Comment on “Giffen Behavior and Subsistence Consumption”: The Giffen Paradox Model

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Abstract

This paper presents a utility function that generates the indifference curves map in Jensen and Miller (2008). This function is the first to model the Giffen paradox and solve the strong Giffen problem simultaneously. I use this function to show an imprecision in Jensen and Miller’s paper: they mention that Giffen demands for staple foods are most likely observed when the staple food is the cheapest source of calories. This utility function shows that the staple food could be a normal good when it is sufficiently cheap, depending on the curvature of the indifference curves.

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1. Introduction

This paper presents a utility function that generates the indifference curves map presented in Figure 1 of Jensen and Miller (2008). It is the first utility function that models the Giffen paradox and solves the strong Giffen problem simultaneously.

The Giffen paradox, according to Marshall (1895), is the following:

As Mr. Giffen has pointed out, a rise in the price of bread makes so large of a drain on the resources of the poorer labouring families and raises the marginal utility of money to them so much that they are forced to curtail their consumption of meat and the more expensive farinaceous goods; and, bread being still the cheapest food which they can get and will take, they consume more, and not less of it.

The strong Giffen problem is to propose a “concrete utility function that is strictly increasing and quasi-concave [...] where the Giffen property can be shown by solving the equation of budget balancedness together with the equation saying that the price ratio equals marginal rate of substitution (Heijman and von Mouche 2012).” In the literature, the equalization between the price ratio and the marginal rate of substitution is also known as Gossen’s second law.

Jensen and Miller use an indifference curves map to derive a “set of conditions under which Giffen behavior is most likely to be observed.” In particular, the third condition in this set says that “the basic good is the cheapest source of calories available.” This paper shows that this condition is not precise. Depending on the curvature of the indifference curves, the staple could be a normal good when it is sufficiently cheap, no matter the consumer’s budget, even when the demand for the staple is upward sloping at higher prices.

The utility function I propose represents a preference relation over two services: flavor and calorie surplus (total amount of calories consumed minus a calorie requirement to enjoy flavor). These services are not available in the markets separately. Consumers must buy bread and meat to obtain these services. Both, the consumption of bread and meat generate calorie surplus; however, only meat provides flavor as well.

The functional form of the utility consists of two pieces: it is CES when the consumption bundle provides a strictly positive value of calorie surplus. Otherwise, the utility is equal to the value of calorie surplus. The latter piece models a consumer that is calorie-deprived (she does not meet the calorie requirement to enjoy flavor.) Thus, the consumer only cares about reducing her calorie deficit.

Appendix 1 shows an example of the indifference curves map that this utility function can generate. It is similar to Figure 1 of Jensen and Miller (2008).
This model follows Lancaster’s (1966) theory, where the consumer does not derive utility from the goods themselves, but rather from their properties or characteristics. Lipsey and Rosenbluth (1971) is the first attempt to employ Lancaster’s theory to explain upward-sloping demands.

This utility function is the first of its kind (Heijman and von Mouche, A Child Garden of Concrete Giffen Utility Functions: A Theoretical Review 2012). Previously, all the models that employed a concrete utility function to address the Giffen paradox used two constraints: the budget constraint and a subsistence constraint (Haagsma, Notes on Some Theories of Giffen Behaviour 2012). In these models, the demand for the staple food shows Giffen behavior when both constraints are active. Thus, the demands do not satisfy Gossen’s second law (Dooley 1988; and van Marrewijk and van Bergeijk 1990). Furthermore, there is no standard textbook of microeconomic theory that shows a utility function that models the Giffen paradox and solves the strong Giffen problem simultaneously. For instance, Mas-Collel, Whinston, and Green (1995) shows that Giffen behavior is possible using an indifference curves map, while Jehle and Reny (2000) just mention it is possible to generate upward-sloping demands using the Slutsky equation.

There are other utilities that induce upward-sloping demands and satisfy Gossen’s second law, but they do not address the Giffen paradox. One example is Haagsma (A Convenient Utility Function with Giffen Behaviour 2012). This utility function models health-damaging products like drinking, smoking, or drug intake. The result is upward-sloping demands for non-health damaging products when the consumer is rich rather than poor, contrary to what the Giffen paradox describes. Another example is Doi, Iwasa, and Shimomura (2012). They introduce a utility function without any microeconomic foundation; its only goal is to induce upward-sloping demands at any income level.

Finally, Sorensen (2007) creates a class of utility functions that do not model the Giffen paradox and do not satisfy Gossen’s second law, but they induce upward-sloping demands. His utility function lacks substitution effect and has an inferior good. Thus, by pure income effect, the demand for the inferior good is upward-sloping.

The rest of this paper proceeds as follows: section 2 presents the model formally. Section 3 derives the demands for bread and meat. Section 4 shows that bread could be a normal good at very low prices, regardless of the consumer’s income, even when the demand for the staple is upward sloping at higher prices. Section 5 shows that the elasticity of substitution causes bread to be a normal good at sufficiently low prices. Section 6 provides the final remarks.
2. The Giffen paradox model

This section presents the functional form of the utility function formally.

The utility function represents a preference relation over two services: calorie surplus and flavor. These services cannot be bought in the markets separately. The consumer must go to the markets to buy bread and meat to obtain these services.

Calorie surplus is defined as total calories consumed from eating bread and meat minus a calorie requirement to enjoy flavor. Therefore, its production function is

\[ c = \alpha_b b + \alpha_m m - \bar{c} \]  

where \( c \) is calorie surplus, \( b \) is the quantity of bread consumed, \( \alpha_b \) is the calories that each unit of bread provides, \( m \) is the quantity of meat consumed, \( \alpha_m \) is the calories that each unit of meat provides, and \( \bar{c} \) is the calorie requirement to enjoy flavor.

Flavor is an abstract service that can only be produced using meat as an input. Its production function is \( \delta M \), where \( \delta \) is the amount of flavor that each unit of meat provides.

The functional form of the utility is

\[ u(b, m) = \begin{cases} 
\left( (\alpha_b b + \alpha_m m - \bar{c})^\rho + (\delta m)^\rho \right)^{\frac{1}{\rho}} & \text{if } c > 0 \\
\alpha_b b + \alpha_m m - \bar{c} & \text{if } c \leq 0 
\end{cases} \]  

where \( \sigma = \frac{1}{(1-\rho)} \) is the elasticity of substitution between calorie surplus and flavor.

Properties of the utility function

When (1) is strictly positive, the utility function is strictly increasing, strictly quasi-concave, and continuously differentiable on bread and meat. It satisfies Inada Conditions on meat, while the marginal utility of bread is finite when the consumption of bread is zero and tends to zero as the consumption of bread grows unboundedly. Thus, the indifference curves are differentiable, downward-sloping, and strictly convex to the origin.

In the whole plane of positive numbers, the utility function is strictly increasing, quasi-concave, and continuous. Thus, the upper-contour sets are closed and convex.

In the next section, I use these properties to find the demands for bread and meat.

3. Demand functions

This section derives the demands for bread and meat.

According to the theory of utility, the demand for bread and meat is the solution to the
consumer problem. The consumer problem is defined as choosing an affordable basket of bread and meat such that maximizes the utility. That is,

$$\max_{\{b,m\}} u(b,m) \quad s.t. \quad p_m m + p_b b \leq i \quad m, b \geq 0$$

(3)

where $i$ stands for income, $p_b$ for the price of bread, and $p_m$ for the price of meat.

There are two different types of solutions to the consumer problem: either the optimal bundle provides a positive amount of calorie surplus or not. The consumer always buys a bundle that provides a positive amount of calorie surplus whenever she has enough income to do so.

First, consider the case when the consumer has enough income to buy a bundle that provides a positive amount of calorie surplus. In this case, there are two possible solutions: interior or corner. The corner solution is when the consumer specializes her consumption in meat. On the other hand, the interior solution is characterized by two equations: the Euler equation (Gossen’s second law) and the budget constraint holding with equality. Hence, after solving this system of equations, the demands for bread and meat are

$$b = \frac{i(\delta - \mu(p_b, p_m) \alpha_m) + \mu(p_b, p_m) p_m \bar{c}}{p_b(\delta - \mu(p_b, p_m) \alpha_m) + \mu(p_b, p_m) p_m \alpha_b}$$

(4)

$$m = \frac{\mu(p_b, p_m) (i\alpha_b - p_b \bar{c})}{p_b(\delta - \mu(p_b, p_m) \alpha_m) + \mu(p_b, p_m) p_m \alpha_b}$$

(5)

where $\mu$ is the following function:

$$\mu(p_b, p_m) = \left(\frac{p_m \alpha_b - p_b \alpha_m}{p_b \delta}\right)^{\frac{1}{1-\gamma}}$$

(6)

Now, consider the case when the consumer cannot afford a bundle that provides a positive amount of calorie surplus. In this case, the consumer spends all her income on the cheapest source of calories.

4. Bread could be a normal good at very low prices

In this section, I show that the demand for bread could be downward-sloping at sufficiently low prices, no matter the income of the consumer, even when her demand is upward-sloping at higher prices. I do this by characterizing the demand for bread when the CES piece of
the utility function is Cobb-Douglas and bread is a cheaper source of calories than meat.\footnote{Appendix 2 shows a numerical example of this case.}

The CES utility is Cobb-Douglas when $\rho = 0$; and bread is a cheaper source of calories when the following inequality holds:

$$\frac{\alpha_m}{p_m} < \frac{\alpha_b}{p_b} \tag{7}$$

I characterize the demand for bread in two steps. First, I show that bread is a normal good when its price is sufficiently low, no matter the income of the consumer. Then, I show that the demand for bread is upward-sloping if and only if its price is sufficiently high and the consumer’s income satisfies the following two ranges:

$$\frac{p_m \bar{c}}{2\alpha_m} < i < \frac{p_m \bar{c}}{\alpha_m} \tag{8}$$

$$\frac{p_b \bar{c}}{\alpha_b} < i \tag{9}$$

In economic terms, (8) and (9) say that Giffen behavior is observed only in consumers who cannot afford the calorie requirement to enjoy flavor eating meat only, but are rich enough to satisfy this requirement eating bread only.

Lastly, notice that the demand for bread is piecewise. (7) implies that the consumer spends all her income on bread when her income does not satisfy (9), because she is calorie-deprived. In that case, the demand for bread is clearly downward-sloping. Thus, for the next two steps, I will only refer to the case when the consumer’s income satisfies (9).

**Step 1: Bread is a normal good when it is sufficiently cheap**

This step shows that the demand for bread is downward-sloping when it is sufficiently cheap in this environment.

To prove this, I apply the Slutsky equation. The Slutsky equation guarantees that the slope of the demand is negative when the good is normal. Thus, it is sufficient to derive the demand for bread with respect to income and show that its value is positive only when the price of bread is sufficiently low.

The derivative of the demand for bread with respect to income is

$$\frac{\partial b}{\partial i} = \frac{p_m \alpha_b - 2p_b \alpha_m}{2p_b (p_m \alpha_b - p_b \alpha_m)} \tag{10}$$

Notice that (7) makes the denominator be positive. Hence, the value of (10) is positive if
and only if the price of bread satisfies the following inequality:

\[ p_b < \frac{p_m \alpha_b}{2\alpha_m} \]  \hspace{1cm} (11)

**Step 2: The demand for bread shows Giffen behavior**

I use the derivative of the demand for bread with respect to its own price to prove this point. I show that the value of the derivative is strictly positive when the price of bread is close enough to its upper bound defined by (7).

The derivative of the demand for bread with respect to its own price is

\[ \frac{\partial B}{\partial P_B} = \frac{P_B^2 \alpha_M (P_M \bar{C} - 2\alpha_M I) + P_M \alpha_B I (2P_B \alpha_M - P_M \alpha_B)}{2P_B^2 (P_M \alpha_B - P_B \alpha_M)^2} \]  \hspace{1cm} (12)

First, notice that the denominator is strictly positive in the whole range of prices allowed by (7). Therefore, (12) is positive whenever its numerator is positive.

Now, notice that (7) and (8) imply that the upper bound of the price of bread is

\[ P_B = \frac{I \alpha_B}{\bar{C}} \]  \hspace{1cm} (13)

Thus, by plugging (13) into (12), we find that the value of (12) is strictly positive because (8) holds (otherwise, the value would be negative).

Finally, by continuity, any price of bread close enough to its upper bound makes (12) be strictly positive.

**5. Impact of the elasticity of substitution**

This section shows that the elasticity of substitution (the curvature of the indifference curves) causes bread to become a normal good at sufficiently low prices, even when its demand is upward sloping at higher prices.

I prove this by setting the elasticity of substitution equal to zero \((\rho = -\infty)\). In this case, the CES utility is Leontief. Thus, \(\mu(P_B, P_M) = 1\).

In this scenario, bread is always an inferior good; it is consumed only when \(i \alpha_m < p_m \bar{c}\).

Giffen property is trivial in this scenario. The demand for bread is upward sloping throughout the whole range defined by (7) whenever the consumer buys bread and her income satisfies (9). In other words, there is not a low enough price of bread that makes bread be a normal good.
Appendix 1

This appendix shows an example of the indifference curves map generated by this utility function. The map shows “elbow” shaped curves, just like the ones presented by Jensen and Miller (2008).

Table 1 shows the values of the parameters employed in this example. Graph 1 shows the map.

Notice that there are only two shapes of indifference curves in graph 1: linear and convex. According to (2), the linear indifference curves are all parallel to the curve labeled as “Calorie deficit”; they characterize the preferences for the bundles that do not meet the requirement of calories to enjoy flavor (the bundles that keep the consumer calorie deprived), and they assign a negative value of utility. In this case, the value of the utility equals to the value of the calorie surplus. The convex indifference curves characterize the preferences for the bundles that satisfy the requirement of calories to enjoy flavor. These curves show the “elbow” shape mentioned in Jensen and Miller.

Appendix 2

This appendix shows a numerical example of the demands for bread that the Cobb-Douglas utility generates; it replicates the inverse u-shape relationship between income and price elasticity documented by Jensen and Miller (2008).

Table 2 shows the value of the parameters used in this appendix. Using those values, this appendix generates two graphs: graph 2 and graph 3. Graph 2 shows an example of the demand curves at different income levels; and graph 3 shows the inverse u-shape curve that this model generates when the price of bread changes from 1.60 to 1.65.
Graph 2 illustrates three different demand curves. The only difference between these demands is the income level. All the demands satisfy (9). The demand curves with the lowest and highest income violate (8); the income levels in these two cases are right in the borders of the interval. As (8) shows, the borders are not included in the rage of incomes that induce Giffen property. The only income in this exercise that satisfies (8) is 0.75 (demand curve labeled as “Giffen behavior”).

The range of prices of bread can be divided into three different subsets: prices that make bread be a normal good, prices that make bread be an inferior good but with a stronger substitution effect than income effect, and prices that induce Giffen behavior.

When income is 0.5 (demand curve labeled as “Low income”), the demand ends right at the border of prices that make bread be inferior. At this price, the consumer is spending all her income in bread and barely meets the requirement to enjoy flavor. Thus, when the price of bread increases, she becomes calorie deprived and her demand disappears because it violates (9). But, if we allowed her income to violate (9), then she would specialize her consumption in bread.

In the range of prices that make bread be inferior, the demand curve of the consumer with an income of 0.75 is higher than the demand curve of the richest consumer (“High income” demand). Yet, the poorer consumer is the only that shows Giffen behavior. The
Giffen property in her demand happens only when the price of bread is at least:

\[
\bar{p}_b = \frac{p_m \alpha_b \left( i \alpha_m + \left[ (i^2 - 2i) \alpha_m^2 + \alpha_m p_m \bar{c} \right]^{\frac{1}{2}} \right)}{\alpha_m (2 \alpha_m i - p_m \bar{c})}
\]  

(14)

\(\bar{p}_b\) is the price of bread that makes (12) equal to zero. Thus, applying the quadratic formula to the numerator of (12), the value of \(\bar{p}_b\) is found.

Graph 3 shows how this model can account for the inverse u-shape documented by Jensen and Miller (2008). It shows the price elasticity of demand for bread at different income levels when the price of bread changes from 1.60 to 1.65.

First, notice that the price elasticity of demand is -1 when income is 0.7 and 0.75. This happens because agents with that income level are calorie deprived. Thus, they spend all their income in bread.

Second, the curve shows positive numbers when the income range is between 0.8 and 0.95. These consumers show Giffen behavior.

Last, the consumers with an income greater or equal to 1 have demands for bread that satisfy the Law of Demand. These consumers, just like the ones with Giffen behavior, perceive bread as an inferior good. But, the least poor consumers are rich enough that the income effect does not dominate the substitution effect.
7. References


