

The Optimum Quantity of Money Revisited

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The accepted wisdom on the optimum quantity of money was first expressed by Friedman (1953,1969): Real money balances represent a service to the economy provided by the government at no cost. The government should maximize the quantity of real balances it provides, since it is costless to do so. It can do this either by means of a deflationary monetary policy or by paying interest on nominal balances. Either policy reduces the cost of holding idle balances and increases the value of the money stock.

Hahn (1971, 1973) has objected to Friedman's analysis because it is not grounded in a fully specified model of an economy with money:

The necessary conditions for Pareto efficiency in a world of uncertainty with intertemporal choice will in general be fulfilled by a market economy only if money plays no role. There are no grounds for supposing that the Friedman rule is either necessary or sufficient for Pareto efficiency since it is of the essence of an explanation for the existence of money that other conventional necessary conditions are violated. Even where money is held only for "transaction purposes," Pareto efficiency relative to a transactions technology is a different animal than the usual textbook one. (Hahn 1971, p. 70)

In this paper we study efficiency of monetary policies in an economy in which money plays an essential role.

Brock (1974) and Benhabib and Bull (1983) have shown that Friedman's intuition is correct in a model in which real balances enter directly into the utility functions of traders. This type of model is fully Walrasian except for the assumption that real balances affect utility; hence, a reduction of the shadow value of real balances to zero suffices to make an equilibrium Pareto efficient. However, such an analysis ignores Hahn's point that the role of money is intimately tied to restrictions on the way transactions can be carried out. Grandmont and Younes (1973) show that Friedman's intuition is also correct in a model where money is required to purchase goods because of the assumption of a cash-in-advance constraint. While this type of model is more explicit in relating the value of money balances to restrictions on the execution of transactions, it is still not responsive to Hahn's concerns: Requiring money to be held to carry out transactions seems no less artificial than having real balances enter the utility function directly and, indeed, is equivalent to a special case of the money-in-the-utility-function model (Lucas and Stokey 1983).

Money-in-the-utility-function and cash-in-advance models ignore the role of money in buffering transactions and smoothing short-term intertemporal variations in purchases and sales. In such models, money balances increase utility even though both the allocation of resources and the real money balances held by different households remain constant over time. As a result, redistributive consequences of monetary policy either are assumed away (in representative-consumer examples) or simply represent transfers to some types of consumers at the expense of others. The idea that monetary policy could substitute for inoperative insurance markets by systematically redistributing resources to households in certain circumstances cannot be analyzed in such models. It is this aspect of monetary policy that we examine here. Our analysis ignores direct utility effects of real money balances and cash-in-advance constraints to focus on the use of money to improve the intertemporal allocation of resources.

Bewley (1980, 1983) and Townsend (1980) have given examples showing that Friedman's intuition is sometimes valid in a model where money serves as a short-run store of value. An excellent summary of this work can be found in Sargent 1987. In these examples, perfectly anticipated changes in endowments (or preferences) give rise to intertemporal trade. Because of problems in collecting debts, currency is the only asset that can be used to carry out these intertemporal trades. In the Townsend interpretation, the difficulty in debt collection arises because of the changing locations of traders. A more general discussion of the way in which limited collectibility of unsecured debt can lead to a role for money can be found in Bryant 1980. Although this type of model is too stylized to represent a real economy, it does capture the idea that in the short run money is held as a buffer between periods in which a trader sells and periods in which he buys. As Friedman suggests, in this setup a steady contraction of the stock of currency reduces the cost of holding real balances, increases intertemporal trade, and makes all traders better off. Conversely, a steady expansion of the stock of currency reduces welfare.

An alternative possibility is discussed by Levine (1988), who argues that a steady expansion of the money supply can improve possibilities for trade. This is because real balances are redistributed from rich sellers to poor buyers. Scheinkman and Weiss (1986) offer a similar argument in favor of a one-time inflation. Here we investigate the strengths of this effect relative to that studied by Friedman, Bewley, and Townsend.

In the intertemporal framework, in contrast with Friedman's aggregative framework, the question of how money is injected into or withdrawn from the economy arises. For simplicity, we suppose that it takes place through lump-sum taxes or subsidies. If arbitrary lump-sum transfers are available as an instrument to the government, it can improve upon any monetary policy simply by calculating the intertemporal allocation of consumption that maximizes its welfare criterion and enforcing that plan by means of lump-sum transfers. In practice, however, the government cannot identify individuals and their preferences sufficiently accurately to carry out short-run trades on their behalf. As a result, it is sensible to impose on the analysis the additional restriction that the only feasible monetary policies give all agents equal lump-sum subsidies, or charge them all the same lump-sum taxes. In other words, the government cannot identify agents sufficiently well to give them differential treatment. Levine (1988) shows how a careful model of private information about preferences can lead to the formulation used by Bewley and Townsend, but with the additional proviso that only equal treatment is possible.

Limiting the way in which money can be injected into or withdrawn from the economy has significant consequences. If we modify the model to allow the possibility that traders cannot perfectly forecast their own future demand, a trader may occasionally find himself in a position where he wishes to buy from another trader who wishes to sell but does not have enough money on hand to make the purchase. An expansionary monetary policy that gives both traders equal amounts of currency effectively redistributes wealth from the relatively rich seller to the relatively poor buyer. This can make possible socially desirable trades that could not otherwise take place. Levine (1988) considers the extreme case in which in equilibrium sellers sell their entire endowment. This is the case, for example, if the marginal utility of buyers exceeds that of sellers throughout the range of socially feasible trades. Since sellers are at a corner, changing the price of money relative to goods does not in general change the amount of goods they trade, and, by virtue of Walras' law, does not change their holdings of real balances either. Friedman's original argument rests essentially on the idea that changing prices affects real balances. As a result, in the boundary case, the Friedman-Bewley-Townsend effect of inflation reducing trade is not present, and inflation, if it is feasible, leads to an unambiguous welfare improvement.

In Townsend's model and Bewley's model, there is no uncertainty and there are interior solutions to agents' optimization problems. In Levine's model, there is uncertainty and there are corner solutions. A model with uncertainty but interior solutions contains both effects; that is, inflation has both positive and negative consequences. Here we try to sort out the costs and benefits of inflation in such a world. The results are mixed: For a broad range of parameter values, deflation is clearly good. On the other hand, these parameter values do not reflect the short-run nature of the model very well. Sensible assumptions about short-term trading are that trade is frequent, that the gains to trade are large, and that the degree of unpredictability is small. Unfortunately, with these realistic parameter values it is less likely that there exist equilibria of the simple kind that we can compute. This is especially the case for parameter values that imply positive net benefits from inflation. That is, we can find sensible examples in which inflation is beneficial. If we push too far in this direction, however, we find that we can no longer use our simple methods of computing equilibria.

Our numerical computations lead to one other significant conclusion: In no case is welfare very sensitive to monetary policy. With reasonable parameter values, we find that if inflation increases by 1 percent this is equivalent to changing GNP by about 0.004 percent. It is conceivable that these numerical effects may be more significant in a model with many types of agents and many assets and in which money plays a more complex role in the economy. At the least, these calculations should serve as a warning against arguments that tell us how to improve upon existing policies but not whether the improvement is worth having.

1 The Model

We study a stylized economy in which agents randomly alternate between being buyers and sellers. There are two types of agents, $i = 1, 2$, and two states of the world, $\eta = 1, 2$. In state 1, type 1 agents are sellers (that is, have low marginal utility of consumption) and type 2 agents are buyers (that is, have high marginal utility of consumption); in state 2, type 2 are sellers and type 1 are buyers. In other words, the state is the same as the type that is a seller. The states themselves form a Markov chain, π being both the conditional probability in state 1 that the subsequent state is 2 and the condi-

tional probability in state 2 that the subsequent state is 1. In short, π is the probability of *reversal*—that buyers and sellers switch roles.

The horizon is unbounded, and periods are $t = 1, 2, \dots$. The state at time t is η_t , and is common knowledge among traders. At time $t = 0$, before the economy begins, each trader has an equal chance of beginning life in period 1 as a buyer or a seller.

There is a single composite consumption good, c , and a single asset called currency, M . Planned holdings by a representative agent of type i in period t are c_t^i and M_t^i . Agents' preferences are given by the expected present value

$$E_0 \sum_{t=1}^{\infty} \delta^t u_i(c_t^i, \eta_t),$$

where $0 \leq \delta < 1$ is a common subjective discount factor and $u_i(c_t^i, \eta_t)$ is a period utility function that depends on consumption and on whether a trader is a buyer or a seller. In other words, $u_i(c_t^i, \eta_t) = u^s(c_t^i)$ if $\eta_t = i$ so that agent i is a seller, and $u_i(c_t^i, \eta_t) = u^b(c_t^i)$ if $\eta_t \neq i$ so that agent i is a buyer. The function u^s and u^b are increasing, concave, smooth, and bounded above. The derivatives are denoted Du^s and Du^b .

Consumption must be non-negative. A representative seller is endowed with ω^s units of the consumption good, a buyer with $\omega^b \leq \omega^s$ units. To ensure that sellers want to sell and buyers to buy, we assume that buyers receive more marginal utility from their endowment than do sellers:

$$Du^b(\omega^b) > Du^s(\omega^s). \quad (1.1)$$

The consumption good is perishable, and there is no production. Social feasibility therefore requires that

$$c_t^1 + c_t^2 \leq \omega^b + \omega^s, \quad (1.2)$$

that is, that total consumption does not exceed the social endowment.

We assume that private debts are prohibitively expensive to collect. This means that currency is the only asset. We also assume that a single type of currency is issued by the government and cannot be forged. Consequently, individual holdings must be non-negative, $M_t^i \geq 0$. We denote the aggregate stock of money in period t by M_t . Social feasibility requires that

$$M_t^1 + M_t^2 \leq M_t. \quad (1.3)$$

To simplify calculations, we investigate only steady-state equilibria; we

assume that initially money balances are distributed as they are in the steady state.

Unlike the consumption good, currency is durable. However, holdings of currency depend not only on past savings, but also on the amount of currency injected into the economy. The impact this has on the economy depends on how currency is injected. If each trader receives new currency in proportion to existing holdings, then each type's share of the total stock is not affected by the injection. This is a neutral policy. We assume, however, that the government cannot distinguish between buyers and sellers and cannot observe currency holdings. Consequently, currency must be injected into the economy in a lump-sum manner so that both buyers and sellers get equal amounts. Each type receives a grant equal to half of the total injection. If g is the rate of growth of currency (possibly negative), then, with an initial stock of M_{t-1} , each type receives $gM_{t-1}/2$ dollars. The total money stock at time t is

$$M_t = (1 + g)M_{t-1}. \quad (1.4)$$

The type of expansionary or contractionary policy being considered is a fiscal policy financed by a monetary expansion or contraction. This corresponds to the class of policies analyzed by Friedman. It is clear that the fiscal transfers involved in the policy are essential to its having the kinds of effects studied below; this type of model cannot, for example, shed light upon what the impact of open market operations between currency and other assets should be. Nonetheless, some insight may be gained into the effects of actual monetary policies insofar as such policies have a redistributive aspect. This is true if the injection of liquidity is not proportional to the existing distribution of liquid assets.

Let p_t denote the price of currency with consumption as the numeraire. In other words, p_t is the reciprocal of the price level. The budget constraint for a representative agent of type i in period t is given by

$$p_t[M_t^i - M_{t-1}^i - gM_{t-1}^i/2] + [c_t^i - \omega_t^i] \leq 0, \quad (1.5)$$

where ω_t^i equals ω^s or ω^b as i is either a seller or a buyer at time t , and where M_0^i is type i 's initial holdings of money.

An *equilibrium* assigns prices, consumption plans, and asset holdings to each history of the states. It must be socially feasible (in other words, satisfy (1.2) and (1.3)), and each type must maximize utility subject to the budget constraint (1.5).

In common with many other monetary models, this model has an equilibrium in which money has no value, where $p_t = 0$: Autarky, where $c_t^i = \omega_t^i$, satisfies social feasibility. Since the budget constraint becomes $c_t^i \leq \omega_t^i$, it also maximizes utility subject to this constraint.

2 Two-State Markov Equilibria

We now introduce a special class of equilibria called *two-state Markov equilibria*. These equilibria have the property that what happens in each period is independent of history. Because these equilibria are relatively easy to compute, the impact of monetary growth can be explicitly studied.

If history is not to matter, then the distribution of currency between the buyers and the sellers at the end of each period must always be the same. There are three possibilities: First, if buyers hold all the currency at the end of each period, then they never consume more than ω^b units of the consumption good. This implies that the combined utility of both buyers and sellers is no more than it would be under autarky. Since each type can guarantee itself the autarkic utility level, buyers holding all the currency is possible only if the equilibrium is autarkic. Second, if buyers and sellers each hold a fixed positive amount of currency at the end of each period, then, throughout time, no trader's holding of currency falls below the smaller of these two amounts. We can argue that this cannot be an optimal policy for the trader: A trader should plan on holding very small amounts of currency under some circumstances. Consequently, equilibria of this sort do not exist. The third and final possibility is that the sellers hold all the currency at the end of each period. This is the only case in which currency can have value, and we now restrict our attention to this case.

If the sellers hold all the currency at the end of each period, then in each state there are only two possibilities: With probability π , a reversal occurs, and the traders holding the currency at the end of the previous period become buyers in the current period. In this case the buyers have the currency from the previous period, as well as their share of newly injected currency, to spend on the consumption good held by the sellers. On the other hand, with probability $1 - \pi$, a reversal fails to occur, and the agents holding the currency at the end of the previous period remain as sellers in the current period. In this case the buyers have only their share of newly injected currency to spend on the consumption good. Although the time periods here are too short to capture business cycles, it is useful to think of

the case where buyers have most of the money as a boom and the case where sellers have most of the money as a recession. In a boom there tends to be a great deal of trade because buyers are wealthy; in a recession there tends to be relatively little because they are poor.

We assume that the initial distribution of currency is consistent with a two-state Markov equilibrium—that is, that the previous sellers hold all the money at the end of the previous period. A two-state Markov equilibrium is like a steady state, and this amounts to assuming that we begin at the steady state. It is not difficult to show that, for a wide range of initial money distributions, the steady state is reached after a single period (Levine 1989a). This means that in the welfare calculations below we are comparing different steady states and are ignoring the initial transitional period.

In addition to requiring that sellers hold all money at the end of each period, in a two-state Markov equilibrium we require that prices and trade in each period depend only on whether the economy is currently in a boom or a recession. To do so, we measure money holdings as a share of the social total, rather than in dollars (which obviously are not stationary if $g \neq 0$). We let $m_t^i = M_t^i/M_t$ be type i 's share of the total money stock and let $\rho_t = p_t M_t$ be the real value of the money stock. The budget constraint (1.5) can then be rewritten as

$$\rho_t \left[m_t^i - \frac{m_{t-1}^i + g/2}{1 + g} \right] + [c_t^i - \omega_t^i] \leq 0. \quad (2.1)$$

The individual optimum can be characterized using the marginal utility of income μ_t^i , which measures the increased present value of an extra dollar. The marginal utility of a dollar must be at least the marginal utility of a dollar's worth of consumption,

$$\mu_t^i \geq Du_i(c_t^i, \eta_t). \quad (2.2)$$

Moreover, exact equality must hold if consumption is positive. In addition, since dollars are durable, the marginal utility of a dollar held today must be at least as great as the marginal utility of a dollar tomorrow. In utility terms, a current dollar depreciates by

$$\Delta = \frac{\delta}{1 + g} \quad (2.3)$$

as a result of subjective discounting and inflation. Consequently,

$$\rho_i \mu_i^i \geq \Delta E_i \rho_{i+1} \mu_{i+1}^i. \quad (2.4)$$

Again, there must be exact equality if money holdings are positive.

In addition to the familiar first-order conditions (2.2) and (2.4), we require that the individual consumption-money holding plan satisfies the transversality condition that μ_i^i is uniformly bounded. (For details see Weitzman 1973 and Levine 1989b.) This requires that $\Delta \leq 1$. To simplify calculations, we assume that $\Delta < 1$. This allows us to consider rates of deflation arbitrarily close to, though not exactly equal to, the discount rate proposed by Friedman. As in money-in-the-utility-function models in which consumers always have positive marginal utility of money and demand infinitely large money balances when the deflation rate is equal to the discount rate, there are technical problems if $\Delta = 1$ (see also Bewley 1983). The restriction to $\Delta < 1$ still allows us to consider whether welfare increases as the money growth rate is lowered.

In a two-state Markov equilibrium, we use overbars to indicate the values variables take on in booms and underbars to indicate the values they take on in recessions. For example, the variable ρ denotes the value of the money stock in a recession. We also define \bar{z} to be the amount of consumption purchased by buyers in a boom and \underline{z} to be the amount purchased by the buyers in a recession. In addition, if i is a buyer, we write $i = b$; if a seller, $i = s$. For example, $\bar{\mu}^b$ denotes the marginal utility of income of a buyer in a boom. With this notation, the marginal conditions (2.2) and (2.4) for a boom are

$$\begin{aligned} \bar{\mu}^b &= Du^b(\omega^b + \bar{z}), \\ \bar{\rho}\bar{\mu}^b &\geq \Delta[(1 - \pi)\rho\bar{\mu}^b + \pi\bar{\rho}\bar{\mu}^s], \\ \bar{\mu}^s &\geq Du^s(\omega^s - \bar{z}), \\ \bar{\rho}\bar{\mu}^s &= \Delta[(1 - \pi)\rho\bar{\mu}^s + \pi\bar{\rho}\bar{\mu}^b]. \end{aligned} \quad (2.5)$$

For a recession they are

$$\begin{aligned} \underline{\mu}^b &= Du^b(\omega^b + \underline{z}), \\ \underline{\rho}\underline{\mu}^b &\geq \Delta[(1 - \pi)\rho\underline{\mu}^b + \pi\underline{\rho}\underline{\mu}^s], \\ \underline{\mu}^s &\geq Du^s(\omega^s - \underline{z}), \\ \underline{\rho}\underline{\mu}^s &= \Delta[(1 - \pi)\rho\underline{\mu}^s + \pi\underline{\rho}\underline{\mu}^b]. \end{aligned} \quad (2.6)$$

Because these conditions are stated in terms of net trades, social feasibility is automatically satisfied. By Walras' law, one budget constraint is redundant. The remaining equilibrium conditions are given by a single budget constraint for each state:

$$\begin{aligned}\bar{z} &= \bar{p}[1 - g/2(1 + g)], \\ \underline{z} &= \underline{p}[g/2(1 + g)].\end{aligned}\tag{2.7}$$

These must satisfy $\bar{z}, \underline{z} \leq \omega^s$. In addition, in the deflationary case, $g < 0$, we require $\underline{z} \geq -\omega^b$. Notice that a two-state Markov equilibrium does not necessarily exist for all values of δ , π , and g .

3 Efficiency

There are two efficiency criteria we can consider. *Ex post* efficiency takes the realization of uncertainty in the first period, but not subsequent periods, as given. In other words, it takes the initial identification of the two types as buyers or sellers as given and asks if both types can be made better off. An *ex post* efficient allocation maximizes a weighted sum of utilities of the two initial types. *Ex ante* efficiency does not take the realization of any uncertainty as given. It recognizes that each type of agent is equally likely to begin life as a buyer or a seller. Following the mechanism-design literature (see, for example, Townsend 1982 and Green 1987), we consider *ex ante* efficiency because, given that agents are *a priori* identical, it provides an unambiguous ranking of any two equilibria even when neither is fully efficient.

No autarkic equilibrium is *ex ante* efficient unless $\pi = 0$. If $g = 0$ and $0 < \pi < 1$, then no monetary equilibrium is efficient in either sense. To see why, consider the case of linear utility, $u^b(c_t^i) = \min(\eta^b c_t^i, \bar{u})$ and $u^s(c_t^i) = \min(\eta^s c_t^i, \bar{u})$ where $\eta^b > \eta^s$, and equal endowments, $\omega^b = \omega^s = 1$. (Here $\bar{u} = \eta^b(2\omega^b + 2\omega^s)$ serves only to bound utility from above.) Maximizing the sum of the two types' utilities shows that any efficient allocation gives all of the consumption (two units' worth) to the type with the higher marginal utility. Can such an allocation be achieved by a monetary equilibrium? Since it can be shown that the price of currency is bounded above, to purchase a single unit of consumption from the other sector requires a minimum expenditure of currency $m^* > 0$ (Levine 1988). This means that a sector cannot purchase the other sector's endowments more than $1/m^*$

times in a row before running out of currency. If, by a stroke of bad luck, the type that is supposed to receive all the consumption when it has the high value happens to have the high value more than $1/m^*$ times in a row, then it cannot purchase any consumption, even though Pareto efficiency requires it to. Notice how setting the rate of monetary expansion $g > 0$ can avoid this problem: By constantly redistributing currency, it ensures that neither type ever runs out.

This argument for inefficiency is closely related to the "classical corridor" property proposed by Leijonhufvud (1973). Leijonhufvud argues that money, or more generally liquidity, acts to cushion small shocks, allowing an economy to follow a classical efficient path. If shocks are too large or too prolonged, however, all liquidity is squeezed out of the system and the economy is stuck in a situation of Keynesian inefficiency. He refers to the range of shocks in which the economy behaves classically as the "classical corridor." In our economy, mutually beneficial trade can be sustained while both sectors have money. A persistent series of shocks can eventually cause one sector to draw its money down to zero, however, forcing it to stop trading. Mutually beneficial trade becomes impossible because money is in the wrong hands. This may be regarded as a recession. In contrast with Leijonhufvud's conception, however, in our model there is no particular tendency of the system to get stuck outside the corridor. Once outside, a reversal of agents' types can easily restore money to the right hands, moving the system back into the corridor.

Let us consider *ex ante* welfare in a two-state Markov equilibrium. Let \bar{u}^b be the momentary utility of a buyer in a boom, \bar{u}^s that of a seller in a boom, \underline{u}^b that of a buyer in a recession, and \underline{u}^s that of a seller in a recession. Let \bar{V}^s , \bar{V}^b , \underline{V}^b , and \underline{V}^s be the corresponding present values. These can be calculated from the following equations:

$$\begin{aligned}\bar{V}^b &= \bar{u}^b + \delta[(1 - \pi)\underline{V}^b + \pi\bar{V}^s], \\ \bar{V}^s &= \bar{u}^s + \delta[(1 - \pi)\underline{V}^s + \pi\bar{V}^b], \\ \underline{V}^b &= \underline{u}^b + \delta[(1 - \pi)\underline{V}^b + \pi\bar{V}^s], \\ \underline{V}^s &= \underline{u}^s + \delta[(1 - \pi)\underline{V}^s + \pi\bar{V}^b],\end{aligned}\tag{3.1}$$

In period 1, the economy has probability $(1 - \pi)$ of beginning in a recession, in which case welfare is $\underline{V}^b + \underline{V}^s$, and probability π of beginning in a boom, in which case welfare is $\bar{V}^b + \bar{V}^s$. Consequently, *ex ante* welfare is

$$W = \pi(\bar{V}^b + \bar{V}^s) + (1 - \pi)(\underline{V}^b + \underline{V}^s). \quad (3.2)$$

We can solve (3.1) and (3.2) to find

$$W = (1 - \delta)^{-1} [\pi(\bar{u}^b + \bar{u}^s) + (1 - \pi)(\underline{u}^b + \underline{u}^s)]. \quad (3.3)$$

4 Costs of Inflation: The Deterministic Case

We now suppose that $\pi = 1$, so that each trader switches back and forth deterministically between being a buyer and a seller. In this case there are no recessions. The only relevant equilibrium conditions are those for booms (equations 2.5), and equations 2.7 serve to define prices. Examining these conditions, we find either that

$$\Delta Du^b(\omega^b + \omega^s) > Du^s(0),$$

in which case $\bar{z} = \omega^s$ and sellers sell all of their endowment to buyers, or that \bar{z} is determined by

$$\Delta Du^b(\omega^b + \bar{z}) + Du^s(\omega^s - \bar{z}).$$

In the latter case, $\Delta Du^b(\omega^b) \geq Du^s(\omega^s)$ must hold if a two-state Markov equilibrium is to exist.

Turning to welfare, we can simplify (3.3) to

$$W = (1 - \delta)^{-1}(\bar{u}^b + \bar{u}^s), \quad (4.1)$$

which implies that

$$\frac{dW}{dg} = (1 - \delta)^{-1} [Du^b(\omega^b + \bar{z}) - Du^s(\omega^s - \bar{z})] \frac{d\bar{z}}{dg}. \quad (4.2)$$

Since $\Delta Du^b(\omega^b + \bar{z}) \geq Du^s(\omega^s - \bar{z})$,

$$Du^b(\omega^b + \bar{z}) - Du^s(\omega^s - \bar{z}) > 0.$$

If $\bar{z} = \omega^s$, then $d\bar{z}/dg = 0$ and inflation has no effect. If, on the other hand, $\bar{z} < \omega^s$, then differentiating

$$\Delta Du^b(\omega^b + \bar{z}) = Du^s(\omega^s - \bar{z})$$

implies that $d\bar{z}/dg < 0$, which implies that $dW/dg < 0$. Consequently, inflation is bad or—if the seller sells his entire endowment—neutral. This is the Freidman-Bewley-Townsend perspective: Inflation lowers real balances and reduces trade.

5 Benefits of Inflation: The Linear Case

To study the benefits of inflation, we must allow for recessions, that is, $\pi < 1$. As an extreme case, let us suppose utility is linear. This ensures that the seller sells everything. In the deterministic case, therefore, inflation is neutral. Let us consider the stochastic case. Specifically, we suppose that $\omega^b = \omega^s = 1$, that $u^b(c_t^i) = \min(\eta^b c_t^i, \bar{u})$, and that $u^s(c_t^i) = \min(\eta^s c_t^i, \bar{u})$ where $\eta^b > \eta^s$. To aid in manipulating the Lagrangian conditions (2.5)–(2.7) we define the constants

$$\underline{\Delta} = \frac{\Delta\pi}{1 - \Delta(1 - \pi)}, \quad (5.1)$$

$$\bar{\Delta} = \frac{\Delta^2\pi(1 - \pi)}{1 - \Delta(1 - \pi) - \Delta^2\pi^2}.$$

Notice that $\underline{\Delta} < 1$ and that $\bar{\Delta} < 1$, provided that $\Delta < 1$. The constant $\underline{\Delta}$, for example, represents the present value in the low-valued state of a dollar received the next time a high-valued state occurs; it serves as an effective discount factor for the low-valued state. Finally, define

$$\gamma = g/(2 + g). \quad (5.2)$$

This variable ranges from -1 to 1 and is increasing in the rate of money growth, g . The Lagrangian conditions (2.5) and (2.6) become

$$\begin{aligned} \bar{\mu}^b &= \eta^b, & \underline{\mu}^b &= \eta^b \\ \underline{\Delta}\bar{\rho} &\geq \bar{\Delta}\rho, & \rho &\geq \underline{\Delta}^2\bar{\rho} \\ \underline{\Delta}\eta^b &\geq \eta^s, & \bar{\rho}\underline{\Delta}\eta^b &\geq \underline{\rho}\eta^s \\ \bar{\mu}^s &= \underline{\Delta}\eta^b, & \underline{\rho}\mu^s &= \bar{\rho}\eta^s. \end{aligned} \quad (5.3)$$

The remaining equilibrium conditions, (2.7), become

$$1 \geq \bar{z} = \bar{\rho}/(1 + \gamma), \quad 1 \geq \underline{z} = \underline{\rho}\gamma/(1 + \gamma) \geq -1. \quad (5.4)$$

To calculate equilibria, we observe that either an inequality in the third line of (5.3) holds with equality or the corresponding \bar{z} or \underline{z} equals 1 . This means that there are always six equations to use in solving for two prices and four marginal utilities of income.

If $\underline{\Delta} < \eta^s/\eta^b$, then $\bar{z} = 1$. Even if $\underline{\Delta} = \eta^s/\eta^b$, we can argue that if there is a two-state Markov equilibrium there is one with $\bar{z} = 1$. There are, there-

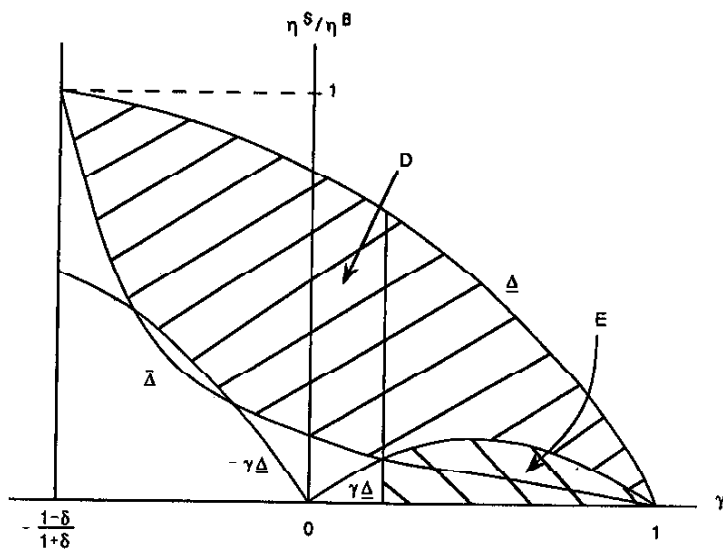


Figure 1

fore, two cases: $z = 1$ and $z < 1$. We refer to the former case as the *efficient case*: The buyers consume all of the consumption good in every state, resulting in an equilibrium which is *ex ante* efficient. The latter case, where $z < 1$, represents an equilibrium that is Pareto dominated, although not necessarily by another equilibrium. We refer to this as the *inefficient case*.

The inequalities characterizing equilibria of the two types are illustrated in figure 1. Here we have used (2.3), (5.1), and (5.2) to calculate

$$\underline{\Delta} = \frac{\delta(1-\gamma)\pi}{1+\gamma-\delta(1-\gamma)(1-\pi)}, \quad (5.5)$$

$$\bar{\Delta} = \frac{\delta^2(1-\gamma)^2\pi(1-\pi)}{(1+\gamma)^2 - \delta(1-\gamma^2)(1-\pi) + \delta^2(1-\gamma)^2\pi^2}.$$

The inefficient equilibria lie in the region labeled D and the efficient equilibria in the region labeled E. For other values of η^s/η^b and γ there is no two-state Markov equilibrium. In the efficient region E, η^s/η^b lies below the curve $\gamma\underline{\Delta}$ and γ is such that $\bar{\Delta}$ curve lies below the $\gamma\underline{\Delta}$ curve. The inefficient region D lies above the $\gamma\underline{\Delta}$ curve for $\gamma > 0$ and above the $-\gamma\underline{\Delta}$ curve

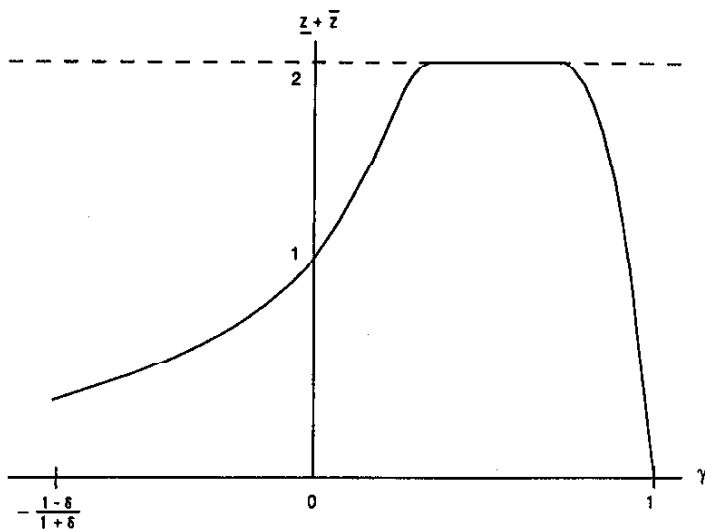


Figure 2

for $\gamma < 0$. It also lies below the $\underline{\Delta}$ curve and above the $\bar{\Delta}$ curve. General qualitative features of the diagrams are that the $\underline{\Delta}$ and $\bar{\Delta}$ curves are downward sloping with $\underline{\Delta} > \bar{\Delta}$, are both equal to zero at $\gamma = 1$, and are both equal to one at $\gamma = -(1 - \delta)/(1 + \delta)$. Finally, the $\bar{\Delta}$ and $\gamma\underline{\Delta}$ curves intersect at a unique point; the $\bar{\Delta}$ and $-\gamma\underline{\Delta}$ curves intersect at two points or not at all.

Within the region where $\eta^a/\eta^b \leq \underline{\Delta}$, it is not always true that a two-state Markov equilibrium exists. When $\eta^a/\eta^b < \bar{\Delta}$ or $\eta^a/\eta^b < -\gamma\underline{\Delta}$ and $\gamma\underline{\Delta} < \bar{\Delta}$; that is, in the lower left corner of figure 1 there is no two-state Markov equilibrium. If η^a/η^b is very small and g is small or negative, then the value of future trade is sufficiently great that the buyer wishes to hold money in the high-valued state. This in no way contradicts the existence of a monetary equilibrium. If the buyer holds money and the price of money is positive, however, the equilibrium cannot be two-state Markov.

It is useful to consider how total trade in both states depends on the rate of money growth. Figure 2 illustrates this dependence. To draw this figure, we use the fact that at an inefficient equilibrium $\underline{z} = \gamma\underline{\Delta}\eta^b/\eta^a$. In other words, the dependence of \underline{z} on γ is proportional to the height of the curve

$\gamma \Delta$. Unless money growth causes a collapse to autarky, the output in a boom is fixed at 1, so only the variation in \underline{z} matters. Trade gradually rises as the inflation rate increases from $\gamma = -(1 - \delta)/(1 + \delta)$, where $g = -(1 - \delta)$, and reaches a peak with $\gamma > 0$. The peak may or may not equal 2, depending on how great η^b/η^s is. Then trade falls, gradually or suddenly, to zero as γ is increased further.

Interpretation of figure 2 would be most straightforward if the different equilibria were Pareto ordered, with larger values of \underline{z} corresponding to more efficient equilibria. This is true with *ex ante* welfare criterion. Unfortunately, it is not true with the *ex post* welfare criterion. Calculating the expected present value of utility to each type shows that \bar{V}^b , V^b , and \bar{V}^s are all increasing in \underline{z} . We can solve (3.1) to calculate

$$\begin{aligned} \frac{dV^s}{d\underline{z}} &= \frac{(1 - \delta + \delta\pi - \delta^2\pi^2)\eta^s - \delta^2\pi(1 - \pi)\eta^b}{(1 - \delta)(1 - \delta + 2\delta\pi)} \\ &= \frac{(1 - \delta + \delta\pi - \delta^2\pi^2)}{(1 - \delta)(1 - \delta + 2\delta\pi)} [\eta^s - \bar{\Delta}\eta^b]. \end{aligned} \quad (5.6)$$

Since the equilibrium conditions imply that $\bar{\Delta}\eta^b \geq \eta^s$, V^s cannot be increasing in \underline{z} . During a boom, both types agree that more \underline{z} is better. During a recession, however, the seller type, who loses immediate consumption if \underline{z} is increased prefers smaller values of \underline{z} , while the buyer who stands to make an immediate gain prefers larger values of \underline{z} . In practice, it is to be expected that during a boom both types will agree that a policy leading to a higher value of \underline{z} in the future is desirable. *Ex post*, after a recession occurs, there is disagreement. The buyer, who is the direct beneficiary of the policy, continues to want the policy to be implemented; the seller, who must give up current consumption, is opposed to the policy.

Using *ex ante* utility as a criterion, the vertical axis in Figure 2 may be interpreted as "efficiency." The optimal monetary policy can also be calculated. For small values of η^s/η^b (large gains to trade), any level of money growth that leads to full efficiency is optimal. For intermediate values of η^s/η^b , the value of γ that maximizes $\gamma \Delta$ should be chosen. For high values of η^s/η^b , the largest value of γ consistent with the existence of an equilibrium should be chosen.

The case in which the largest possible value of γ should be chosen points up a possible problem with an expansionary policy: The government flirts with disaster. A little extra expansion causes the economy to collapse. As γ

is gradually increased, welfare increases. Suddenly, however, there is a catastrophe and welfare drops radically. In this situation, taking into account the government's uncertainty about the nature of the parameters, it might be better to use a more conservative policy and keep the level of monetary expansion well away from the growth rate that would lead to collapse, even though a small increase may bring a small benefit.

The possibility of catastrophic collapse due to an overexpansionary monetary policy can be illustrated in the previous case where $\pi = 1$. If a two-state Markov monetary equilibrium exists, even with $g = 0$, recall that it is fully efficient. A two-state Markov equilibrium exists if and only if

$$(\underline{\Delta} =) \Delta \geq \frac{\eta^a}{\eta^b}. \quad (5.7)$$

It is instructive to consider what happens when (5.7) holds with exact equality. In this case, sellers are exactly indifferent between autarky and the monetary equilibrium. Buyers strictly prefer the monetary equilibrium, however; next period they are indifferent, but now they receive an extra η^b that they would not receive in autarky. If (5.7) is violated by a small amount, then sellers marginally prefer autarky, causing the monetary equilibrium to collapse. This marginal gain to sellers causes a substantial loss (of η^b) to the buyers. In this sense the collapse to autarky is catastrophic.

6 Inflation Tradeoffs: The Logarithmic Case

In the previous two sections we have studied extreme cases. In the deterministic model, inflation has unambiguously negative effects. In the linear model, it has unambiguously positive effects. We now study a model in which there are both costs and benefits to inflation. Suppose that

$$u^s(c_t^i) = u^b(c_t^i) = \min(\log c_t^i, \bar{u})$$

and that

$$\omega^s > \omega^b.$$

In this case consumption must always be positive, and conditions (2.5) and (2.6) become

$$\begin{aligned}\bar{z} &= \frac{\Delta\omega^s - \omega^b}{\Delta + 1}, \\ z &= \frac{g\omega^s\bar{z}}{2\omega^s + g\omega^s - 2\bar{z}}, \\ \bar{\rho} &= \frac{\bar{z}(2 + 2g)}{2 + g}, \\ \underline{\rho} &= \frac{\Delta\bar{\rho}(\omega^s - z)}{\omega^b + \bar{z}}.\end{aligned}\tag{6.1}$$

Here Δ is defined as in (5.5). To compute an equilibrium, we solve these four equations for \bar{z} , z , $\bar{\rho}$, and $\underline{\rho}$. To be an equilibrium, the solution must further satisfy the following inequalities:

$$\begin{aligned}\frac{\bar{\rho}}{\omega^b + \bar{z}} &\geq \Delta \left[\frac{(1 - \pi)\underline{\rho}}{\omega^b + \bar{z}} + \frac{\pi\bar{\rho}}{\omega^s - \bar{z}} \right], \\ \frac{\underline{\rho}}{\omega^b + \bar{z}} &> \Delta \left[\frac{(1 - \pi)\underline{\rho}}{\omega^b + \bar{z}} + \frac{\pi\bar{\rho}}{\omega^s - \bar{z}} \right], \\ \bar{\rho} &\geq 0, \\ \underline{\rho} &\geq 0, \\ \frac{1 - \Delta\Delta\pi}{\Delta\Delta(1 - \pi)} &\geq \frac{\omega^s - \bar{z}}{\omega^b + \bar{z}} \geq \frac{1}{\Delta}.\end{aligned}\tag{6.2}$$

Our goal is to analyze dW/dg at $g = 0$: Does welfare increase or decrease as the rate of growth of money is slightly increased from zero? Simple but tedious calculations using (3.1) and the equilibrium conditions (6.1) yield

$$\begin{aligned}\frac{dW}{dg} &= \frac{1}{(1 - \delta)(1 - \delta + \delta\pi)} \\ &\times \left(-\frac{\pi(1 - \delta)}{1 - \delta + 2\delta\pi} + \frac{(1 - \pi)(\omega^s/\omega^b - 1)(\delta\pi\omega^s/\omega^b - 1 + \delta - \delta\pi)}{2(\omega^s/\omega^b + 1)} \right).\end{aligned}\tag{6.3}$$

This expression has two terms, the first negative and the second positive. The first term corresponds to the derivative of welfare in a boom. Since we can show that $d\bar{z}/dg < 0$ and welfare is increasing in \bar{z} , increasing the

growth rate of money increases the cost of holding money balances and consequently lowers their value. This reduces trade during a boom, \bar{z} , and reduces welfare. This is the Bewley-Townsend effect; in the deterministic case they consider, the economy is always in a boom. The second term is positive because $d\bar{z}/dg > 0$ and welfare is increasing in \bar{z} . In other words, the reduction in the value of real balances is more than offset by the redistribution of money to the low-endowment sector, making it possible for trade to take place and increasing welfare. This is the only effect in the linear case.

In addition to determining how dW/dg depends on values of the parameters, we must check that a two-state Markov equilibrium actually exists. The inequalities in (6.2) in the case where $g = 0$ reduce to

$$\frac{1 - \delta + \delta\pi - \delta^2\pi^2}{\delta^2\pi(1 - \pi)} \geq \frac{\omega^s}{\omega^b} \geq \frac{1 - \delta + \delta\pi}{\delta\pi}, \quad (6.4)$$

which is necessary and sufficient for a two-state Markov equilibrium to exist near $g = 0$ and to validate the use of (6.3) for welfare computations.

More interesting than dW/dg is $(1 - \delta)dW/dg$, the elasticity of welfare in consumption units with respect to the money supply. We compute this various parameter values in table 1. Notice that two-state Markov

Table 1

| | | $(1 - \delta) \frac{dW}{dg}$ ω^s/ω^b | | |
|------------------------|---|---|---------|---------|
| | | 2 | 4 | 8 |
| $\delta = \frac{1}{2}$ | $\pi = \begin{cases} \frac{1}{2} \\ \frac{3}{4} \\ \frac{7}{8} \end{cases}$ | * | -0.2833 | -0.0093 |
| | | * | -0.2893 | -0.1067 |
| | | * | -0.3069 | -0.2065 |
| $\delta = \frac{3}{4}$ | $\pi = \begin{cases} \frac{1}{3} \\ \frac{3}{4} \\ \frac{7}{8} \end{cases}$ | -0.1833 | * | * |
| | | -0.1518 | -0.0351 | * |
| | | -0.1451 | -0.0834 | * |
| $\delta = \frac{7}{8}$ | $\pi = \begin{cases} \frac{1}{2} \\ \frac{3}{4} \\ \frac{7}{8} \end{cases}$ | * | * | * |
| | | -0.0514 | * | * |
| | | -0.0592 | * | * |

* No two-state Markov equilibrium exists.

equilibria often do not exist. Notice too that for a broad range of parameter values inflation decreases welfare. This is not to say that inflation always decreases welfare: At $\delta = \frac{1}{2}$, $\pi = \frac{1}{2}$ and $\omega^s/\omega^b = 10$, for example, $(1 - \delta)dW/dg = 0.1439$.

It is evident from (6.3) that, for fixed δ and π , dW/dg is a monotonically increasing function of ω^s/ω^b : The greater the fluctuations in endowments (and hence the greater the need for intertemporal trade), the more likely that the optimal growth rate of currency is positive rather than negative. It can also be seen from (6.3) that when ω^s/ω^b takes the lowest value consistent with (6.4), that is, when $\omega^s/\omega^b = (1 - \delta + \delta\pi)/\delta\pi$, the second term is 0 while the first is negative. Consequently, $dW/dg < 0$. Furthermore, dW/dg would eventually become positive—indeed unboundedly large for ω^s/ω^b large enough—if a two-state Markov equilibrium continued to exist for such large values of ω^s/ω^b .

For fixed δ and π , it may or may not be possible to have $dW/dg > 0$ when ω^s/ω^b is in the interval specified by (6.4). Indeed, since dW/dg is increasing in ω^s/ω^b , this is possible if and only if it is true when ω^s/ω^b is chosen equal to the upper bound in (6.4). Substituting this value into (6.3) shows that $dW/dg > 0$ if and only if

$$(1 - \delta)^2(1 + \delta\pi) > 2\delta^2\pi(1 - 2\pi + \delta\pi). \quad (6.5)$$

This condition holds for some, but not all, choices of δ and π that satisfy $0 < \delta < 1$ and $0 < \pi < 1$. The region where it holds is shown in figure 3. Notice that, for fixed π , (6.5) is satisfied for δ small enough but is violated for δ close enough to 1. Since the case of δ close to 1 (where time periods are very short) is of great interest, this may make it seem unlikely that there are interesting parameter values for which a two-state Markov equilibrium exists and $dW/dg > 0$.

Nonetheless, if π approaches either 0 or 1 rapidly enough as δ approaches 1, the inequality (6.5) continues to hold. If $\pi = k(1 - \delta)^2$, where $0 < k < \frac{1}{2}$, then (6.5) holds for all δ close enough to 1. Similarly, if $\pi = 1 - \theta(1 - \delta)^k$, where $1 > \theta > 0$ and $k \geq 1$, then (6.5) holds for all δ close enough to 1. In either of these cases, it is possible for a two-state Markov equilibrium to exist and for dW/dg to be positive, even with δ arbitrarily close to 1.

In the case where $\pi = 1$, Bewley and Townsend conclude that $dW/dg < 0$ for all values of δ and ω^s/ω^b . How robust is this conclusion? Suppose that δ and ω^s/ω^b are fixed and that $\omega^s/\omega^b > 1/\delta$, so that a two-state Markov

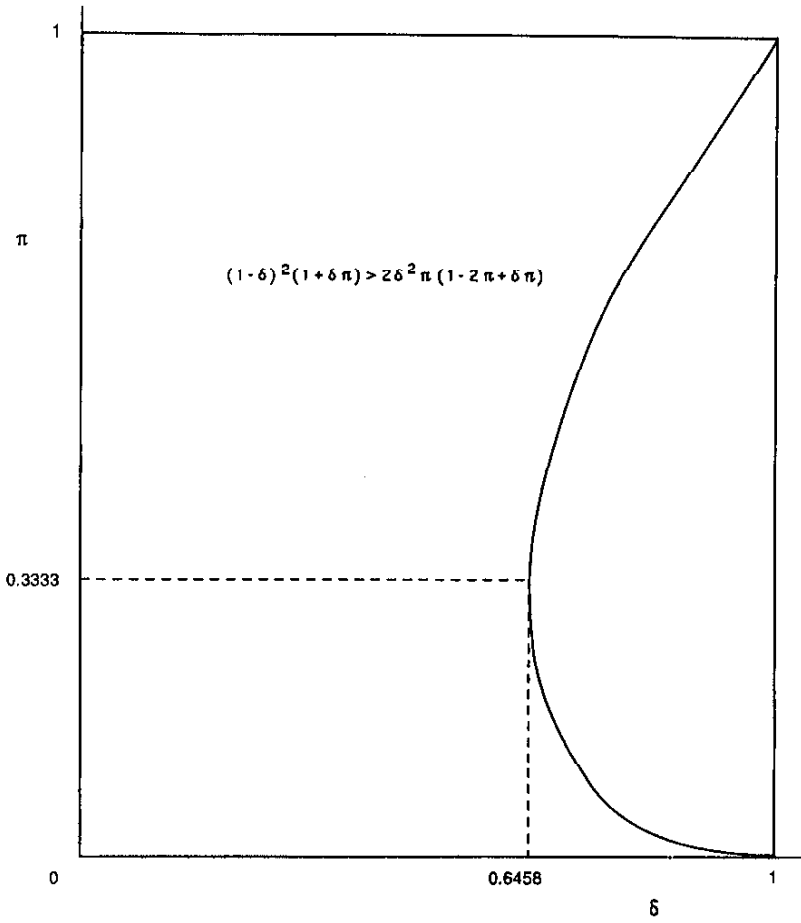


Figure 3

Table 2

| | | $(1 - \delta) \frac{dW}{dg}$ ω^a/ω^b | | |
|------------------|---|---|---------|---------|
| | | 50 | 200 | 500 |
| $\delta = 0.97$ | $\pi = \begin{cases} 0.999 \\ 0.9999 \\ 0.999999 \end{cases}$ | 0.0076 | * | * |
| | | -0.0129 | -0.0057 | 0.0089 |
| | | -0.0150 | -0.0143 | -0.0128 |
| $\delta = 0.99$ | $\pi = \begin{cases} 0.999 \\ 0.9999 \\ 0.999999 \end{cases}$ | * | * | * |
| | | -0.0027 | 0.0047 | * |
| | | -0.0048 | -0.0041 | -0.0026 |
| $\delta = 0.999$ | $\pi = \begin{cases} 0.999 \\ 0.9999 \\ 0.999999 \end{cases}$ | * | * | * |
| | | * | * | * |
| | | -0.0003 | 0.0005 | * |

* No two-state Markov equilibrium exists.

equilibrium exists in the limit. This is also the case in which the monetary steady state studied by Townsend exists. As π approaches 1, the first term in (6.3) approaches $-1/(1 + \delta)$, the second term approaches 0, and $dW/dg < 0$. On the other hand, the Bewley-Townsend welfare result is non-robust in the sense that for π arbitrarily close to 1, there are values of ω^a/ω^b for which $dW/dg > 0$. This is established in the previous paragraph. Welfare elasticities for a range of cases of this type are displayed in table 2.

Thus far, we have considered only the possible benefits from a steady expansion. It is clear, however, that a policy that expands only in recessions would have the positive effects of the expansions considered here but would mitigate the negative incentive effects. This is an important rationale for a central bank or an international monetary system as a lender of last resort. By providing cash to liquidity-constrained individuals and organizations, such an institution provides socially desirable insurance.

7 Measuring the Costs and Benefits of Inflation

One problem with the discussion of the previous section is that when δ is near 1 the actual magnitude of the welfare loss or gain is very small. Some numerical examples bear out this point. Suppose that periods are a month long, and that the monthly real interest rate is 1 percent. Then $\delta = 0.99$.

Suppose also that $\omega^a/\omega^b = 200$ and that $\pi = 0.9999$. A two-state Markov equilibrium exists when $g = 0$. The welfare elasticity is $(1 - \delta)dW/dg = 0.0047$. Suppose that $g = 0.0020$, corresponding to a 2.4 percent annual growth rate of money. This results in a decline in W that is equivalent to reducing consumption forever and for sure by only 0.00067 percent—about what is predicted by $(1 - \delta)dW/dg$. As the annual growth rate of currency is increased above 5 percent, the welfare gain falls, and at a 10 percent annual rate welfare is roughly the same as at 0 percent. At a 400 percent annual inflation rate, there is a welfare loss equal to about 1 percent of consumption forever. An annual deflation of 0.12 percent leads to a minuscule welfare loss of 0.000048 percent of consumption. Higher rates of deflation are not consistent with the existence of a two-state Markov equilibrium. A simple calculation shows why the actual magnitude of welfare loss is so small when δ is near 1. If π equals 1 as δ approaches 1, dW/dg approaches $-\frac{1}{2}$. This is the Bewley-Townsend case. To convert welfare changes into units of consumption, we must multiply by $1 - \delta$; the elasticity of consumption lost due to inflation or deflation goes to 0.

These results suggest that the welfare costs and benefits of inflation are small. Unfortunately, the simple structure of our model and the restrictive nature of the equilibria we can analytically compute limit the force of our results. Nevertheless, our analysis points out some factors that any serious attempt to measure the costs and benefits of inflation should take into account.

The first framework developed for measuring the costs of inflation was that of Bailey (1956), who suggested computing the area of the triangle under the demand curve for money. Fischer (1981), using this approach, finds the welfare costs of 10 percent inflation to be 0.3 percent of GNP. Lucas (1981), using M1 rather than currency as the measure of the money supply, modifies this estimate to 0.45 percent. Cooley and Hansen (1989) impose a cash-in-advance constraint on a real business cycle model of the sort developed by Kydland and Prescott (1982). They find that the welfare cost of 10 percent inflation is 0.4 percent of GNP. It comes as no surprise that inflation is bad in the welfare triangle and the cash-in-advance approach. In each case the optimal monetary policy is to set $g = \delta - 1$ and deflate.

An interesting approach to measuring the costs of inflation, which uses a model similar in many respects to ours, is taken by Imrohroglu (1988). In her model money is also held to self-insure. All risk is idiosyncratic, how-

ever; there is no aggregate uncertainty. Imrohoroglu finds that the welfare costs of inflation are substantially higher than do Bailey, Lucas, and Cooley and Hansen. In a model where the welfare costs of a 10 percent inflation would be 0.42 percent if measured using the welfare triangle approach, the correct measure of the cost is 1.09 percent.

Like the model of this paper, Imrohoroglu's model has a tradeoff between costs and benefits of inflation because seignorage revenues, which are redistributed in equal lump-sum transfers to agents, improve insurance possibilities. There are two possible explanations for the large disparity in results: First, as noted, Imrohoroglu's model has no aggregate uncertainty, whereas our model relies on it. Second, in her model the transition probability of going from a low-endowment state to a high-endowment state is much higher than that of going from a high-endowment state to a low-endowment state. This implies that the probability of being in a low-endowment state for two consecutive periods, where the insurance benefits of inflation are most valuable, is relatively insignificant. Since our model has only two types of agents, who are always in different states, these two probabilities need to be equal. To explore the differences and their consequences would require more powerful techniques for computing equilibria of this type model: Imrohoroglu's approach is heavily dependent on there being no aggregate uncertainty. Our approach is heavily dependent on there being only two types of uncertainty and on equilibria being two-state Markov. A more general approach to computing equilibria is provided by Kehoe and Levine (1985).

To measure the costs and benefits of inflation in a serious way, we would want to consider the whole range of alternative government policies for financing expenditures: money financing, bond financing, and distortionary taxation. Woodford (1990) provides a summary of much of the research in this area. Judd (1989) stresses the importance of the interactions among these alternatives. Simple calculations suggest, for example, that the effect of inflation on the economy through capital taxation because of nonindexation of capital depreciation allowances is far greater than those calculated in the traditional Friedman framework by Bailey (1956).

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References

- Bailey, M. J. 1956. "The welfare cost of inflationary finance." *Journal of Political Economy* 64: 93-110.
- Benhabib, J., and C. Bull. 1983. "The optimal quantity of money: A formal treatment." *International Economic Review* 24: 101-111.
- Bewley, T. 1980. "The optimum quantity of money." In *Models of Monetary Economies*, ed. J. H. Kareken and N. Wallace. Federal Reserve Bank of Minneapolis.
- Bewley, T. 1983. "A difficulty with the optimum quantity of money." *Econometrica* 51: 1485-1504.
- Brock, W. A. 1974. "Money and growth: The case of long run perfect foresight." *International Economic Review* 15: 750-777.
- Bryant, J. (1980): "Transaction demand for money and moral hazard." In *Models of Monetary Economies*, ed. J. H. Kareken and N. Wallace. Federal Reserve Bank of Minneapolis.
- Cooley, T. F., and G. D. Hansen. 1989. "The inflation tax in a real business cycle model." *American Economic Review* 79: 733-748.
- Fischer, S. 1981. "Towards an understanding of the costs of inflation: II." In *Carnegie-Rochester Conference Series on Public Policy: The Costs and Consequences of Inflation*, ed. K. Brunner and A. H. Meltzer. North-Holland.
- Friedman, M. 1953. *Essays in Positive Economics*. University of Chicago Press.
- Friedman, M. 1969. *The Optimum Quantity of Money and Other Essays*. Aldine.
- Grandmont, J.-M., and Y. Younes. 1973. "On the efficiency of a monetary equilibrium." *Review of Economic Studies* 40: 149-165.
- Green, E. J. 1987. "Lending and the smoothing of uninsurable income." In *Contractual Arrangements for Intertemporal Trade*, ed. E. C. Prescott and N. Wallace. University of Minnesota.
- Hahn, F. H. 1971. "Professor Friedman's views on money." *Economica* 38: 61-80.
- Hahn, F. H. 1973. "On the foundations of monetary theory." In *Essays in Modern Economics*, ed. M. Parkin and A. R. Nobay. Longman.
- Imrohroglu, A. 1988. *The Welfare Costs of Inflation*. Unpublished manuscript, USC.
- Judd, K. L. 1989. *Optimal Taxation in Dynamic Stochastic Economies: Theory and Evidence*. Unpublished manuscript, Hoover Institution, Stanford University.
- Kehoe, T. J., and D. K. Levine. 1985. *Approximate Markov Equilibria of Stationary Rational Expectations Models*. Unpublished manuscript, UCLA.
- Kydland, F. E., and E. C. Prescott. 1982. "Time to build and aggregate fluctuations." *Econometrica* 50: 1345-1370.
- Leijonhufvud, A. 1973. "Effective demand failures." *Swedish Journal of Economics* 75: 27-48.

- Levine, D. K. 1988. Asset Trading Mechanisms and Expansionary Policy. Research Department Working Paper 388, Federal Reserve Bank of Minneapolis.
- Levine, D. K. 1989a. "Efficiency and the value of money." *Review of Economic Studies* 56: 77-88.
- Levine, D. K. 1989b. "Infinite horizon equilibrium with incomplete markets." *Journal of Mathematical Economics* 18: 357-376.
- Lucas, R. E., Jr. 1981. "Discussion of the Fischer paper." In *Carnegie-Rochester Conference Series on Public Policy: The Costs and Consequences of Inflation*, ed. K. Brunner and A. H. Meltzer. North-Holland.
- Lucas, R. E., Jr., and N. L. Stokey. 1983. "Optimal fiscal and monetary policy in an economy without capital." *Journal of Monetary Economics* 12: 55-93.
- Sargent, T. J. 1987. *Dynamic Macroeconomic Theory*. Harvard University Press.
- Scheinkman, J. A., and L. Weiss. 1986. "Borrowing constraints and aggregate economic activity." *Econometrica* 54: 23-45.
- Townsend, R. M. 1980. "Models of money with spatially separated agents." In *Models of Monetary Economies*, ed. J. H. Kareken and N. Wallace. Federal Reserve Bank of Minneapolis.
- Townsend, R. M. 1982. "Optimal multiperiod contracts and the gain from enduring relationships under private information." *Journal of Political Economy* 90: 1166-1186.
- Townsend, R. M. 1987. Optimal Activist Currency Rules. Unpublished manuscript, University of Chicago.
- Weitzman, M. L. 1973. "Duality theory for infinite horizon convex models." *Management Science* 19: 783-789.
- Woodford, M. 1990. "The optimum quantity of money." In *Handbook of Monetary Economics*, ed. B. Friedman and F. H. Hahn. North-Holland.