Game Theory Notes:
Examples of Games with Dominant Strategy Equilibrium or Nash Equilibrium

Below are two different games. The first game has a dominant strategy equilibrium. The second game has two Nash equilibria. This handout goes through the process of how to find these Nash Equilibria.

Example 1 – Dominant Strategy Equilibrium:

Consider the following game:

- **Players:** Firm 1, Firm 2
- **Strategies:** Low Output (Q), High Output (Q)
- **Payoffs:** given by

We can think of this as a duopoly game – a game between two firms. These firms can decide to collude – keep output low so that the price is high, in effect they would be acting like a monopolist. However, it is possible for each firm to decide to produce a high output. Each firm will want the other firm to produce a low output in order to keep prices low, while it actually produces a high output in order to take advantage of the low output of its competitor.

We want to find a possible solution to this game. Let’s start by looking to see if any player has a dominant strategy.
**Def:** A strategy is a **dominant strategy** for a player if it is that player’s best strategy (yields the highest payoff) no matter what the other players do.

Let’s look at the strategies of each player.

First consider Firm 1 (F1). If Firm 2 (F2) is playing “Low Output” then F1 is better off playing “High Output” (F1 will get 10 if it plays “High Output” versus 6 if it were to play “Low Output”). Now, if F2 were to play “High Output” then F1 is better off playing “High Output” (F1 will get 3 if it plays “High Output” versus 1 if it were to play “Low Output”).

We can see that F1 is better off playing “High Output” *no matter* what F2 is doing. This means that for F1, “High Output” is a **dominant strategy**.

Now consider Firm 2 (F2). If Firm 1 (F1) is playing “Low Output” then F2 is better off playing “High Output” (F2 will get 10 if it plays “High Output” versus 6 if it were to play “Low Output”). Now, if F1 were to play “High Output” then F2 is better off playing “High Output” (F2 will get 3 if it plays “High Output” versus 1 if it were to play “Low Output”).

We can see that F2 is better off playing “High Output” *no matter* what F1 is doing. This means that for F2, “High Output” is a **dominant strategy**.

**Def:** A **dominant strategy equilibrium** is a set of strategies such that all players are playing a dominant strategy.

In this game there is a dominant strategy equilibrium since each player has a dominant strategy to play.

The **dominant strategy equilibrium for this game** is: (Firm 1 plays “High Output,” Firm 2 plays “High Output”).

Remember an equilibrium is a set of strategies for each player.
Example 2 – Two Nash Equilibria:

Consider the following game:

- **Players:** Firm 1, Firm 2
- **Strategies:** Price 1, Price 2
- **Payoffs:** given by

We can think of an intuitive economic story to motivate the above game. For example, in a duopoly one firm’s price decision impacts the other firm. Let’s assume that Price 2 is higher than Price 1. We see that each firm is better off if it is playing the same strategy as the other firm. This makes sense, since we are dealing with a duopoly; if the only two firms that sell a good charge a high price, quantity sold will not suffer too much.

We see that when the two firms are offering different prices, the firm that is offering Price 2 does worse than the firm offering Price 1. This implies that Price 2 is higher than Price 1. Since payoffs for firms does not change a lot between (Price 1, Price 1) and (Price 2, Price 2) it seems that the demand curve is pretty close to unit elastic (slightly inelastic, because increasing prices seems to increase payoffs (profits)).

We can see there are not dominant strategies.

First consider Firm 1 (F1). If Firm 2 (F2) plays Price 1, it is best for F1 to play Price 1 (P1) (If F2 plays P1, then F1 gets a payoff of 4 if it plays P1, and only a payoff of 3 if it plays P2. Thus F1 will play P1 if F2 plays P1). However, F1 is best off playing P2 if F2 plays P2 (If F2 plays P2, then F1 gets a payoff of 6 if it plays P2, and only a payoff of 5 if it plays P1. Thus F1 will play P2 if F2 plays P2). Thus F1’s best strategy changes as F2’s strategy changes. This means F1 does not have a dominant strategy.

Now consider F2. If F1 plays P1, then F2 is best off playing P1 (then F2 will get a payoff of 5 instead of a payoff of 4). However, F2 is best off playing P2 if F1 plays P2 (then F2
will get a payoff of 5 instead of a payoff of 4). F2’s best strategy changes as F1’s strategy changes. This means F2 does not have a dominant strategy.

Thus there will be no dominant strategy equilibrium. We need another idea for an equilibrium.

Is there any strategy set (“box”) that if we started there we would stay there?

- Consider:
  - If F2 play P1, F1 plays P1
  - If F1 plays P1, F2 plays P1

- And
  - If F2 plays P2, F1 plays P2
  - If F1 plays P2, F2 plays P2

- This idea – if both agents are playing a strategy such that its strategy is the best response given the other agent plays its best response, then we have a Nash equilibrium

**Def:** A Nash Equilibrium is a set of strategies such that each player correctly believes that it is doing the best it can given the other players’ strategies.

Example: Above game has 2 Nash Equilibrium

1. (F1 plays P1, F2 plays P1)
   a. If F1 plays P1, it is in F2’s best interest to play P1
   b. If F2 plays P1, it is in F1’s best interest to play P1

2. (F1 plays P2, F2 plays P2)
   a. If F1 plays P2, it is in F2’s best interest to play P2
   b. If F2 plays P1, it is in F1’s best interest to play P2

So a Nash equilibrium is a set of strategies such that each player correctly believes that it is doing the best it can taking the other players’ strategies as given.

We will look at two methods for finding all Nash equilibria in a game.
The first method will be to use the following algorithm.

Algorithm for finding Nash equilibria:

1. Start with a specific player (1) and a specific strategy
2. Given player (1) plays this strategy, determine the other player (2) best response
3. Now suppose player (2) plays this strategy, what is player (1)’s best response?
4. If player (1)’s best response is the same as the original strategy in step 1, we have found a Nash Equilibrium. If not move on.
5. Try player (1)’s alternative strategy, redo steps 2-4

One specific set of steps is as follows. Start with Firm 1 playing the strategy Price 1.

2. Given Firm 1 plays Price 1, then Firm 2’s best response would be to play Price 1.
4. We get “Firm 1 plays Price 1” in both Step 1 and Step 3, thus we know that a Nash Equilibrium is (Firm 1 plays Price 1, Firm 2 plays Price 1).

To make sure we have found all Nash Equilibria, consider Firm 1 playing Price 2.

2. Given Firm 1 plays Price 2, then Firm 2’s best response would be to play Price 2.
4. We get “Firm 1 plays Price 2” in both Step 1 and Step 3, thus we know that a Nash Equilibrium is (Firm 1 plays Price 2, Firm 2 plays Price 2).

We know that there are two Nash Equilibria, (Price 1, Price 1) and (Price 2, Price 2).

Notice that there could be more than one Nash equilibria – so you need to be sure to check for all equilibria.
Another (possibly simpler) method will be something I call the “underlining” method.

1. For Player 1, take Player 2’s strategy as given
   o (pick a particular column)
   o In this column, underline the payoff resulting from Player 1’s best strategy
   o Do this for both columns

2. For Player 2, take Player 1’s strategy as given
   o (pick a particular row)
   o In this row, underline the payoff resulting from Player 2’s best strategy
   o Do this for both rows

3. Any box with both payoffs underlined is a Nash. Remember the set of strategies played is the equilibrium (not the payoff numbers).

The reason the above strategy finds the Nash equilibria is because above is finding each player’s best response to the other player’s strategy. When both players are playing their best responses at the same time, that is a Nash equilibria.

We can apply the above strategy to the game above:

1. For Firm 1, take Firm 2’s strategy as given
   o When Firm 2 plays Price 1, Firm 1 is best off playing Price 1, getting payoff of 4 (underline this payoff)
   o When Firm 2 plays Price 2, Firm 1 is best off playing Price 2, getting payoff of 6 (underline this payoff)

2. For Firm 2, take Firm 1’s strategy as given
   o When Firm 1 plays Price 1, Firm 2 is best off playing Price 1, getting payoff of 5 (underline this payoff)
   o When Firm 1 plays Price 2, Firm 2 is best off playing Price 2, getting payoff of 5 (underline this payoff)

3. The pairs of strategies (Firm 1 plays Price 1, Firm 2 plays Price 1) and (Firm 1 plays Price 2, Firm 2 plays Price 2) are both Nash equilibria (both of those boxes have both sets of strategies underlined).
Either of the two methods presented gives the same two Nash equilibria.