A Theory of Business Transfers

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Motivation

Privately-owned firms

- Account for 1/2 of US business net income
- Dominate discussions on growth, wealth inequality, tax policy
- But pose challenge for
  - theory: technology of capital accumulation and transfer
  - measurement: no reliable data on private wealth

This Paper

- Theory of private firm dynamics and capital reallocation
- Characterization of competitive equilibrium
- Administrative IRS data to discipline theory (*in progress*)
  - estimate share of transferable value and investment technology
- Business transfers allow us to
  - back out model-based measure of business valuation
  - study effect of taxing business income, capital gains, wealth on investment, entry, welfare

Business Transfers are Taxable Events

- Buyers and sellers both report sale
  - seller has to pay capital gains
  - buyer has to report depreciable assets
- Price allocated across asset types
  - seller wants to allocate to long-term
  - buyer wants to allocate to short-term

$\Rightarrow$ Conflict of interest and thus consistent reporting

What do we know about Private Business Capital?

Form **8594** (Rev. November 2021) Department of the Treasury Internal Revenue Service

Asset Acquisition Statement Under Section 1060

▶ Attach to your income tax return.

▶ Go to *www.irs.gov/Form8594* for instructions and the latest information.

OMB No. 1545-0074

Attachment Sequence No. **169**

Name as shown on return	Identifying number as shown on return

Check the box that identifies you:

☐ Purchaser ☐ Seller

Part I General Information

1 Name of other party to the transaction	Other party's identifying number

Address (number, street, and room or suite no.)

City or town, state, and ZIP code

2 Date of sale	3 Total sales price (consideration)

Part II Original Statement of Assets Transferred

4 Assets	Aggregate fair market value (actual amount for Class I)	Allocation of sales price
Class I	$	$
Class II	$	$
Class III	$	$
Class IV	$	$
Class V	$	$
Class VI and VII	$	$
Total	$	$

← Cash/securities

← Inventories

← Fixed assets

← Sec. 197 intangibles

5 Did the purchaser and seller provide for an allocation of the sales price in the sales contract or in another written document signed by both parties? . . . . . ☐ Yes ☐ No

If "Yes," are the aggregate fair market values (FMV) listed for each of asset Classes I, II, III, IV, V, VI, and VII the amounts agreed upon in your sales contract or in a separate written document? . . . . . ☐ Yes ☐ No

6 In the purchase of the group of assets (or stock), did the purchaser also purchase a license or a covenant not to compete, or enter into a lease agreement, employment contract, management contract, or similar arrangement with the seller (or managers, directors, owners, or employees of the seller)? . . . . . ☐ Yes ☐ No

If "Yes," attach a statement that specifies **(a)** the type of agreement and **(b)** the maximum amount of consideration (not including interest) paid or to be paid under the agreement. See instructions.

What do we know about Private Business Capital?

- Transferred assets are primarily **intangible** (from form 8594 $\approx$ 70%)
  - Customer bases and client lists, non-compete covenants
  - Licenses and permits, trademarks, tradenames
  - Workforce in place
  - Goodwill and on-going concern value
- Assets are **sold as a group**
- Sale **requires time** to find buyers/negotiate (from brokered data $\approx$ 290 days)

$\Rightarrow$ Add intangible investment and transfers to Hopenhayn-style model

Related Literature

- Firm Dynamics
  - Hopenhayn (1992), Hsieh and Klenow (2009, 2014), Sterk et al. (2021)
- Capital Reallocation
  - Holmes and Schmitz (1990), Ottonello (2014), Guntin and Kochen (2020), Gaillard and Kankanamge (2020), David (2021)
- Entrepreneurship and Private Wealth
  - Cagetti and De Nardi (2006), Saez and Zucman (2016), Smith et al. (2019)
- Capital Gain Taxes and Wealth Taxes
  - Chari et al. (2003), Scheuer and Slemrod (2020), Guvenen et al. (2021), Agersnap and Zidar (2021)

Environment

- Infinite horizon, continuous time
- Demographics:
  - total population N: workers and business owners
  - newborns enter the economy, choose occupation, exit at rate δ
- Preferences: risk-neutral
- Workers supply labor inelastically

Environment

- Entry technology: entry cost $n_0 w$, draw $s \sim G(s)$, where $s = (z, k)$
- Productivity, z
  - non-transferable
  - evolves according to $\mathrm{dz} = \mu z \mathrm{dt} + \sigma z \mathrm{d}\mathcal{B}$
- Business capital, k
  - built through investment: $\mathrm{dk} = \theta - \delta_k$, convex cost $C(\theta)$
  - bilaterally traded
- Production technology:

$$y(s, n) = z(s) k(s)^{\alpha} n^{\gamma}$$

  where n is a rentable input (today: labor)

Markets

Bilateral trade of capital:

- Firms access market at rate η
- Allocation between s and $\tilde{s}$: $k^m(s,\tilde{s}) \in \{k(s)+k(\tilde{s}),0\} \Rightarrow$ indivisibility
  (extension w/ costly divisibility)
- Price paid by s to $\tilde{s}$: $p^m(s,\tilde{s})$, negative if selling
  (extension w/ financing constraints: $p^m(s,\tilde{s}) \leq \xi y(s,n)$)

Spot market for labor:

- Labor demand for
  i) production: $n(s;w) = \left(\frac{\gamma z(s) k(s)^{\alpha}}{w}\right)^{\frac{1}{1-\gamma}}$
  ii) entry: n_0 workers per firm

Owner's Value

- The owner's value solves the following HJB

$$(r+\delta)V(s) = \underbrace{\max_n y(s,n) - wn}_{\text{production}} + \underbrace{\max_\theta \partial_k V(s)(\theta-\delta_k) - C(\theta)}_{\text{investment}}$$

$$+ \underbrace{\mu z \partial_z V(s) + \frac{1}{2}\sigma^2 z^2 \partial_{zz} V(s)}_{\text{evolution of productivity}} + \underbrace{\max_\lambda \eta W(s;\lambda)}_{\text{trade}}$$

where

$$W(s;\lambda) = \Sigma_{\tilde{s}}[V(z,k^m(s,\tilde{s})) - V(z,k) - p^m(s,\tilde{s})]\lambda(s,\tilde{s})$$

and

$$\Sigma_{\tilde{s}}\lambda(s,\tilde{s}) + \lambda(s,0) = 1$$

Free Entry and Law of Motion

- Occupational Choice (”free-entry”)

$$\int V(s)dG(s) - n_0 w \leq \frac{w}{r+\delta}, \quad \phi_e \geq 0, \quad \text{w/ c.s.}$$

- Labor supply in production: $N - n_0\phi_e$
- Distribution over the state space ϕ evolves according to the Kolmogorov Forward (KF) equation

$$\dot{\phi} = \Gamma(\theta, \lambda; \phi) + \phi_e$$

- Evolution of ϕ induced by
  - investment
  - trade
  - entry/exit
  - individual productivity process

Definition of Recursive Equilibrium

A (stationary) equilibrium is a set of value functions $V(s)$, policy functions for investment $\theta(s)$ and trade $\lambda(s, \tilde{s})$, terms of trade $(k^m(s, \tilde{s}), p^m(s, \tilde{s}))$, wage w, and distribution over the state space $\phi(s)$ that satisfy

- business owners' optimality
- no-arbitrage in occupational choice
- market clearing
- consistency of measures

Discussion of the Capital Trading Protocol

$\Rightarrow$ Trade of multiple differentiated goods

- Standard approach:
  - CES demand/monopolistic competition
  - frictional market with fixed point on matching set
- Our model:
  - frictionless matching: competitive equilibrium + stochastic trade opportunity
  - block structure, flexible trade module: $(\phi, V) \to (\lambda, p^m, k^m) \to (\phi', V')$

Trading Protocol: Implementation

- With transferable utility, solution is linear programming problem
  - maximize (static) social surplus s.t. adding up constraints
- Delivers equilibrium allocation (λ, k^m) and prices (p^m)
- Gains from trade $W(s)$ from envelope theorem

details

- Easy to extend to non-transferable utility environment

Properties of the Equilibrium

- **Competitive prices** are independent of seller's z

$$p^m(s, \tilde{s}) = \mathcal{P}(\kappa(\tilde{s}))$$

  Intuition: competitive nature of the equilibrium, same good sold at same price

- **Pairwise stability**: $\nexists(s, \tilde{s})$ and feasible trade that makes the pair (strictly) better off
- **Competitive allocation** solves the planner's problem starting at $\phi(s, 0) = \phi^{ss}(s)$

Using the Model

- Calibration using data on
    - firm dynamics
    - business transfers
- Model deliverables
    - dispersion in mpk
    - business price and value
- Tax Policy Analysis

Calibration

Parameter	**Value**
Discount rate	$r = 0.06$
Share of rentable input	$\gamma = 0.70$
Entry distribution, G	mass point at $z = z_0, k = 1$
Death rate, depreciation rate	$\delta = 0.1, \delta_k = 0.058$
Investment cost, $C(\theta) = A\theta^{\rho}$	$A = 25, \rho = 2$
Trading rate	$\eta = 1$
Returns to scale	$\alpha = 0.1$
Productivity process	$\mu_z = 0, \sigma_z = 0.25$

Identification Strategy

- Life-cycle firm dynamics $\Rightarrow$ productivity process, rentable input share, exit rate
- Transaction data $\Rightarrow$ production, investment, meeting technology

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- Transaction data $\Rightarrow$ production, investment, meeting technology

Key parameters

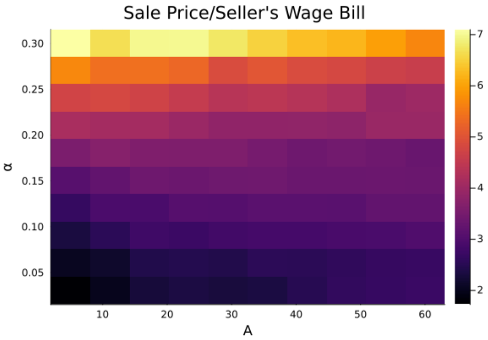
- meeting rate η
- investment cost $C(\theta) = A\theta^{\rho}$
- output elasticity wrt k, $y(z, k, n) = zk^{\alpha} n^{\gamma}$

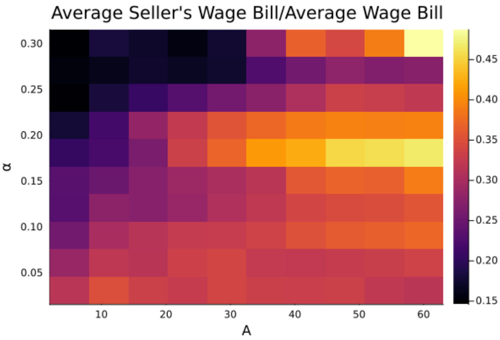
Key moments from data

- brokered sales: time to sell
- IRS filings
  - trade volume
  - sale price/wage bill
  - selection into selling

Identification of return to scale (α) and investment cost (A)

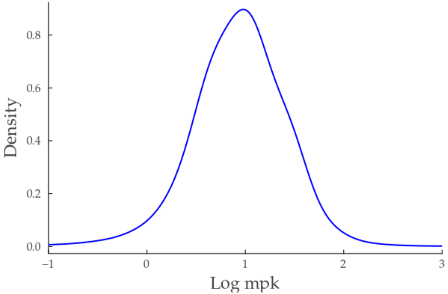
- $\alpha \uparrow \Rightarrow k$ share of seller's output $\uparrow \Rightarrow$ price to seller's wage bill, $\frac{p}{nw} \uparrow$
- $A \uparrow \Rightarrow$ quality of marginal seller $\uparrow \Rightarrow$ seller's wage bill, $\frac{nw}{\overline{nw}} \uparrow$





Dispersion in MPK

- Idiosyncratic change in productivity → input reallocation toward higher MPK
- Dispersion in marginal product of capital induced by
  - decentralized trading
  - indivisibility of asset sold
- Standard deviation of log-mpk: 45%



Business Wealth

- Finance textbook: Present value of owner's dividend
  - Model counterpart: $V(s)$
- SCF respondent: Answer to the survey question–"What could you sell it for?"
  - Model counterpart: $\mathcal{P}(k)$

Model Predictions for Business Wealth

- Heterogeneity in transferable share and returns
- Inputs to analysis of capital and wealth taxation

Distribution Pctile	Transferable Share	Income Yield
	$\frac{\mathcal{P}(k)}{V}$	$\frac{y-wn-C(\theta)}{V}$
5	0.00	-0.16
25	0.16	0.07
50	0.25	0.09
75	0.36	0.10
95	0.49	0.11
99	0.64	0.13

Business Taxation

- Recent debate on business taxation
- What to tax
  - flows: business income
  - stocks: business capital or wealth
  - transfers: capital gains
- Our model is well-suited to address this question

details

Tax Instruments and Welfare

Comparison of

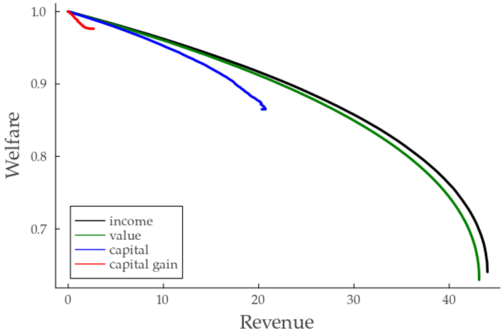
- **capital gains**: $\tau_c \mathcal{P}(k)$ [capital transaction]
- **business income**: $\tau_b(y - wn)$
- **business capital**: $\tau_k \mathcal{P}(k)$ [capital ownership]
- **wealth**: $\tau_v V$

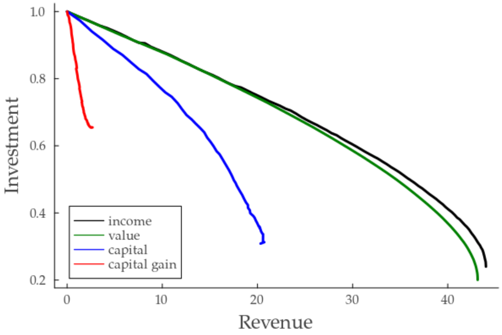
Welfare measure: steady-state value at entry conditional on raising revenue R

- by indifference at entry, all agents' ex-ante value is proportional to w

Main Results

- For most levels of R, exclusively use τ_b





Intuition

$\Rightarrow$ 3 margins: entry, investment, reallocation

Compared with tax on income,

- tax on capital gains
  - distorts capital reallocation across firms
  - decreases investment to sell
- tax on business capital plot
  - increases taxes on low z, high $k \Rightarrow$ investment $\downarrow$
  - lowers taxes on high z, low k (but z is inelastic!)
- tax on wealth $\approx$ tax on income + tax on option value of selling capital

Practical implementation: k and V are not observed

Next Steps

- Estimation using IRS data
  - life-cycle dynamics
  - production and investment technology
- Full study of tax policy
  - undiversifable risk
  - financial constraints
  - alternative instruments

Auxiliary Problem: Static Planner

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Lemma

- $W(s) = \frac{\partial Q}{\partial \phi(s)} = \frac{\mu^a(s)+\mu^b(s)}{2} \equiv \mu(s) \Rightarrow$ HJB
- $\lambda(s,\tilde{s}) = \frac{2\pi(s,\tilde{s})}{\phi(s)} \Rightarrow$ KF
- $k^m(s,\tilde{s}) = \arg\max X(s,\tilde{s}) \qquad p^m(s,\tilde{s}) = V(z,k^m(s,\tilde{s})) - V(z,k) - W(s) \Rightarrow$ Data

$$X(s,\tilde{s}) = \max_{k^m \in \{k(s)+k(\tilde{s}),0\}} \{V(z(s),k^m) + V(z(\tilde{s}),k(s)+k(\tilde{s})-k^m)\} - (V(s)+V(\tilde{s}))$$

$$Q(\phi) = \max_{\pi \geq 0} \Sigma_{s,\tilde{s}} X(s,\tilde{s})\pi(s,\tilde{s})$$

$$s.t. \quad \Sigma_{\tilde{s}}\pi(s,\tilde{s}) + \pi(s,0) = \frac{\phi(s)}{2} \quad \forall s \quad [\mu^a(s)]$$

$$\Sigma_{\tilde{s}}\pi(\tilde{s},s) + \pi(0,s) = \frac{\phi(s)}{2} \quad \forall s \quad [\mu^b(s)]$$

Dual Problem

back

- From the minimax thm, the solution of the primal problem is equal to the solution of the dual
- The multipliers in the primal are equal to the choice variable in the dual, and vice versa

$$Q(\phi) = \min_{\mu^a \geq 0, \mu^b \geq 0} \Sigma_s \left(\mu^a(s) + \mu^b(s) \right) \frac{\phi(s)}{2}$$

$$s.t. \;\; \mu^a(s) + \mu^b(\tilde{s}) \geq X(s, \tilde{s}) \;\; \forall s, \tilde{s} \quad [\pi(s, \tilde{s})]$$

Trade with Preference Shocks

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- After-trade values for buyers (v_b) and sellers (v_s)
  - $v_b(s, \hat{k}; p)$: value from buying $\hat{k}$
  - $v_s(s, 0; p)$: value from selling $k(s)$
- Matching probability

$$\lambda(s, \hat{k}; p) = \exp\left(\frac{v_b(s, \hat{k}; p) - W(s)}{\sigma}\right)$$

$$\lambda(s, 0; p) = \exp\left(\frac{v_s(s, 0; p) - W(s)}{\sigma}\right)$$

where $W(s) = \mathbb{E}\max\{v_b(s, \hat{k}; p), v_s(s, 0; p)\}$

- Find $\{p(s)\}$ such that $\forall \hat{k}$

$$\underbrace{\int \lambda(s, \hat{k}; p)}_{\text{demand}} = \underbrace{\int \lambda(s, 0; p)\mathbb{I}\{k(s) = \hat{k}\}}_{\text{supply}}$$

Price Cap and Taxes

- Under capital gain tax τ,

$$\begin{aligned} v_b(s;\hat{k}) &= V(z, k(s)+\hat{k}) - p(\hat{k}) \\ v_s(s) &= V(\tilde{s}, 0) + (1-\tau)p(k(s)) \end{aligned}$$

- Under cap on paid price equal to $\xi y(s,n)$

$$\begin{aligned} v_b(s;\hat{k}) &= \begin{cases} V(z, k(s)+\hat{k}) - p(\hat{k}) & \text{if } p(\hat{k}) \le \xi y(s,n) \\ -\infty & \text{o/w} \end{cases} \\ v_s(s) &= V(\tilde{s}, 0) + p(k(s)) \end{aligned}$$

Feasibility and Pair-wise stability

Terms of trade $\{p^m, k^m\}$ satisfy

- feasibility

$$
\begin{aligned}
&k^m(s,\tilde{s}) \in \{k(s) + k(\tilde{s}), 0\} \\
&k^m(s,\tilde{s}) + k^m(\tilde{s}, s) \leq k(s) + k(\tilde{s}) \\
&p(s,\tilde{s}) + p(\tilde{s}, s) \geq 0
\end{aligned}
$$

- pair-wise stability: $\nexists(s,\tilde{s})$ and feasible trade that makes the pair (strictly) better off

Taxation and Firm Entry

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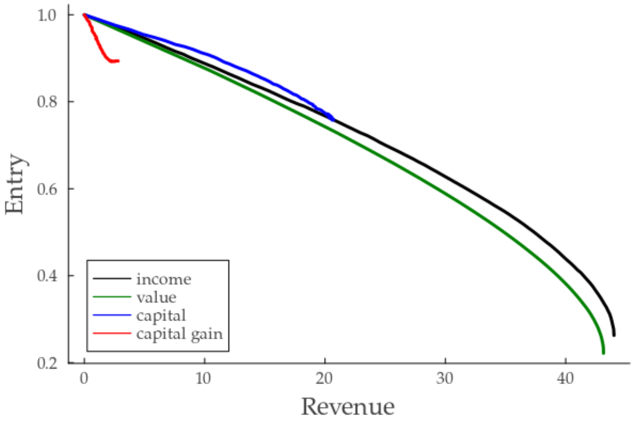

Understanding the Tax Results

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log-ratio of taxes paid: $\log\left(\frac{\tau_b(y-wn)}{\tau_k \mathcal{P}(k)}\right)$