle theory. Without entering into the polemics, let me simply report that Judd is highly critical of how most economists have used these methods. For an economist, it is this part of the book that contains the least familiar material.

The fourth part of the book reports on how some of the methods presented in the second and third parts have been used in solving both deterministic and stochastic dynamic economic models. This part of the book is disappointing for a student trying to learn exactly how to set up and simulate a model. For that, a student (or even a professor) would prefer *Recursive Macroeconomic Theory* by Lars Ljunquist and Thomas Sargent (MIT Press, 2000) or even some of the chapters in collected volumes like *Frontiers in Business Cycle Theory* (Princeton University Press, 1995) edited by Thomas Cooley or *Computational Methods for the Study of Dynamic Economies* edited by Ramon Marimon and Andrew Scott (Oxford University Press, 1999). These other books contain much more textbook-like presentations of applications. (I am not implying that Judd would approve of the way that numerical methods are used in these other books.)

An obvious use for Judd's book would be as the principal textbook for a graduate course on numerical methods in dynamic economics. An internet search confirms that the book is indeed being used in such courses in a number of top graduate programs. It would also be useful as a general reference and should be on any bookshelves that contain Stokey-Lucas-Prescott, Sargent-Ljunquist, Cooley, or Marimon-Scott.

Two aspects of this book disturb me. The first of these is the attitude that Judd takes towards proofs of existence of equilibrium. In this book and even more explicitly in his 1997 *Journal of Economic Dynamics and Control* article, Judd judges the emphasis that economists have placed on existence proofs to be misplaced. He is entirely right when he notes that physicists do not share this emphasis: As a participant in the early days of the Economics Program at the Sata Fe Institute, I recall that some of the physicists found my reliance on formal mathematics and on the theorem-proof approach to problems of computation to be entertaining (at best). Perhaps, Judd is right, and the approach to computation followed by such economists as Herbert Scarf should be replaced by approaches more like those in physics. I have my doubts, however, and I am particularly worried about the application of recursive techniques to models where we do not have proofs of existence of recursive equilibrium. This is the case of many models with taxes and externalities, where an equilibrium does not solve a dynamic programming problem.

The second problem that I have involves Judd's website, which is advertised as containing a large archive of supplemental material, especially computer code and solved examples. Unfortunately, this website, whose server seems to be down more often than not, contains very little material beyond some useful links to public domain software. Anyone thinking of using this book as a textbook should check out the website ahead of time to see what is currently available. This problem points to a fundamental question about textbooks and their relation to material on the internet: What is there to assure potential buyers of a textbook that any associated website will have a life span that is as long as that of the book?

TIMOTHY J. KEHOE
University of Minnesota and Federal Reserve Bank of Minneapolis