A Theory of Business Transfers

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Motivation

- Privately-owned firms
  - Account for 1/2 of US business net income
  - Relevant for growth, wealth, tax policy/compliance
- But pose challenge for theory and measurement

This Paper

- Proposes theory of firm dynamics and capital reallocation
- Characterizes properties of competitive equilibrium
- Uses administrative IRS data to discipline theory
- Studies transfers, wealth, and impact of capital gains tax

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- Characterizes properties of competitive equilibrium

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- Studies transfers, wealth, and impact of capital gains tax

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Private Business Capital: What is Known?

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Form **8594** (Rev. November 2021) Department of the Treasury Internal Revenue Service

Asset Acquisition Statement
Under Section 1060

▶ Attach to your income tax return.
▶ Go to *www.irs.gov/Form8594* for instructions and the latest information.

OMB No. 1545-0074
Attachment Sequence No. **169**

Name as shown on return	Identifying number as shown on return

Check the box that identifies you:
☐ Purchaser ☐ Seller

Part I General Information

1 Name of other party to the transaction	Other party's identifying number

Address (number, street, and room or suite no.)

City or town, state, and ZIP code

2 Date of sale	3 Total sales price (consideration)

Part II Original Statement of Assets Transferred

4 Assets	Aggregate fair market value (actual amount for Class I)	Allocation of sales price
Class I	$	$
Class II	$	$
Class III	$	$
Class IV	$	$
Class V	$	$
Class VI and VII	$	$
Total	$	$

← Cash/securities
← Inventories
← Fixed assets
← Sec. 197 intangibles

5 Did the purchaser and seller provide for an allocation of the sales price in the sales contract or in another written document signed by both parties? ☐ Yes ☐ No

If "Yes," are the aggregate fair market values (FMV) listed for each of asset Classes I, II, III, IV, V, VI, and VII the amounts agreed upon in your sales contract or in a separate written document? ☐ Yes ☐ No

6 In the purchase of the group of assets (or stock), did the purchaser also purchase a license or a covenant not to compete, or enter into a lease agreement, employment contract, management contract, or similar arrangement with the seller (or managers, directors, owners, or employees of the seller)? ☐ Yes ☐ No

If "Yes," attach a statement that specifies **(a)** the type of agreement and **(b)** the maximum amount of consideration (not including interest) paid or to be paid under the agreement. See instructions.

Private Business Capital: What is Known?

- Transferred assets are primarily intangible

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  $\Rightarrow$ evidence in IRS Forms 8594, 8883 data
  shows intangible, non-liquid share is $\approx 60\%$

Private Business Capital: What is Known?

- Transferred assets are primarily intangible
  - Customer bases and client lists
  - Non-compete covenants
  - Licenses and permits
  - Franchises, trademarks, tradenames
  - Workforce in place
  - IT and other know-how in place
  - Goodwill and on-going concern value

$\Rightarrow$ Classified as *Section 197 intangibles* by IRS

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    $\Rightarrow$ evidence in brokered sale data is $\approx$ 290 days

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  - Exchanged after timely search and brokered deals

$\Rightarrow$ Existing models unsuitable for studying business transfers

Today's Talk

- Study firm dynamics
- Characterize competitive equilibrium
- Estimate wealth and impact of capital gains tax

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Today's Talk

- Study firm dynamics with
  - Indivisible capital
  - Bilaterally traded
  - Requiring time to reallocate
- Characterize competitive equilibrium
  - Who trades with whom?
  - How are terms of trade determined?
  - What are the properties?
- Estimate wealth and impact of capital gains tax

Theory

Environment: A Helicopter View

- Infinite horizon with discrete time
- Preferences: (for today) owners are risk-neutral
- Technology:
  - Firms indexed by $s = (z, \kappa)$
  - Produce $y(s) = z(s)\kappa(s)^{\alpha} = \max_n \hat{z}(s)\kappa(s)^{\hat{\alpha}} n^{\gamma} - wn$

    z: non-transferable capital with $z'|z$ exogenous

    κ: transferable capital

    n: all external rented factors
  - Investment: $\theta = P\{\kappa(s') = \kappa(s) + 1\}$ at cost $C(\theta)$
- Birth/death: draw from $G(s)$ at cost c_e and die at rate δ

Timing of Decisions

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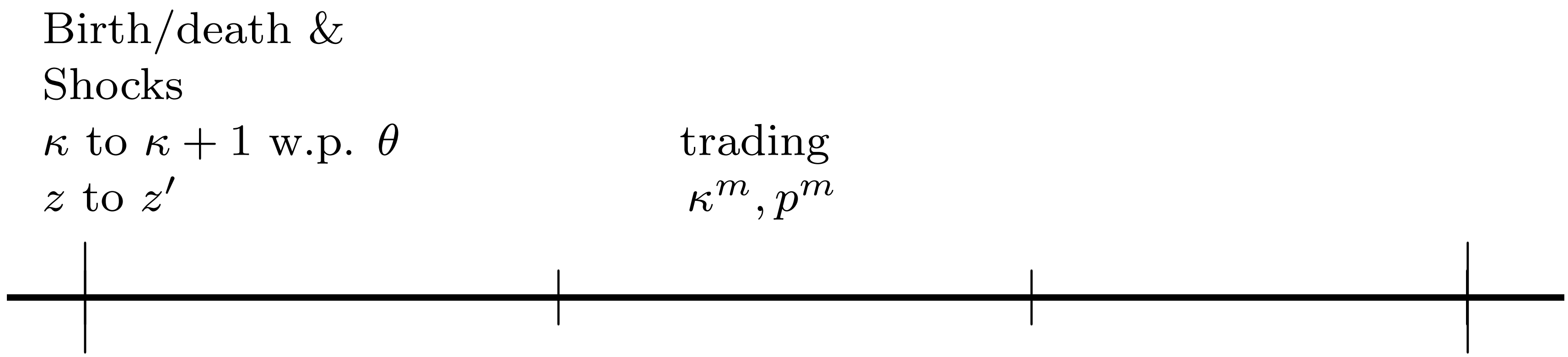
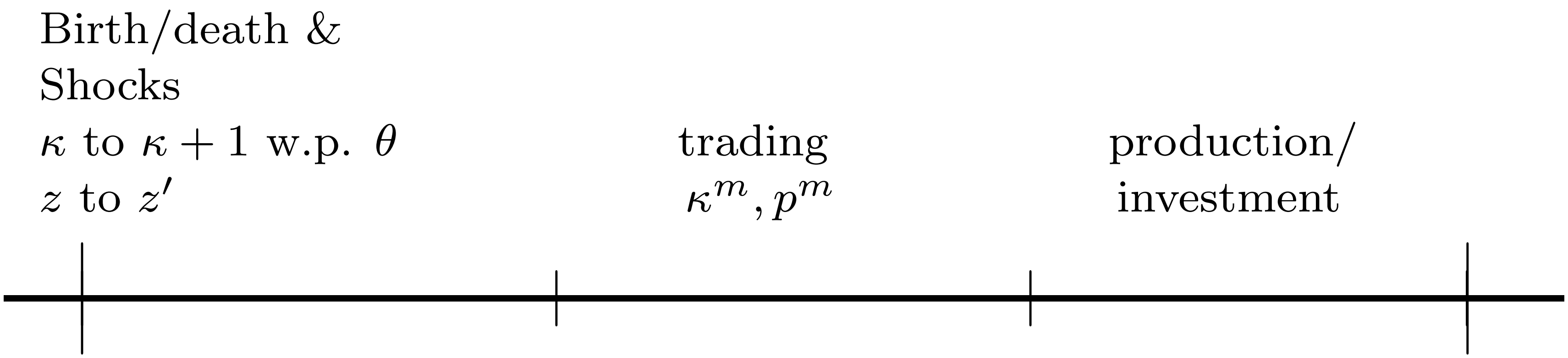
Birth/death &
Shocks
κ to $\kappa + 1$ w.p. θ
z to z'

Timing of Decisions



- Terms of trade for pair $(s, \tilde{s})$
  - Allocations: $\kappa^m(s, \tilde{s})$ is post-trade capital for s
  - Prices: $p^m(s, \tilde{s})$ is payment by s to $\tilde{s}$

Timing of Decisions



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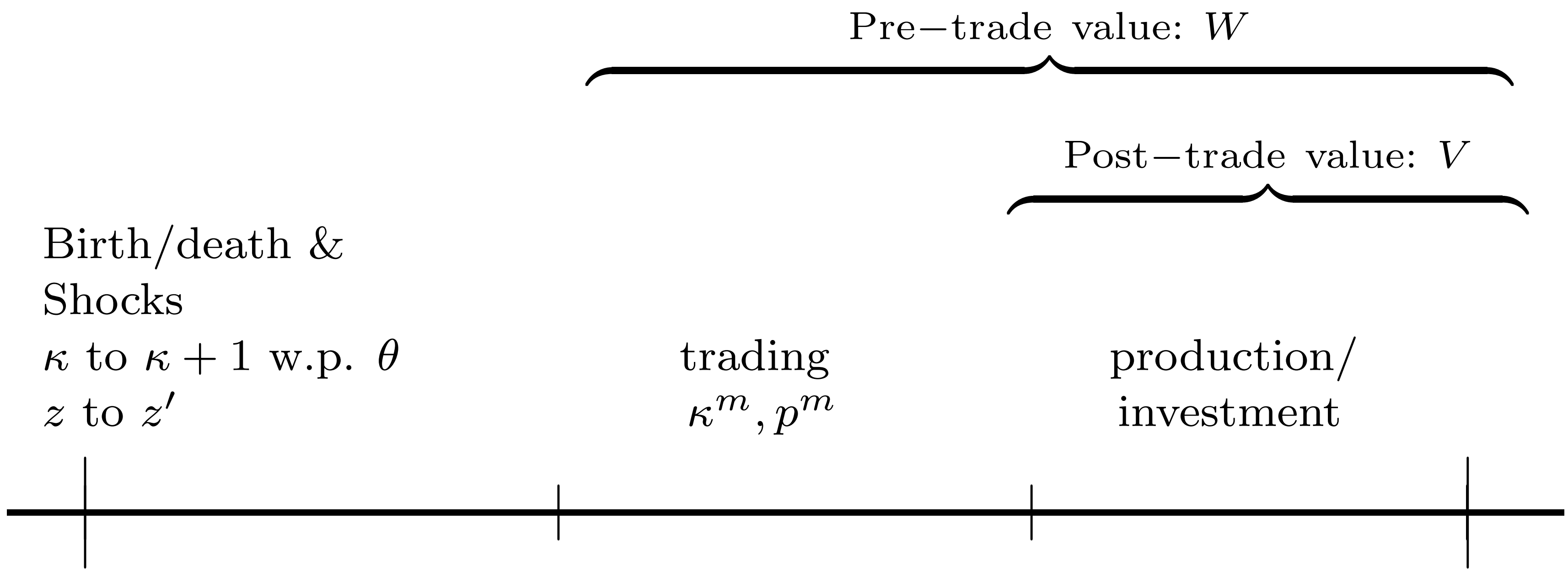

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Dynamic Program of Incumbent Firms

- Given prices and allocations $\{p^m(s,\tilde{s}), \kappa^m(s,\tilde{s})\}_{s,\tilde{s}}$

- Compute values:

$$V(s) = \max_{\theta \in [0,1]} \; z(s)\kappa(s)^\alpha - C(\theta) + (1-\delta)\beta\, \mathbb{E} W(s')$$

$$W(s') = \max_{\substack{\lambda(\tilde{s}) \geq 0 \\ \lambda_o \geq 0}} \int \underbrace{\left[V(z(s'), \kappa^m(s',\tilde{s})) - p^m(s',\tilde{s})\right] \lambda(\tilde{s})}_{\text{value of trading with } \tilde{s}} + \underbrace{V(s')\,\lambda_o}_{\text{being alone}}$$

where $\{\lambda(\cdot), \lambda_o\}$ are probabilities over trading options

Aggregate Measures

- Measures:
  - $\phi(s)$: firms of type s
  - $\phi_e(s)$: entrants of type s
  - $\Lambda(s, \tilde{s}) = \lambda(\tilde{s}|s)\phi(s)$: matches between s,$\tilde{s}$
  - $\Lambda_o(s) = \lambda_o(s)\phi(s)$: unmatched firms of type s
- Law of motion for ϕ:

$$\phi'(s) = \Gamma(\phi; \lambda, \lambda_o, \theta, \phi_e, k^m)$$

Recursive Equilibrium with Pairwise Stability

$$\text{Objects: } \{ \underbrace{V, W,}_{\substack{\text{value}\\ \text{functions}}} \underbrace{\kappa^m, p^m,}_{\substack{\text{terms of}\\ \text{trade}}} \underbrace{\phi, \Lambda, \Lambda_o, \phi_e}_{\text{measures}} \}$$

such that

1. firms optimize and entrants make zero profits
2. bilateral trades are feasible and pairwise stable
3. measures are consistent with decisions and stationarity

Conditions 1) and 3) are standard. Next, consider 2)

Feasibility and Pairwise Stability

- Terms of trade satisfy
  - Feasibility:

$$\kappa^m(s,\tilde{s}) + \kappa^m(\tilde{s},s) \leq \kappa(s) + \kappa(\tilde{s})$$
$$p^m(s,\tilde{s}) + p^m(\tilde{s},s) \geq 0$$

$$\text{where} \quad \kappa^m(s,\tilde{s}) \in \Big\{ \underbrace{\kappa(s)+\kappa(\tilde{s})}_{\text{buy}}, \underbrace{\kappa(s)}_{\text{no trade}}, \underbrace{0}_{\text{sell}} \Big\}$$

  - Pairwise stability:

    $\nexists$ feasible trade for $(s,\tilde{s})$ increasing pair's welfare

Back to RE Definition

$$\text{Objects: } \{ \underbrace{V, W,}_{\substack{\text{value}\\\text{functions}}} \underbrace{\kappa^m, p^m,}_{\substack{\text{terms of}\\\text{trade}}} \underbrace{\phi, \Lambda, \Lambda_o, \phi_e}_{\text{measures}} \}$$

such that

1. V, W solve firms problems and entrants make zero profits
2. κ^m, p^m are feasible and pairwise stable
3. $\phi, \Lambda, \Lambda_o, \phi_e$ satisfy for all $A \subseteq \mathcal{S}$, $m \geq 0$:

$$\phi(A) = \int \Lambda(ds \in A, d\tilde{s} \in \mathcal{S}) + \Lambda_o(ds \in A)$$

$$\phi(A) = \int \Lambda(d\tilde{s} \in \mathcal{S}, d\tilde{s} \in A) + \Lambda_o(ds \in A)$$

$$\phi_e(A) = G(ds \in A)m$$

$$\phi'(A) = \Gamma(\phi; \lambda, \lambda_o, \theta, \phi_e, k^m)(A)$$

Discussion

- Relative to models with
  - CES demand/ monopolistic competition
  - Frictional labor or asset markets
- Framework delivers (with few a priori restrictions)
  - Differentiated goods
  - Rich heterogeneity in market participants
  - Endogenously evolving matching sets

Characterizing Equilibria

Who Trades with Whom?

- Intuitive example:
  - Productivity types: 20 with $z_H = 1$, 10 with $z_L = 0$
  - Capital pre-trade: all have $\kappa = 1$
- Efficient reallocation:
  - 10 low types sell to 10 of the high types

How are Terms of Trade Determined?

- Intuitive example:
  - Productivity types: 20 with $z_H = 1$, 10 with $z_L = 0$
  - Capital pre-trade: all have $\kappa = 1$
- Price leaves high types indifferent between:
  - Trading, with $\kappa = 2$ post-trade
  - Not trading, with $\kappa = 1$ post-trade

Equilibrium Policy Functions

- Intuitive example:
  - Productivity types: 20 with $z_H = 1$, 10 with $z_L = 0$
  - Capital pre-trade: all have $\kappa = 1$
- Capital allocations: $k^m(s_H, s_L) = 2, k^m(s_L, s_H) = 0$
- Prices: $p^m(s_H, s_L) = 1, p^m(s_L, s_H) = -1$
- Choice probabilities:

$$\lambda(s_H|s_L) = 1,\ \lambda(s_L|s_H) = 1/2,\ \lambda_o(s_L) = 0,\ \lambda_o(s_H) = 1/2$$

More Generally Given (ϕ, V)

- Who trades with whom?
  - Solve assignment problem maximizing total gains
- How are terms of trade determined?
  - Compute shadow prices from assignment problem
- Can solve dynamic program iteratively
  - Update: $(\phi, V) \rightarrow$ equilibrium objects $\rightarrow (\phi, V)$

Monge-Kantorovich Assignment Problem

- Imagine splitting our businesses in two

$$5\left\{\begin{matrix} z_L & z_L \\ z_L & z_L \\ z_L & z_L \\ z_L & z_L \\ z_L & z_L \end{matrix}\right\}5$$

$$10\left\{\begin{matrix} z_H & z_H \\ z_H & z_H \\ z_H & z_H \\ \vdots & \vdots \\ z_H & z_H \end{matrix}\right\}10$$

Monge-Kantorovich Assignment Problem

$$Q(\phi, V) = \max_{\substack{\pi_{s,\tilde{s}} \geq 0 \\ \pi_o, \tilde{\pi}_o \geq 0}} \int X(s, \tilde{s})\pi_{s,\tilde{s}}(ds, d\tilde{s}) + V(s)\pi_o(ds) + V(\tilde{s})\tilde{\pi}_o(d\tilde{s})$$

$$s.t. \int \pi_{s,\tilde{s}}(ds \in A, d\tilde{s} \in \mathcal{S}) + \pi_o(ds \in A) = \phi(A)/2$$

$$\int \pi_{s,\tilde{s}}(ds \in \mathcal{S}, d\tilde{s} \in A) + \tilde{\pi}_o(ds \in A) = \phi(A)/2$$

where the gains to trade are

$$X(s, \tilde{s}) = \max\{\underbrace{V(z(s), \kappa(s)+\kappa(\tilde{s})),}_{s \text{ buys}}\underbrace{V(s)+V(\tilde{s}),}_{\text{no trade}}\underbrace{V(z(\tilde{s}), \kappa(s)+\kappa(\tilde{s}))}_{\tilde{s} \text{ buys}}\}$$

Role of MK Lagrange Multipliers

- Multipliers $\mu = \mu^a = \mu^b$ capture gains from trade

$$\mu = \nabla_\phi Q$$

- Prices implement optimal gains from trade:

$$\underbrace{\mu(s)}_{\text{social}} = \underbrace{V(z(s), k^m(s,\tilde{s})) - p^m(s,\tilde{s})}_{\text{=private gains}}$$

- Updates of ϕ, V are easy to compute:

$$V(s) = \max\ y(s) - C(\theta) + (1-\delta)\beta\,\mathbb{E}\mu(s')$$

$$\phi'(s) = \Gamma(\phi; \pi, \pi_o, \theta, \phi_e, k^m)$$

Properties of Equilibrium

- Competitive allocations maximize

$$\sum_t \beta^t \int \phi_t(s)[y(s) - C(\theta(s)) - m_t c_e]$$

- Competitive prices independent of z

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$$\sum_t \beta^t \int \phi_t(s)[y(s) - C(\theta(s)) - m_t c_e]$$

- Competitive prices independent of z, eg,

$$p^m(\tilde{s}, s) = V(z(\tilde{s}), \kappa(s) + \kappa(\tilde{s})) - \mu(\tilde{s})$$

$$p^m(\tilde{s}, s') = V(z(\tilde{s}), \kappa(s') + \kappa(\tilde{s})) - \mu(\tilde{s})$$

$\Rightarrow$ $p^m(\tilde{s}, s')$ depends on κ but not z

Properties of Equilibrium

- Competitive allocations maximize

$$\sum_t \beta^t \int \phi_t(s)[y(s) - C(\theta(s)) - m_t c_e]$$

- Competitive prices independent of z

$$p^m(s, \tilde{s}) = \mathcal{P}(\kappa(s))$$

Quantitative Results

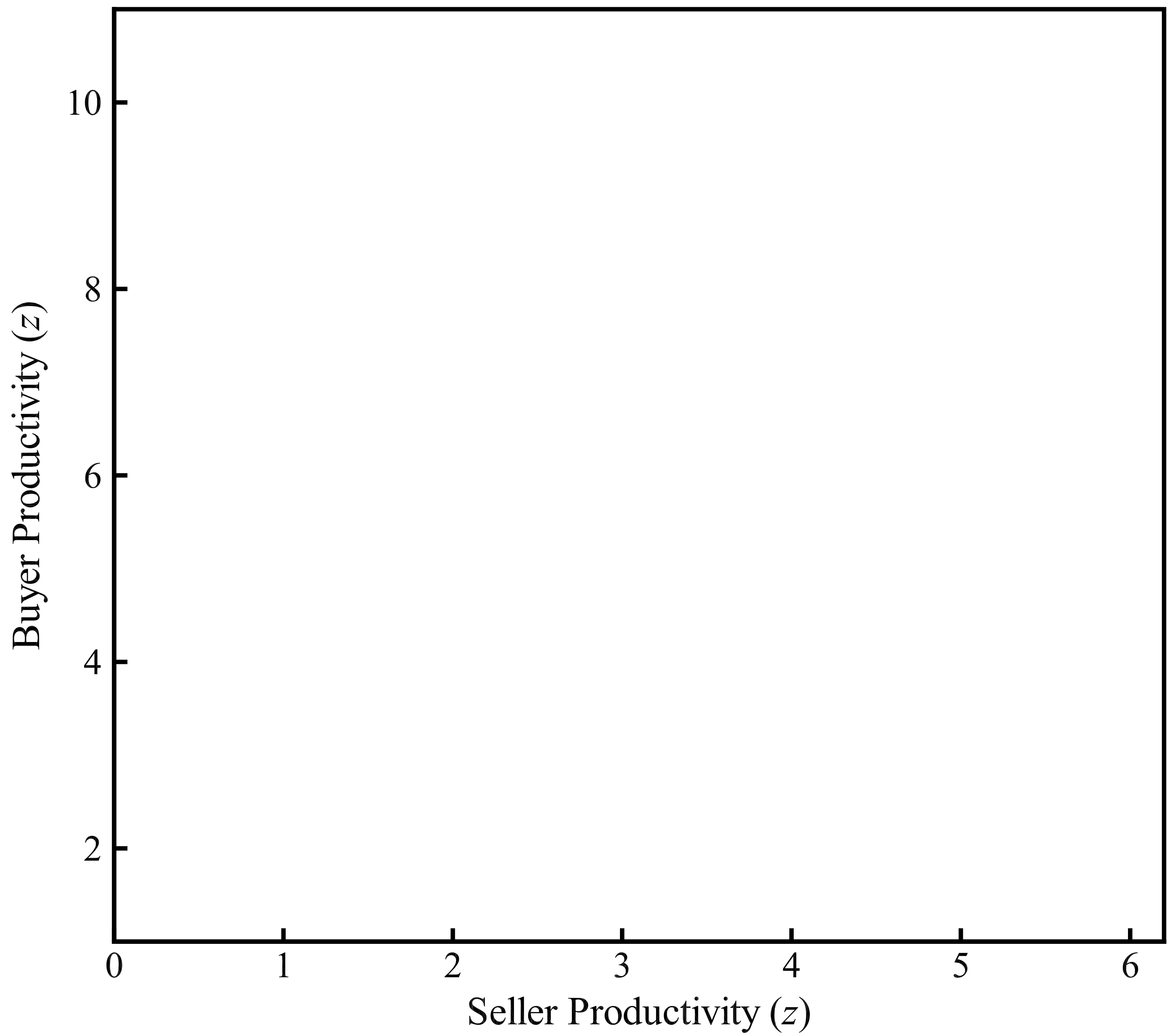
Model Parameters

Description	Values
Returns to scale	$\alpha = 0.50$
Discount rate	$\beta = 0.95$
Investment cost, $C(\theta) = A\theta^{\rho}$	$A = 10, \rho = 2.0$
Productivity, $z'\|z$ AR(1)	$\rho_z = 0.90, \sigma_z = 0.30$
Entrant distribution, Zipf(z)	tail $= 1.20$
Death rate	$\delta = 0.20$

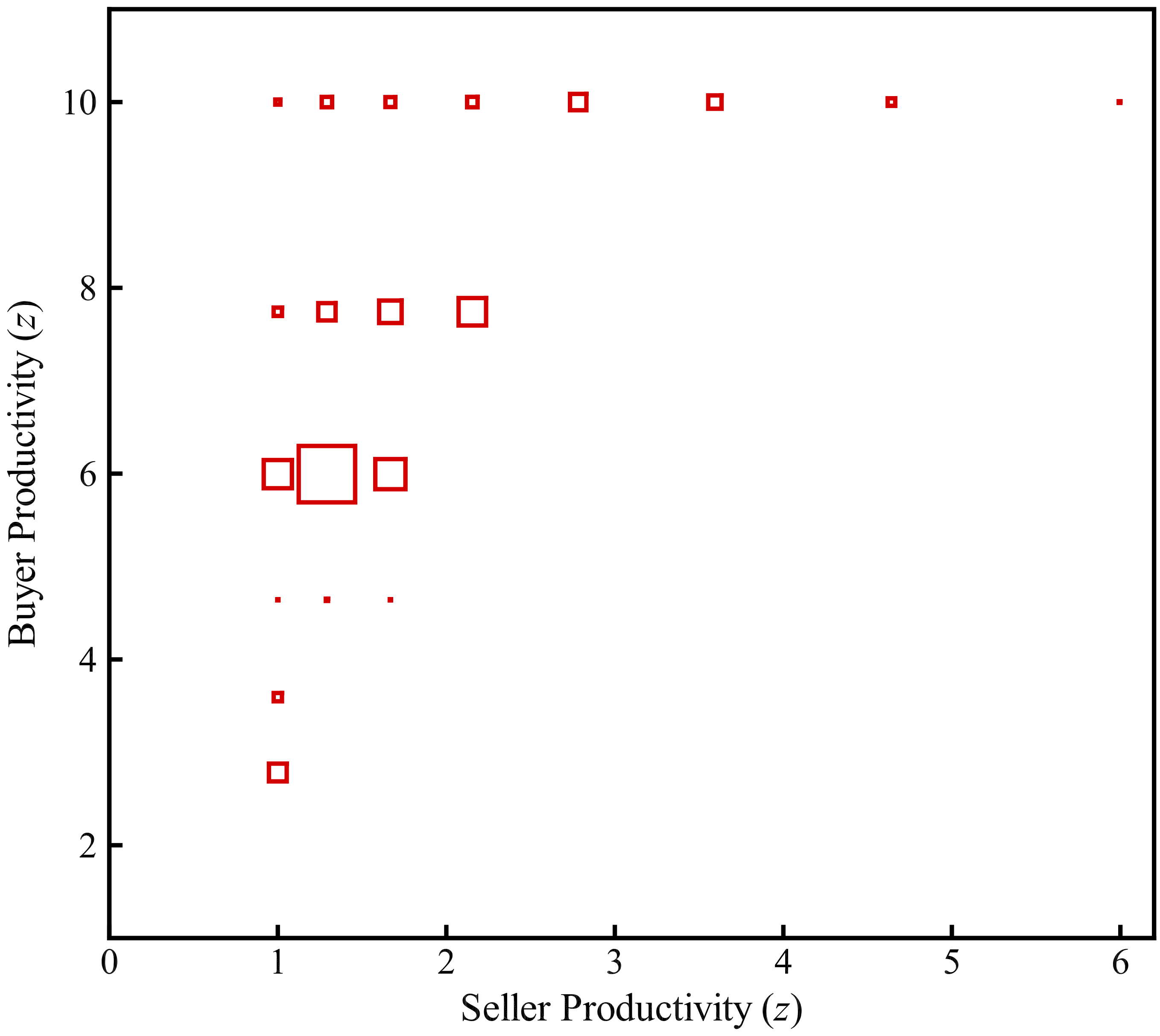
Patterns of Trade

- Statistics to be matched to IRS data:
  - Roughly 4% of κ units traded each period
  - Price is 4 to 7 times seller's income
  - Buyer's income is 2 to 4 times seller's income
- Who trades with whom?

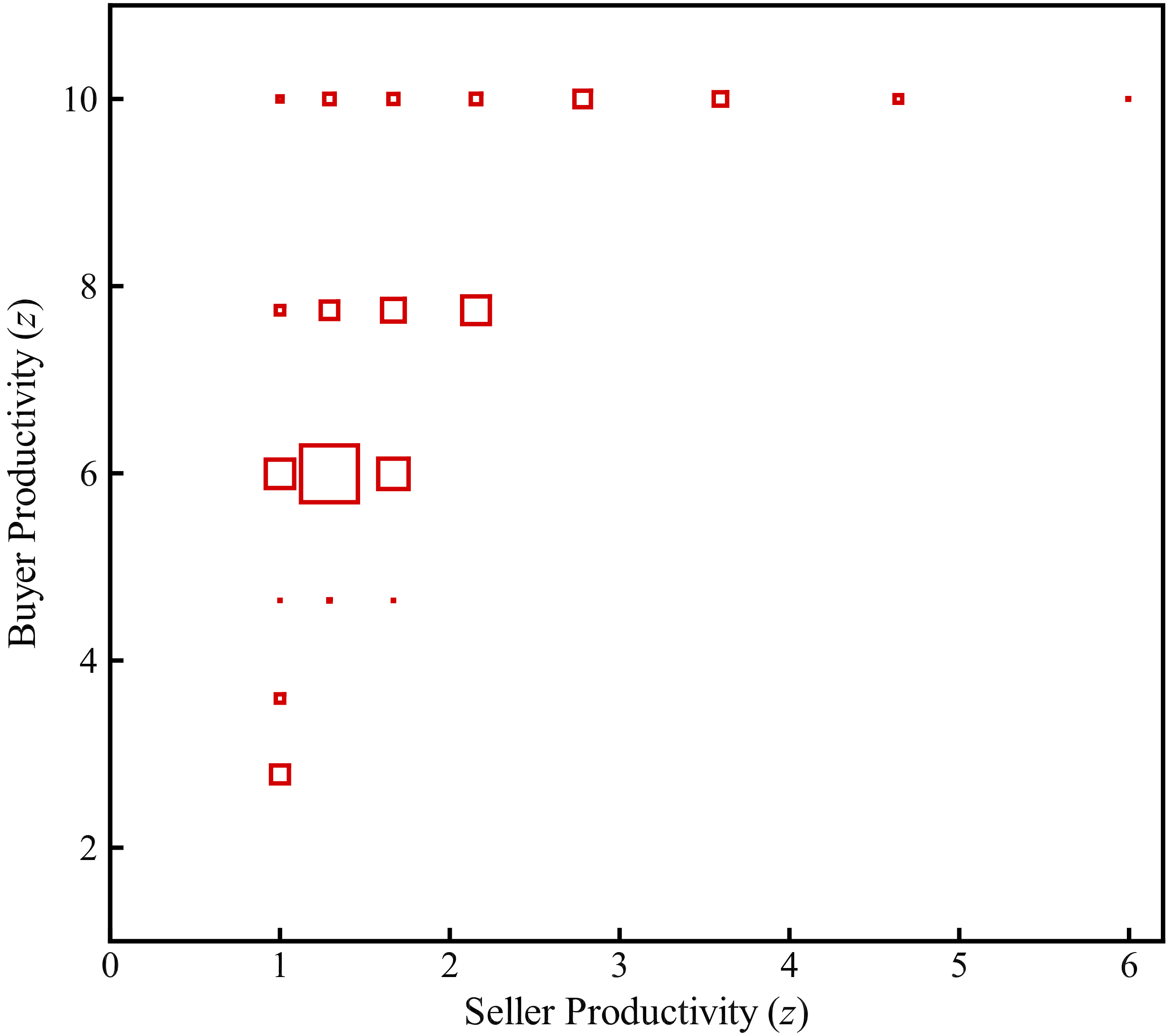
Patterns of Trade



Patterns of Trade



Capital Trades Upward in MPK Sense

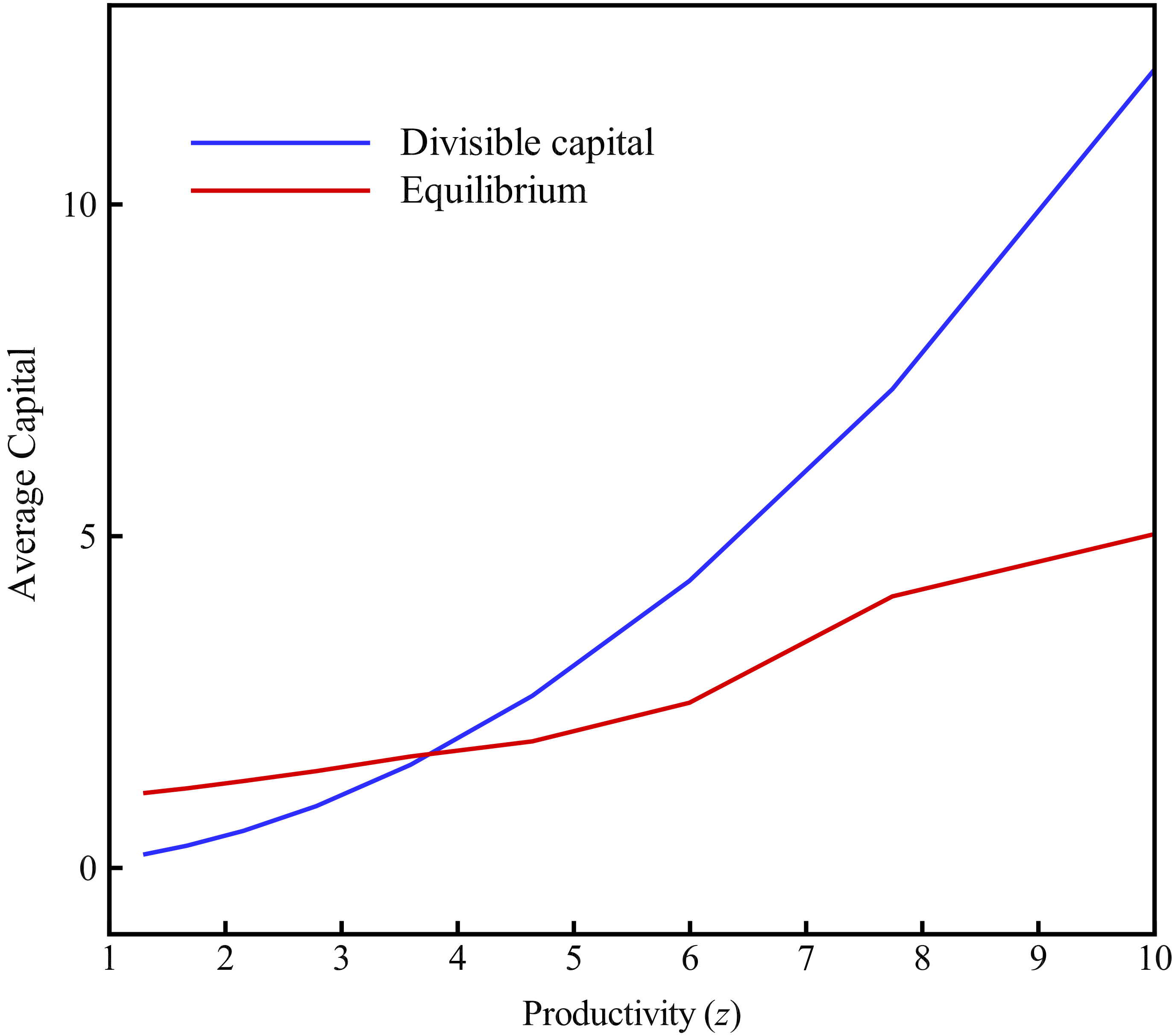


Allocation of Capital

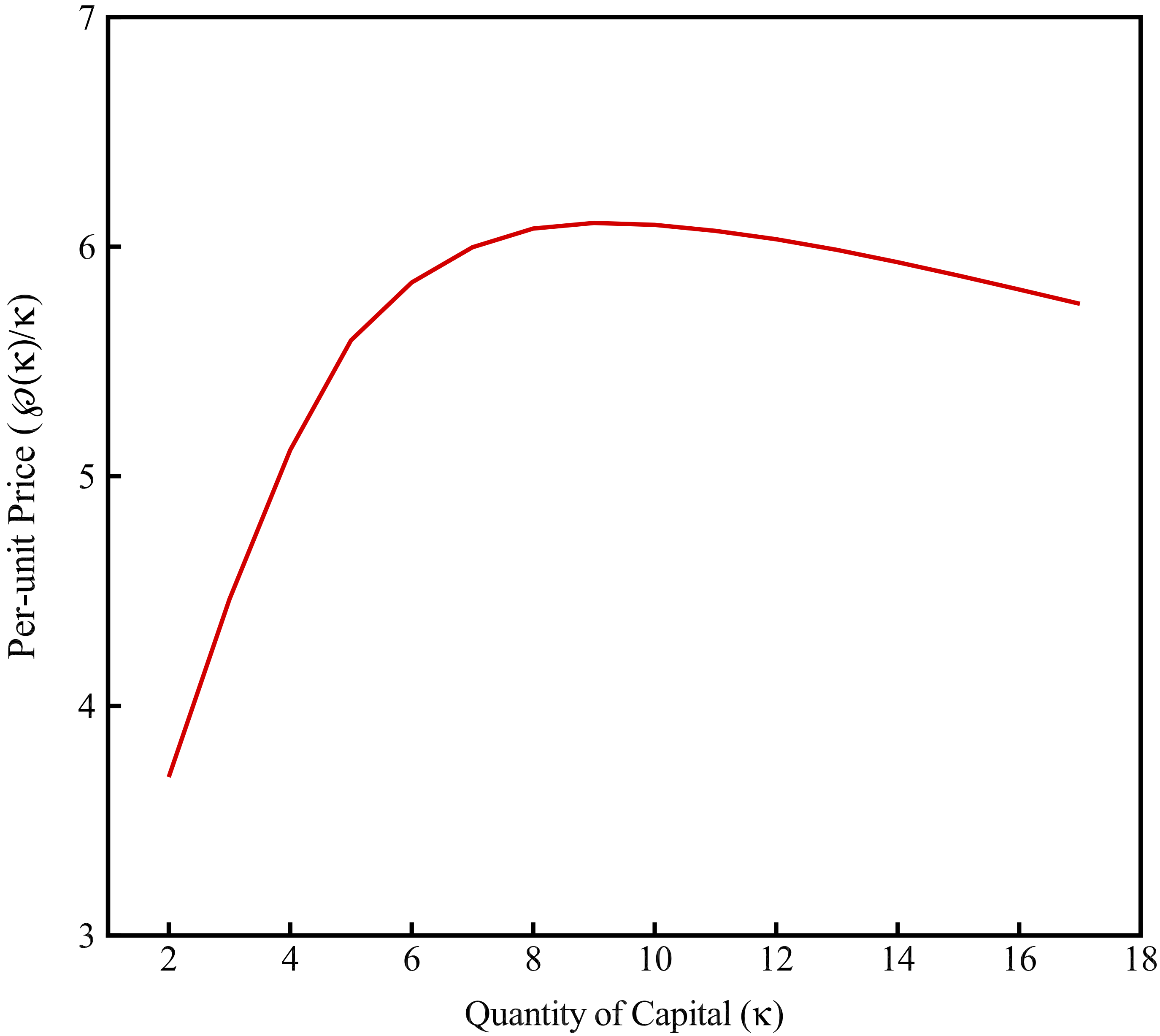
- Compare to "misallocation" literature benchmark
  - Divisible versus indivisible capital
  - Rental versus no rental markets
- Compute *first-best*:

$$\kappa^{FB}(s) \in \operatorname{argmax} \int z(s)[\kappa^{FB}(s)]^{\alpha}\phi(s)ds$$
$$\int \phi(s)\kappa^{FB}(s)ds = \int \phi(s)\kappa(s)ds$$

Dispersion in MPKs without Frictions



Dispersion in Prices without Frictions



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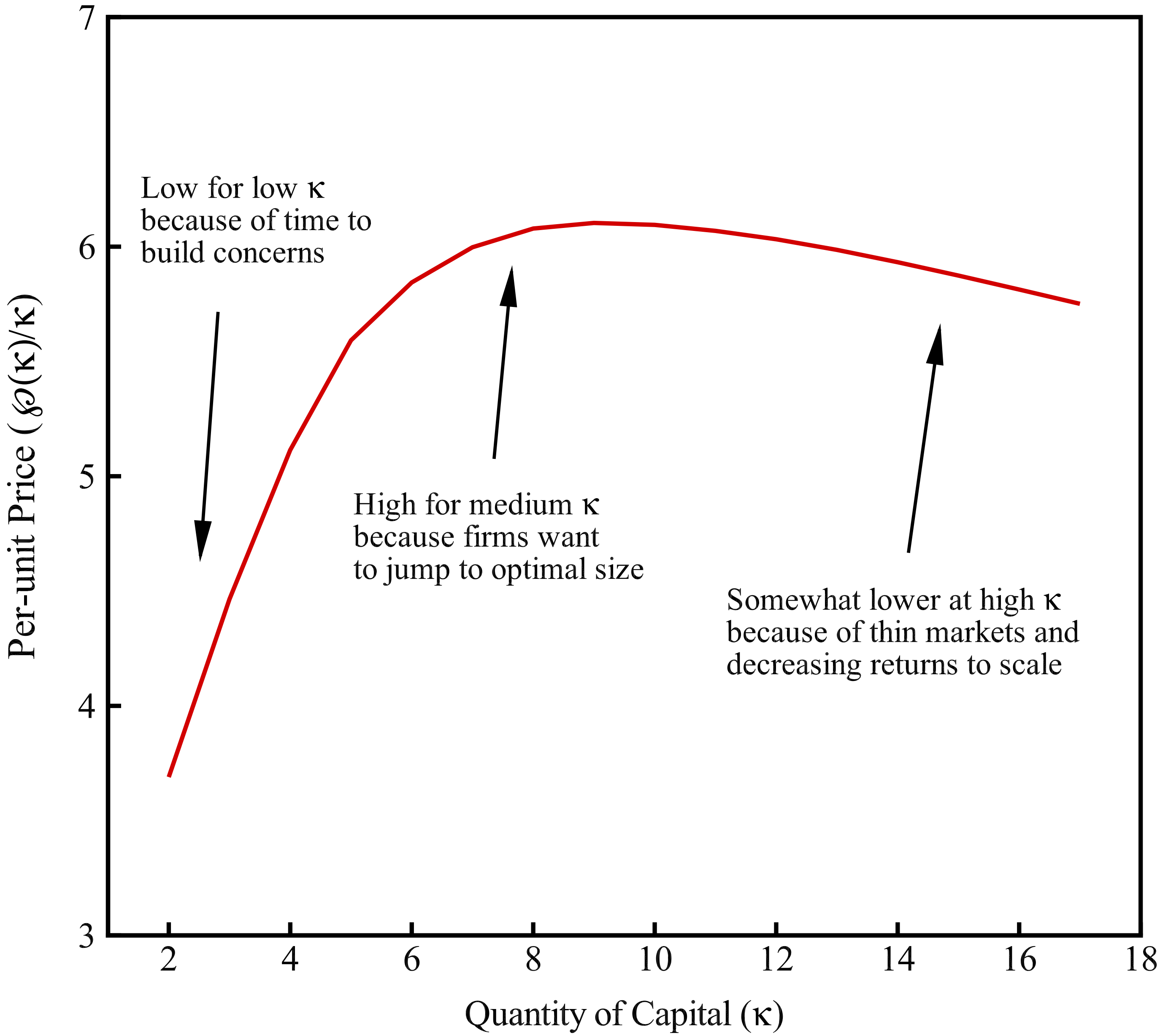


Estimating Business Wealth

- Finance textbook: present value of owner dividends
- SCF survey: price if sold business today
- Both have clear model counterparts

Estimating Business Wealth

- Finance textbook: present value of owner dividends, $V(s)$
- SCF survey: price if sold business today, $\mathcal{P}(\kappa(s))$
- Both have clear model counterparts

Estimating Business Wealth

Productivity Level (z)	Transferable Share $\mathcal{P}(\kappa(s))/V(s)$	Income Yield $[y(s) - C(\theta(s))]/V(s)$

Estimating Business Wealth

Productivity Level (z)	Transferable Share $\mathcal{P}(\kappa(s))/V(s)$	Income Yield $[y(s) - C(\theta(s))]/V(s)$
1.00	0.54	0.13
1.29	0.47	0.14
1.67	0.42	0.16
2.15	0.37	0.17
2.78	0.34	0.19
3.59	0.31	0.20
4.64	0.32	0.21
5.99	0.41	0.23
7.74	0.38	0.24
10.0	0.33	0.23
Avg	0.43	0.17

Taxing Capital Gains

Capital Gains Tax

- Introduce tax τ on gains
  - Seller receives $(1-\tau)p^m(s,\tilde{s})$
  - Government receives $\tau p^m(s,\tilde{s})$
- Use tricks to handle nontransferable utility case

Effects of Tax

- Fewer trades (obvious)
  - Tax eliminates trades where gains are small
- Heterogeneity in tax incidence
  - Larger on buyer if transacted quantity small
  - Larger on seller if transacted quantity large

Eliminates Small-Gain Trades

- With tax, find larger distance between buyers/sellers
- For example, ratio of MPKs of buyer to seller:

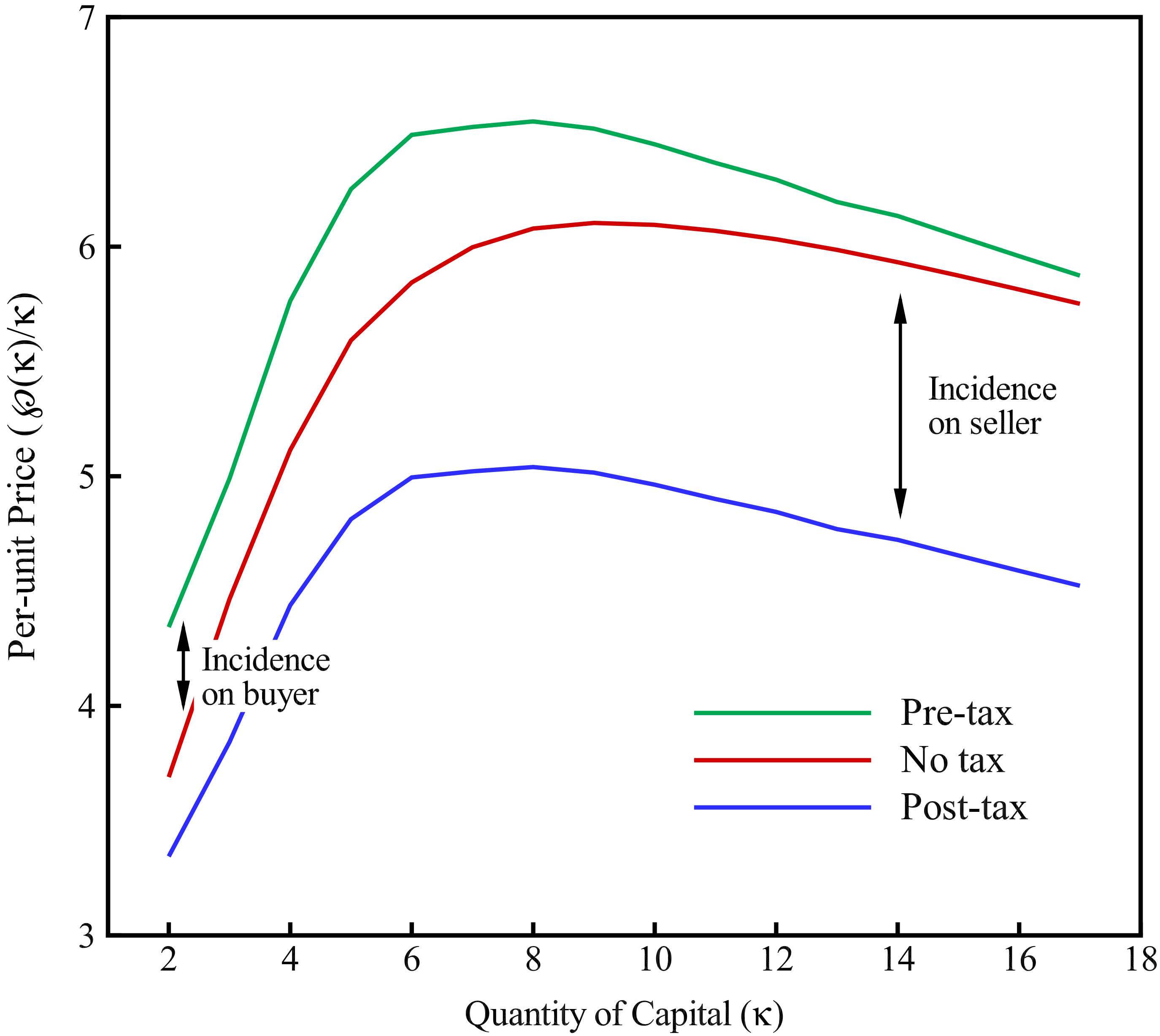
Moments	$\tau = 0\%$	$\tau = 20\%$
Mean		
Standard deviation		
5^{th} percentile		
25^{th}		
50^{th}		
75^{th}		
95^{th}		

Eliminates Small-Gain Trades

- With tax, find larger distance between buyers/sellers
- For example, ratio of MPKs of buyer to seller:

Moments	$\tau = 0\%$	$\tau = 20\%$
Mean	8.2	10.7
Standard deviation	1.8	1.7
5^{th} percentile	5.9	8.0
25^{th}	7.0	9.5
50^{th}	8.0	10.4
75^{th}	9.3	12.0
95^{th}	12.0	13.4

Heterogeneity in Tax Incidence



Next Steps

- Theory: add curvature and financing constraints
- Estimation: continue work with IRS data
- Applications: continue work studying capital taxation

Appendix: Galichon-Kominers-Weber

Galichon-Kominers-Weber Tricks

- Without capital gains tax
  - Labeling buyers/sellers a priori not necessary
  - Exploiting symmetry possible with MK
- With capital gains tax
  - Labeling buyers/sellers a priori is necessary
  - Exploiting MK requires complicated outer loop
- GKW's trick is to introduce small "preference shocks"
  - All types are buyers and sellers
  - Numerical objects are equations not inequalities