Below is an example dealing with algebra and taxes. Beginning with demand and supply equations we can algebraically determine the quantity and prices after a tax, along with surpluses for consumers, producers and the government.

In lecture we saw how to model a tax with a tax wedge, find quantity after a tax, prices after a tax and tax burdens. Also, we saw a quick example of dealing with taxes and algebraic equations. Now we will look another example of algebra and taxes, including how to find surpluses after a tax.

We will have specific functions for supply and demand, and given a certain tax we need to determine the prices after the tax ($P_D^T$, the price paid by consumers after a tax, and $P_S^T$, the price received by producers after a tax), and $Q_T$, the quantity bought and sold after a tax.

Example

- Demand is given by $P_D(Q) = 50 - Q$
- Supply is given by $P_S(Q) = 10 + 3Q$
Algebraically we can solve for equilibrium as follows

\[ P_D(Q) = P_S(Q) \]
\[ 50 - Q = 10 + 3Q \]
\[ 40 = 4Q \]
\[ Q = 10 \]

Now we can calculate the price using either the Demand or Supply equations.

\[ P_D(10) = 50 - 10 = $40 \]
\[ P_S(10) = 10 + 3 \times 10 = $40 \]

Thus, we can define equilibrium \( Q^* = 10 \) and \( P^* = $40 \)

Now suppose there is a per unit tax of \( T = $4 \) levied on this good. (For every unit bought or sold, the government must receive $4. We have discussed how it does not matter who the tax is levied on, either consumers or producers.)

In class we saw that to find \( Q_T \), the quantity bought and sold after a tax, we need to find a quantity to the left of \( Q^* \) such that the vertical distance between Demand and Supply is equal to per unit tax, \( T \).

\( \text{(Since the government is collecting } T \text{ per unit, the } Q_T \text{ must be selected in the manner described above. Take the amount of money consumers are paying, } P_D^T \text{, subtract off the per unit tax, } T \text{, and get the amount producers are receiving, } P_S^T .) \)

On the graph, we can find the point on the diagram left of \( Q^* \) where Demand and Supply are \( T = 4 \) unit apart, mark this as \( Q_T \). Where \( Q_T \) hits the Demand curve is \( P_D^T \), where \( Q_T \) hits the supply curve is \( P_S^T \).

We can go from the diagram to the algebra. We know that the difference between Demand and Supply at \( Q_T \) must be equal to \( T = 4 \).

Algebraically we can solve for after-tax quantity as follows

\[ P_D(Q) - P_S(Q) = T \]
\[ 50 - Q - (10 + 3Q) = 4 \]
\[ 40 - 4Q = 4 \]
\[ 4Q = 36 \]
\[ Q = 9 \]

Now we need to calculate the price paid by the consumer and the price received by the supplier

\[ P_D(9) = 50 - 9 = $41 \]
\[ P_S(9) = 10 + 3 \times 9 = $37 \]

**Note:** The difference between the price paid and the price received is equal to the per unit tax.
Given this information, we can calculate consumers and producers burdens (how much does each group pay of the tax) and also total tax revenue.

First we know that since 9 units are bought/sold after the tax, and the per unit tax is $4, the total tax revenue raised is $36.

For consumers burden, we know that consumers are now paying $1 more per unit than they were before \( P_D^T - P^* = 41 - 40 = 1 \). Since consumers are purchasing 9 units, total consumer burden is $9.

For producers burden, we know that producers are now paying $3 more per unit than they were before \( P^* - P_S^T = 40 - 37 = 3 \). Since 9 units are being sold, we know producers total burden are paying $27.

Note that if we add the two burdens together we get $36, which is total tax revenue. We need to get this result (total burden equal to total revenue).

We can see that the consumer burden is a bit smaller than the producer burden. We should not be surprised by this. We can see from the diagram that the demand curve is a bit flatter than the supply curve, meaning supply is more inelastic than demand. We expect the more inelastic party to bear more tax burden.

Now we want to look at calculating surpluses before and after the tax. Lets go back to the market diagram. We can see the Consumers Surplus (CS) and Producers Surplus (PS).

We can calculate the surplus as follows:

\[
CS = \frac{1}{2}(b)(h) = \frac{1}{2}(10 - 0)(50 - 40) = \frac{1}{2}(10)(10) = $50
\]

\[
PS = \frac{1}{2}(b)(h) = \frac{1}{2}(10 - 0)(40 - 10) = \frac{1}{2}(10)(30) = $150
\]
Now we want to look at how CS and PS could change after a market distortion. Specifically, we want to see what happens to surpluses if the government adds a tax to this market (per unit tax of T=\$4).

We want to look at what CS and PS will be after a tax. We should look at a diagram with the tax wedge, $Q_T$, $P^D_T$ and $P^S_T$ illustrated. We need to figure out what the CS will be on the diagram - we should reference the definition. Now the consumers are paying $P^D_T$ and purchasing $Q_T$ - we want the area below the Demand curve and above $P^D_T$ and going up to $Q_T$.

What will PS be - again use definition. We want area below $P^S_T$, above S curve, up to $Q_T$. We can see these areas on a market diagram.

We can calculate the Surplus and Deadweight Loss as follows:

\[
CS = \frac{1}{2}(b)(h) = \frac{1}{2}(9 - 0)(50 - 41) = \frac{1}{2}(9)(9) = \$40.50
\]

\[
PS = \frac{1}{2}(b)(h) = \frac{1}{2}(9 - 0)(37 - 10) = \frac{1}{2}(9)(27) = \$121.5
\]

\[
Gov't\ Rev = T \times Q_T = 4 \times 9 = \$36
\]

\[
DWL = \frac{1}{2}(b)(h) = \frac{1}{2}(10 - 9)(41 - 37) = \frac{1}{2}(1)(4) = \$2
\]

In class we defined the DWL as the amount Total Surplus changes after a market distortion. Above I directly calculated the DWL directly, but we could have calculated the DWL as follows.

The Government Revenue from tax is represented as a rectangle above with area, 4*9=\$36. If we were to add CS, PS and Tax Rev after the tax, we get \$198. Before the tax CS and PS was \$200. The loss in total surplus due to the tax, which is the deadweight loss of the tax, is \$2.