A Theory of Business Transfers

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Motivation

- Privately-owned firms
  - Account for 1/2 of US business net income
  - Relevant for growth, wealth, tax policy/compliance
- But pose challenge for theory and measurement

This Paper

- Proposes theory of firm dynamics and capital reallocation
- Characterizes properties of competitive equilibrium
- Uses administrative IRS data to discipline theory
- Studies transfers, wealth, and impact of capital gains tax

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- Characterizes properties of competitive equilibrium

† Uses administrative IRS data to discipline theory

- Studies transfers, wealth, and impact of capital gains tax

† Still in progress

Private Business Capital: What is Known?

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- Transferred assets are primarily intangible

  $\Rightarrow$ evidence in IRS Forms 8594, 8883 data shows intangible share is $\approx 60\%$

Private Business Capital: What is Known?

Form **8594** (Rev. November 2021) Department of the Treasury Internal Revenue Service

Asset Acquisition Statement Under Section 1060

▶ Attach to your income tax return.

▶ Go to *www.irs.gov/Form8594* for instructions and the latest information.

OMB No. 1545-0074

Attachment Sequence No. **169**

Name as shown on return | Identifying number as shown on return

Check the box that identifies you:

☐ Purchaser ☐ Seller

Part I General Information

1 Name of other party to the transaction | Other party's identifying number

Address (number, street, and room or suite no.)

City or town, state, and ZIP code

2 Date of sale | 3 Total sales price (consideration)

Part II Original Statement of Assets Transferred

4 Assets	Aggregate fair market value (actual amount for Class I)	Allocation of sales price
Class I	$	$
Class II	$	$
Class III	$	$
Class IV	$	$
Class V	$	$
Class VI and VII	$	$
Total	$	$

← Cash/securities
← Inventories
← Fixed assets
← Sec. 197 intangibles

5 Did the purchaser and seller provide for an allocation of the sales price in the sales contract or in another written document signed by both parties? ☐ Yes ☐ No

If "Yes," are the aggregate fair market values (FMV) listed for each of asset Classes I, II, III, IV, V, VI, and VII the amounts agreed upon in your sales contract or in a separate written document? ☐ Yes ☐ No

6 In the purchase of the group of assets (or stock), did the purchaser also purchase a license or a covenant not to compete, or enter into a lease agreement, employment contract, management contract, or similar arrangement with the seller (or managers, directors, owners, or employees of the seller)? ☐ Yes ☐ No

If "Yes," attach a statement that specifies **(a)** the type of agreement and **(b)** the maximum amount of consideration (not including interest) paid or to be paid under the agreement. See instructions.

Private Business Capital: What is Known?

- Transferred assets are primarily intangible
  - Customer bases and client lists
  - Non-compete covenants
  - Licenses and permits
  - Franchises, trademarks, tradenames
  - Workforce in place
  - IT and other know-how in place
  - Goodwill and on-going concern value

$\Rightarrow$ Classified as *Section 197 intangibles* by IRS

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    $\Rightarrow$ evidence in seller's business tax filings shows little activity after sale

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  - Exchanged after timely search and brokered deals

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    $\Rightarrow$ evidence in brokered sale data is $\approx$ 290 days

Private Business Capital: What is Known?

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  - Sold as a group that makes up a business
  - Exchanged after timely search and brokered deals

$\Rightarrow$ Existing models unsuitable for studying business transfers

Today's Talk

- Study firm dynamics

- Characterize competitive equilibrium

- Estimate wealth and impact of capital gains tax

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- Study firm dynamics with
  - Indivisible capital
  - Bilaterally traded
  - Requiring time to reallocate

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Today's Talk

- Study firm dynamics with
  - Indivisible capital
  - Bilaterally traded
  - Requiring time to reallocate
- Characterize competitive equilibrium
  - Who trades with whom?
  - How are terms of trade determined?
  - What are the properties?
- Estimate wealth and impact of capital gains tax

Theory

Environment: A Helicopter View

- Infinite horizon with continuous time
- Business type indexed by $s = (z, \kappa)$
  - z: non-transferable capital/owner productivity
  - κ: transferable and accumulable capital
- Business value $V(s)$ depends on
  - Production, $y(s)$
  - Firm dynamics $(s \to s')$

Production

- Technology:

$$y(s,n) = \max_n \hat{z}(s)\kappa(s)^{\hat{\alpha}} n^{\gamma} - wn$$
$$\equiv z(s)\kappa(s)^{\alpha}$$

  where

  $\hat{z}$: non-transferable capital/owner productivity

  κ: transferable and accumulable capital

  n: all external rented factors

- *Idea*: $\hat{z}$ is owner-specific, κ is self-created intangibles

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Firm Dynamics, $s \to s'$

- Entry $\to (z, \kappa)$
- Shocks to productivity $z \to z'$
- Investment $\kappa \to \kappa'$
- Capital transfer $\kappa \to \kappa'$
- Exit $(z, \kappa) \to$

Firm Dynamics: Some notation

- Entry and exit:

$$\begin{aligned} G(s) &= \text{initial distribution of type} \\ c_e &= \text{entry cost} \\ \delta &= \text{exit rate} \end{aligned}$$

- Shocks to productivity:

$$dz = \mu(z)dt + \sigma(z)d\mathcal{B}$$

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Note: just standard Hopenhayn so far

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Next: add self-created intangibles and transfers

Firm Dynamics: Build or Buy Capital?

- Investment: $d\kappa = \theta - \delta_\kappa$ with convex cost $C(\theta)$
- Trades between s, $\tilde{s}$:
  - Bilateral meeting rate: η
  - Allocation: $\kappa^m(s, \tilde{s}) \in \{\kappa(s) + \kappa(\tilde{s}), 0\}$
  - Price: $p^m(s, \tilde{s})$
  - Distribution of partners: $\sum_{\tilde{s}} \lambda(s, \tilde{s}) + \lambda(s, 0) = 1$
  - Expected gain:

$$W(s; \lambda) = \sum_{\tilde{s}} \Big\{ V([z, \kappa^m(s, \tilde{s})]) - V(s) - p^m(s, \tilde{s}) \Big\} \lambda(s, \tilde{s})$$

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† More general specifications also explored

Adding it up: Owner's Value

$$(r+\delta)V(s) = \underbrace{\max_n \; y(s,n) - wn}_{\text{production}} + \underbrace{\mu(z)\partial_z V(s) + \frac{1}{2}\sigma^2(z)\partial_{zz}V(s)}_{\text{shocks to productivity}}$$

$$+ \underbrace{\max_\theta \; \partial_\kappa V(s)(\theta - \delta_k) - C(\theta)}_{\text{investment}} + \underbrace{\max_\lambda \eta W(s;\lambda)}_{\text{trade}}$$

Closing the Model

- Free entry condition

$$\int V(s)dG(s) \leq c_e$$

where measure of entrants is $\phi_e(s) = mG(s) > 0$

- Evolution of types:

$$\dot{\phi} = \Gamma(\theta, \lambda; \phi) + \phi_e$$

induced by drivers of firm dynamics

Recursive Equilibrium

$$\text{Objects: } \{ \underbrace{V,}_{\substack{\text{value}\\\text{function}}} \underbrace{\kappa^m, p^m, \theta, \lambda,}_{\substack{\text{policy}\\\text{functions}}} \underbrace{\phi, \phi_e,}_{\text{measures}} \underbrace{w}_{\text{wage}} \}$$

that satisfy

1. business owners' optimality
2. market clearing
3. consistency of measures

Discussion of Trading Protocol

- Relative to models with
  - CES demand/ monopolistic competition
  - Frictional labor or asset markets
- Framework delivers (with few a priori restrictions)
  - Differentiated goods
  - Rich heterogeneity in market participants
  - Endogenously evolving matching sets

Characterizing Equilibria

Who Trades with Whom?

- Intuitive example:
  - Productivity types: 20 with $z_H = 1$, 10 with $z_L = 0$
  - Capital pre-trade: all have $\kappa = 1$
- Efficient reallocation:
  - 10 low types sell to 10 of the high types

How are Terms of Trade Determined?

- Intuitive example:
  - Productivity types: 20 with $z_H = 1$, 10 with $z_L = 0$
  - Capital pre-trade: all have $\kappa = 1$
- Price leaves high types indifferent between:
  - Trading, with $\kappa = 2$ post-trade
  - Not trading, with $\kappa = 1$ post-trade

Equilibrium Policy Functions

- Intuitive example:
  - Productivity types: 20 with $z_H = 1$, 10 with $z_L = 0$
  - Capital pre-trade: all have $\kappa = 1$
- Capital allocations: $k^m(s_H, s_L) = 2, k^m(s_L, s_H) = 0$
- Prices: $p^m(s_H, s_L) = 1, p^m(s_L, s_H) = -1$
- Choice probabilities:

$$\lambda(s_H|s_L) = 1,\ \lambda(s_L|s_H) = 1/2,\ \lambda_o(s_L) = 0,\ \lambda_o(s_H) = 1/2$$

More Generally Given (ϕ, V)

- Who trades with whom?
  - Solve planner problem maximizing total gains
- How are terms of trade determined?
  - Compute shadow prices from planner problem
- Can solve dynamic program iteratively
  - Update: $(\phi, V) \rightarrow$ static planner $\rightarrow (\phi, V)$

Static Planner Problem

- Let $X(s,\tilde{s})$ be match surplus given by

$$\max_{\kappa^m\in\{\kappa(s)+\kappa(\tilde{s}),0\}} \Big\{V([z(s),\kappa^m]) + V([z(\tilde{s}),\kappa(s)+\kappa(\tilde{s})-\kappa^m])\Big\} - V(s) - V(\tilde{s})$$

- Define total gains $Q(\phi)$ as

$$Q(\phi) = \max_{\pi\geq 0} \sum_{s,\tilde{s}} \pi(s,\tilde{s}) X(s,\tilde{s})$$

$$\text{s.t.} \quad \sum_{\tilde{s}} \pi(s,\tilde{s}) + \pi(s,0) = \phi(s)/2 \quad \forall s \qquad [\mu^a(s)]$$

$$\sum_{\tilde{s}} \pi(\tilde{s},s) + \pi(0,s) = \phi(s)/2 \quad \forall s \qquad [\mu^b(s)]$$

Deliverables from Planner Problem

- Multipliers $\mu = \mu^a = \mu^b$ capture gains from trade

$$\mu(s) = \frac{\partial Q}{\partial \phi(s)}$$

- Prices implement optimal gains from trade:

$$\underbrace{\mu(s)}_{\text{social}} = \underbrace{V([z, \kappa^m(s,\tilde{s})]) - V(s) - p^m(s,\tilde{s})}_{=\text{private gains}}$$

- Updates of ϕ, V are then easy to compute

Properties of Equilibrium

- Competitive allocations maximize

  $$\int e^{-rt} \sum_s [y(s) - C(\theta(s,t)) - m(t)c_e]\phi(s,t)dt$$

  $\Rightarrow$ achieves efficiency

- Competitive prices independent of z

  $$p^m(s, \tilde{s}) = \mathcal{P}(\kappa(\tilde{s}))$$

  $\Rightarrow$ same good sold at same price

- Bilateral trades are pairwise stable

  $\nexists$ feasible trade for $(s, \tilde{s})$ making pair strictly better off

Quantitative Results

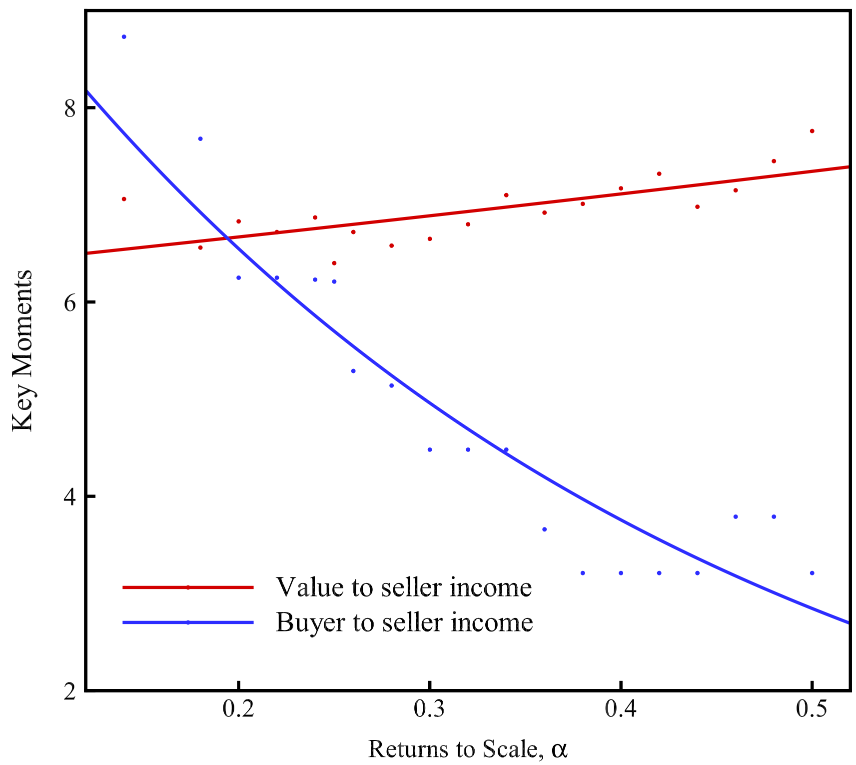
Model Parameters

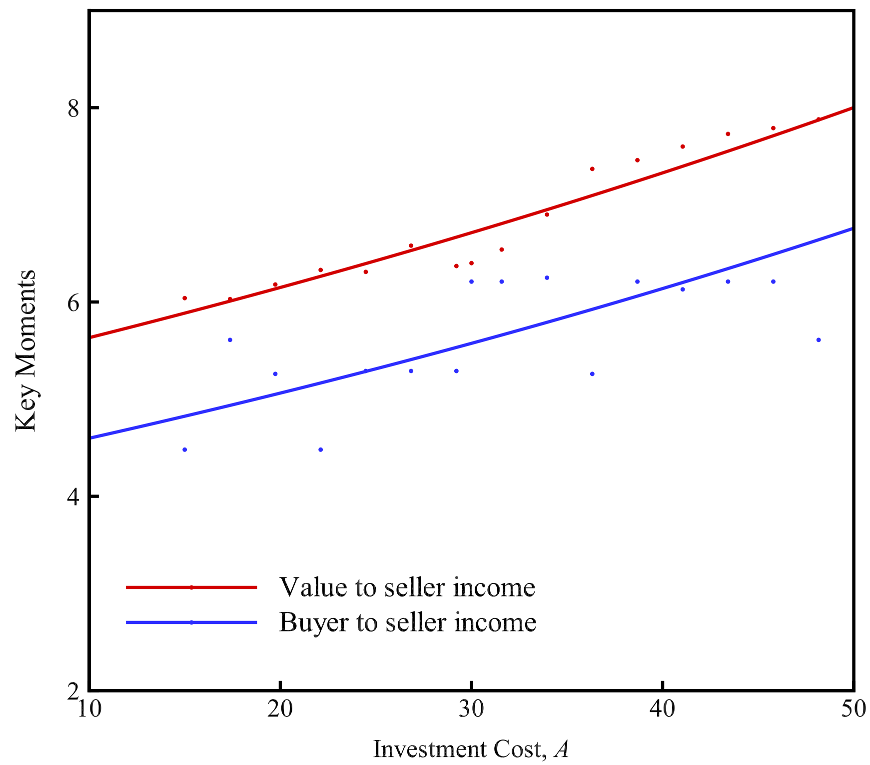
Description	Values
Returns to scale	$\alpha = 0.45$
Discount rate	$r = 0.06$
Investment cost, $C(\theta) = A\theta^\rho$	$A = 30, \rho = 2.0$
Productivity	$\mu = 0, \sigma = 0.25$
Entrant distribution	mass at $z = z_0, \kappa = 1$
Death rate	$\delta = 0.10$
Depreciation rate	$\delta_\kappa = 0.058$
Bilateral meeting rate	$\eta = 0.20$

Identifying Key Parameters

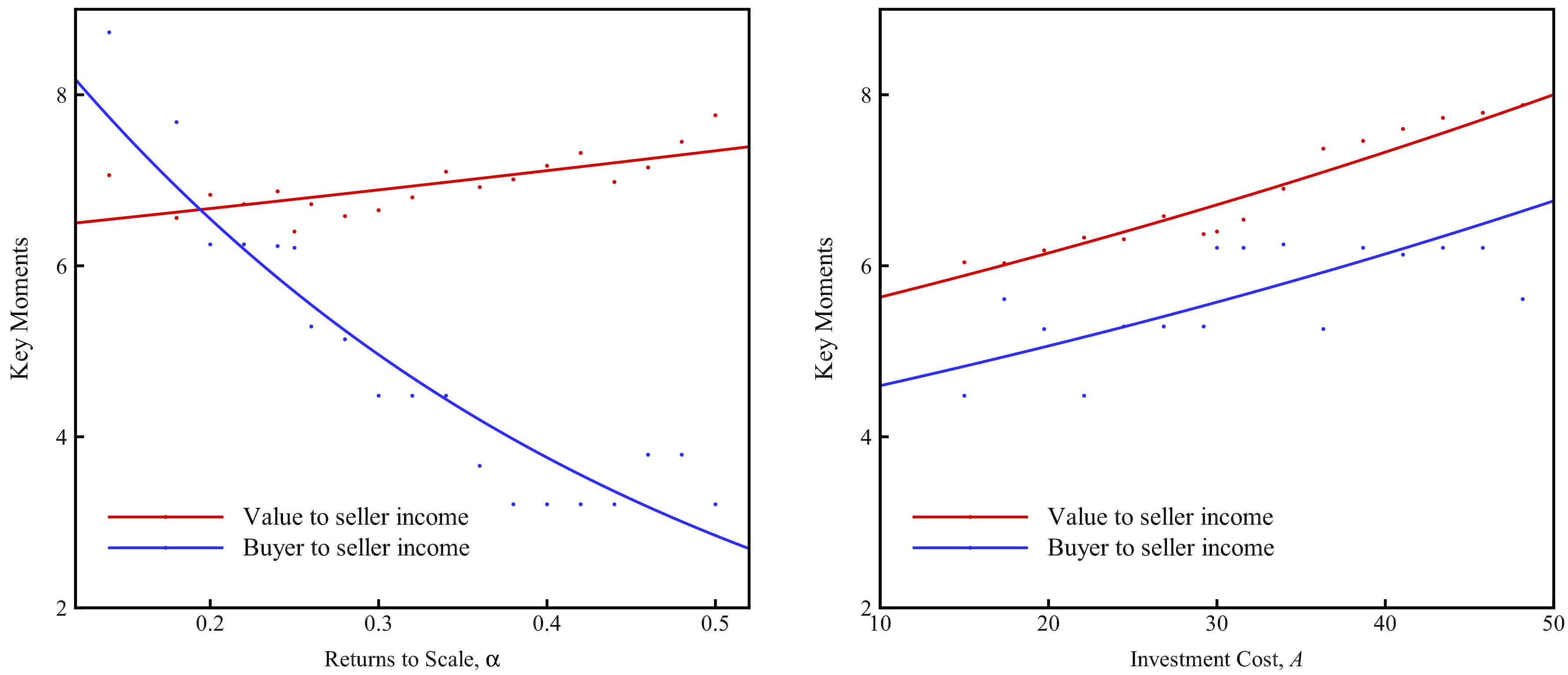
- Key parameters
  - Investment costs $C(\theta) = A\theta^\rho$
  - Returns to scale in $y = z\kappa^\alpha$
- Key moments from IRS (8594 and annual filings)
  - Ratio of business price to seller income
  - Ratio of buyer to seller income

Identifying Key Parameters





Identifying Key Parameters



Next: Use IRS data to validate model

Two Striking Patterns

- Varying age of buyer:
  - Ratio of business price to seller income constant
  - Ratio of buyer to seller income rising

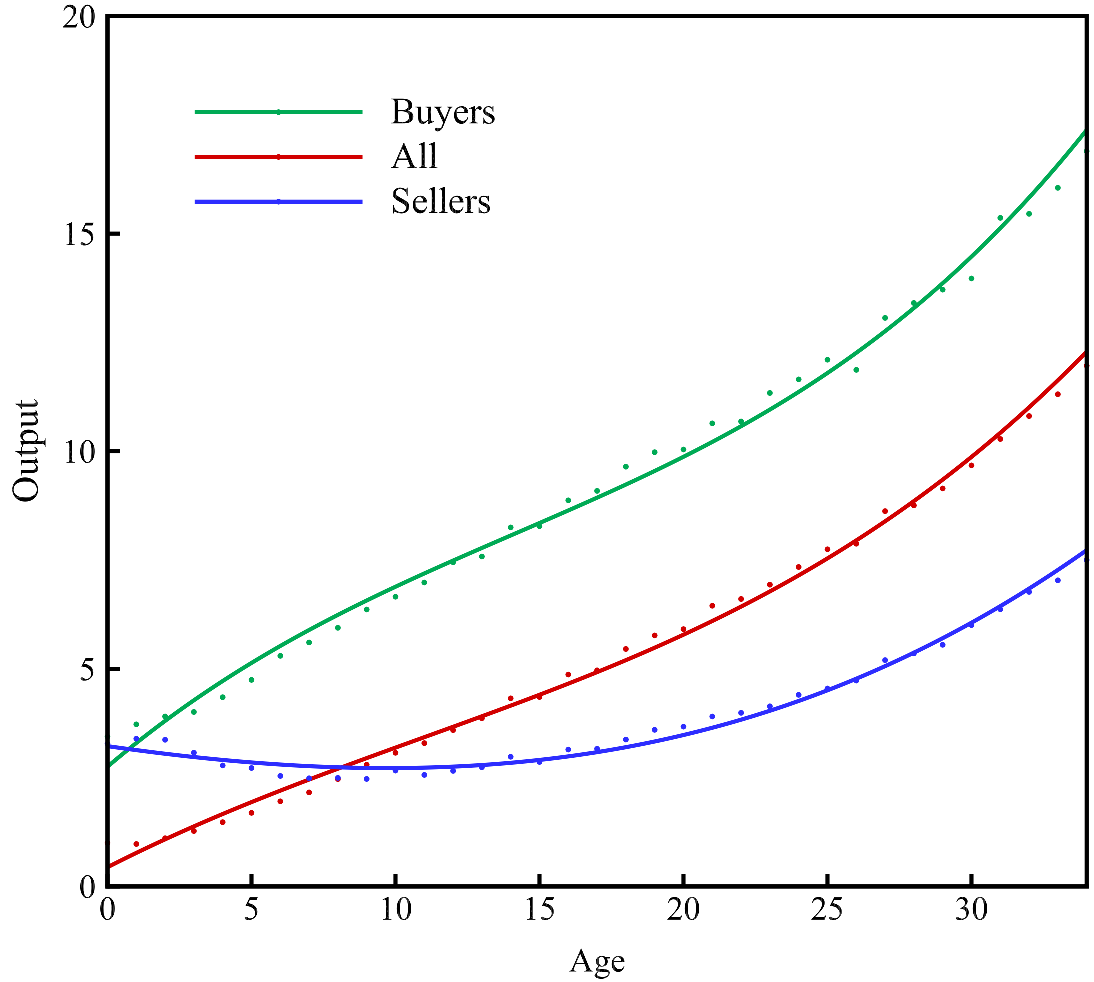
$\Rightarrow$ same in model and data

Moments from the Model

	Age (years)			
	1-5	5-10	10-25	25+
		Buyer		
Price to seller income	6.9	7.5	7.1	6.9
Relative buyer/seller size	2.8	3.8	4.9	5.3
		Seller		
Price to seller income	5.9	7.3	8.6	9.6
Relative buyer/seller size	2.8	3.9	4.3	3.9

- Model: older sellers have high κ and low z
- Data: still investigating reasons for sale

Moments from the Model

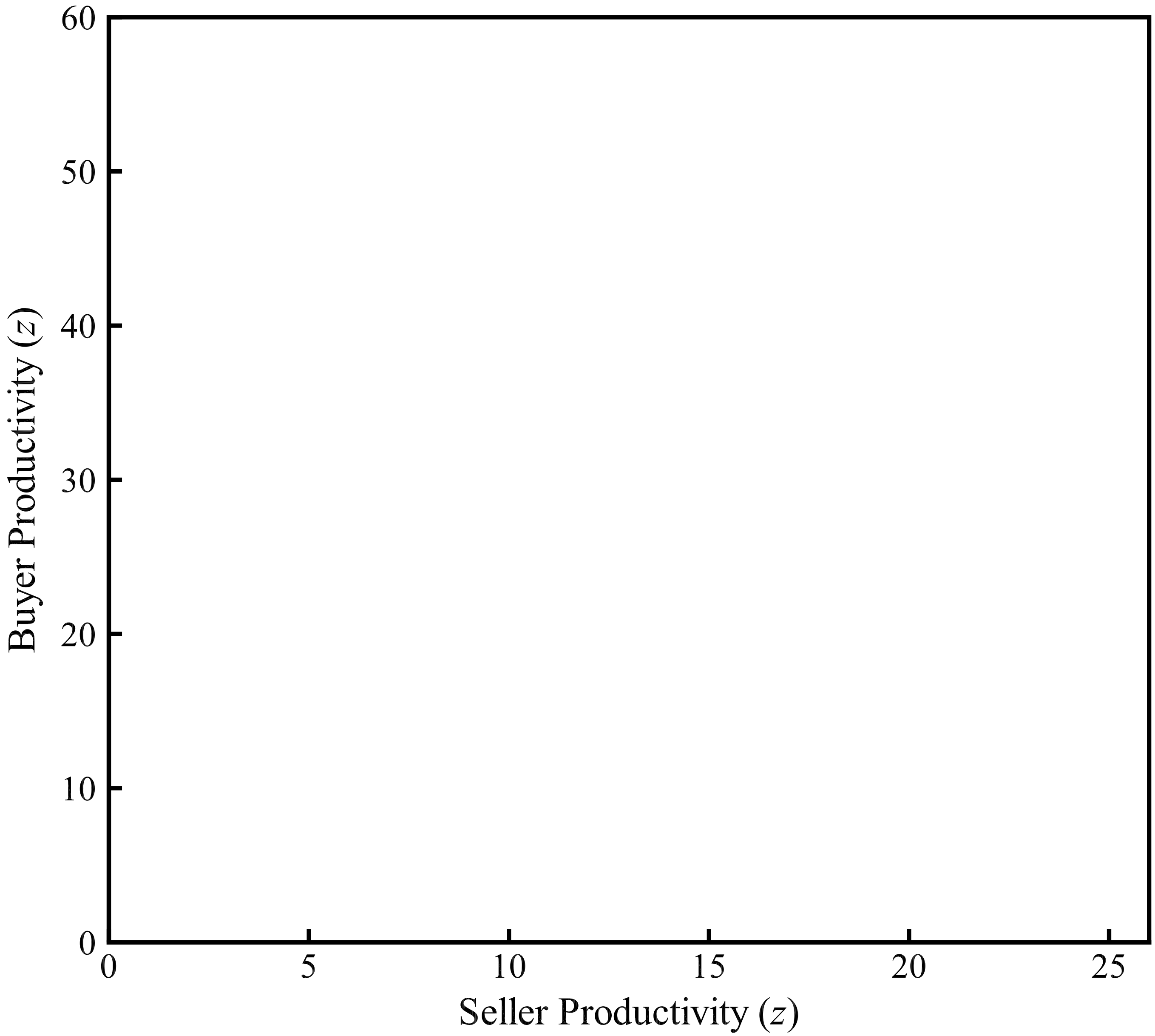


⇒ Buyers larger than average firm

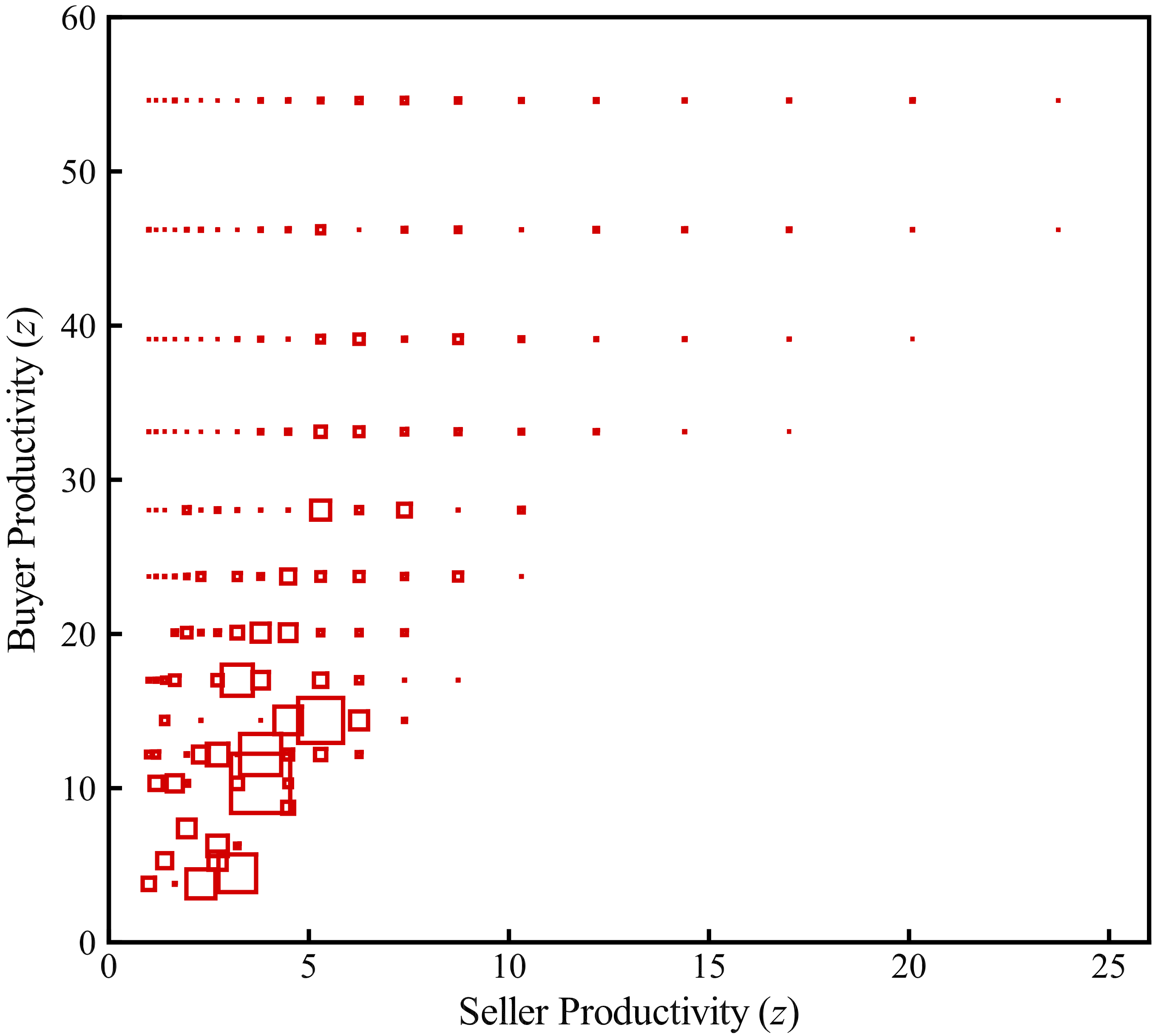
Sellers profile relatively flat

Patterns of Trade

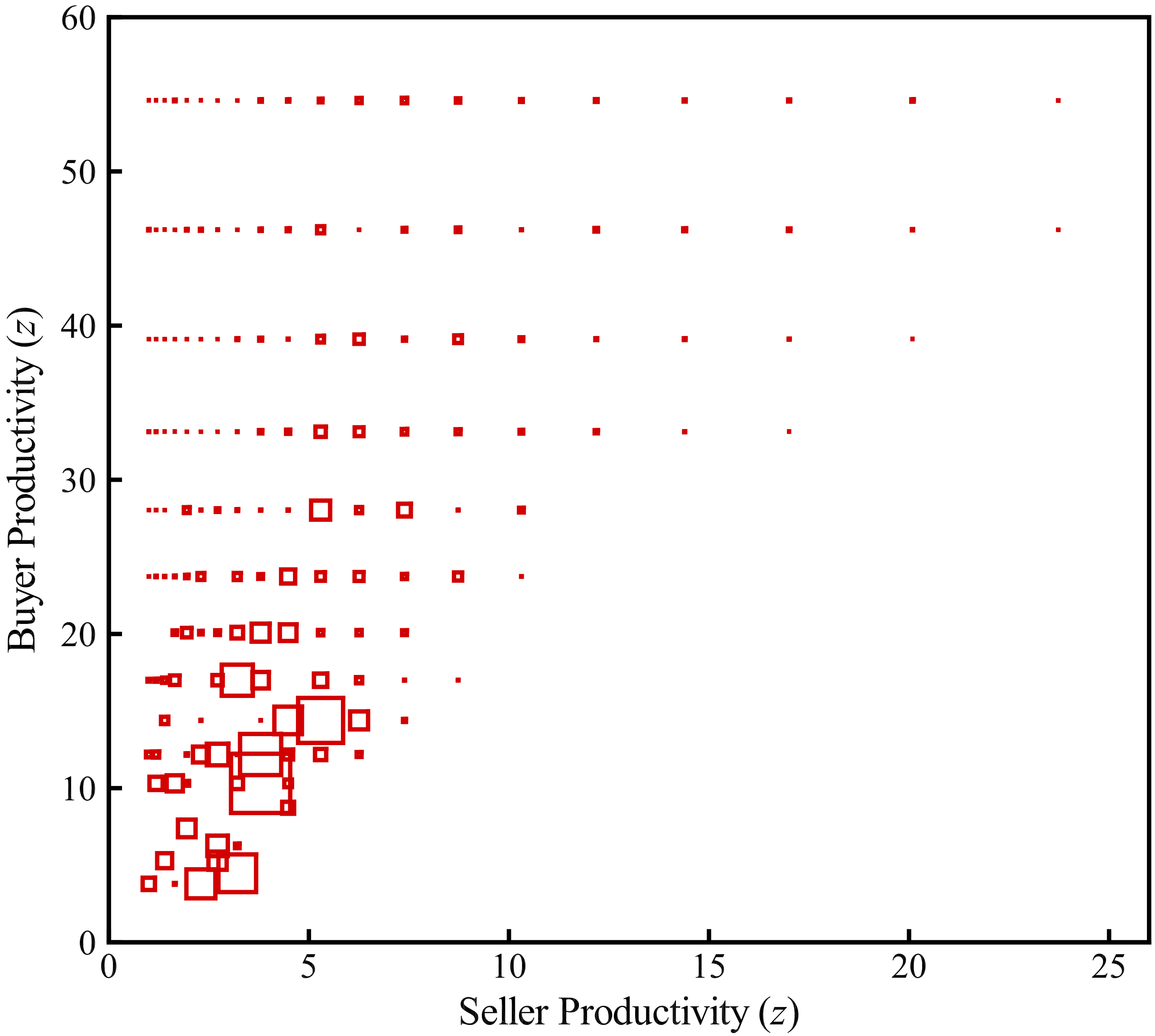
Patterns of Trade



Patterns of Trade



Capital Trades Upward in MPK Sense



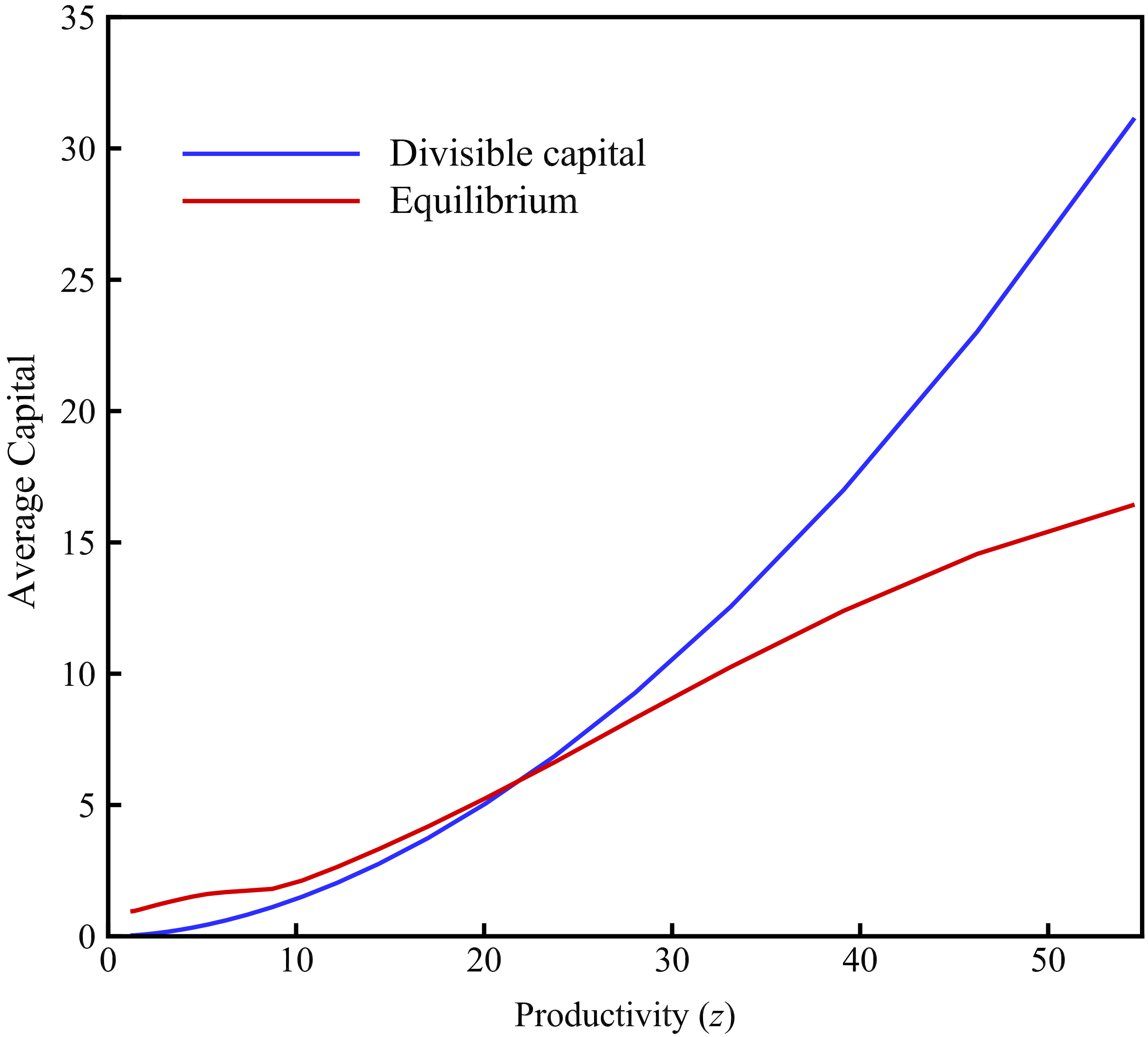
Allocation of Capital

- Compare to "misallocation" literature benchmark
  - Divisible versus indivisible capital
  - Rental versus no rental markets
- Compute *first-best*:

$$\kappa^{FB}(s) \in \text{argmax} \int z(s)[\kappa^{FB}(s)]^{\alpha}\phi(s)ds$$

$$\int \phi(s)\kappa^{FB}(s)ds = \int \phi(s)\kappa(s)ds$$

Dispersion in MPKs without Frictions



Estimating Business Wealth

- Finance textbook: present value of owner dividends

- SCF survey: price if sold business today

$\Rightarrow$ Both have clear model counterparts

Estimating Business Wealth

- Finance textbook: present value of owner dividends, $V(s)$
- SCF survey: price if sold business today, $\mathcal{P}(\kappa(s))$

Estimating Business Wealth

Productivity Level (z)	Transferable Share $\mathcal{P}(\kappa(s))/V(s)$	Income Yield $[y(s) - C(\theta(s))]/V(s)$

Estimating Business Wealth

Productivity Level (z)	Transferable Share $\mathcal{P}(\kappa(s))/V(s)$	Income Yield $[y(s) - C(\theta(s))]/V(s)$
1	0.51	
2	0.50	
4	0.44	
8	0.30	
40	0.34	

Estimating Business Wealth

Productivity Level (z)	Transferable Share $\mathcal{P}(\kappa(s))/V(s)$	Income Yield $[y(s) - C(\theta(s))]/V(s)$
1	0.51	-0.09
2	0.50	-0.03
4	0.44	0.04
8	0.30	0.07
40	0.34	0.16

Estimating Business Wealth

Productivity Level (z)	Transferable Share $\mathcal{P}(\kappa(s))/V(s)$	Income Yield $[y(s) - C(\theta(s))]/V(s)$
1	0.51	−0.09
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40	0.34	0.16

$\Rightarrow$ Significant transferable share and heterogeneity in returns

Taxing Capital Gains

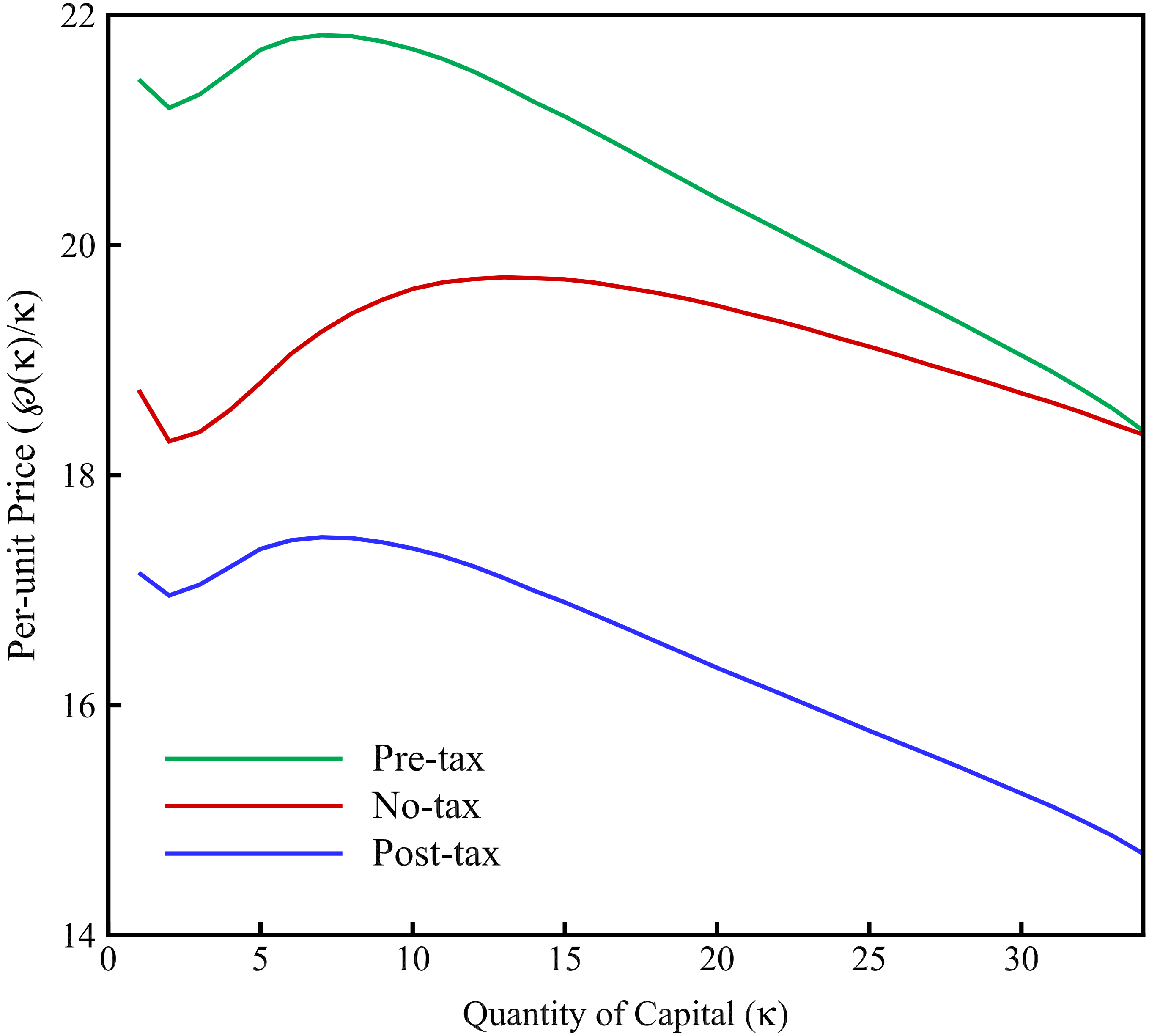
Capital Gains Tax

- Introduce tax τ on gains
  - Seller receives $(1-\tau)p^m(s,\tilde{s})$
  - Government receives $\tau p^m(s,\tilde{s})$
- Positive tax base due to κ (not in Hopenhayn)

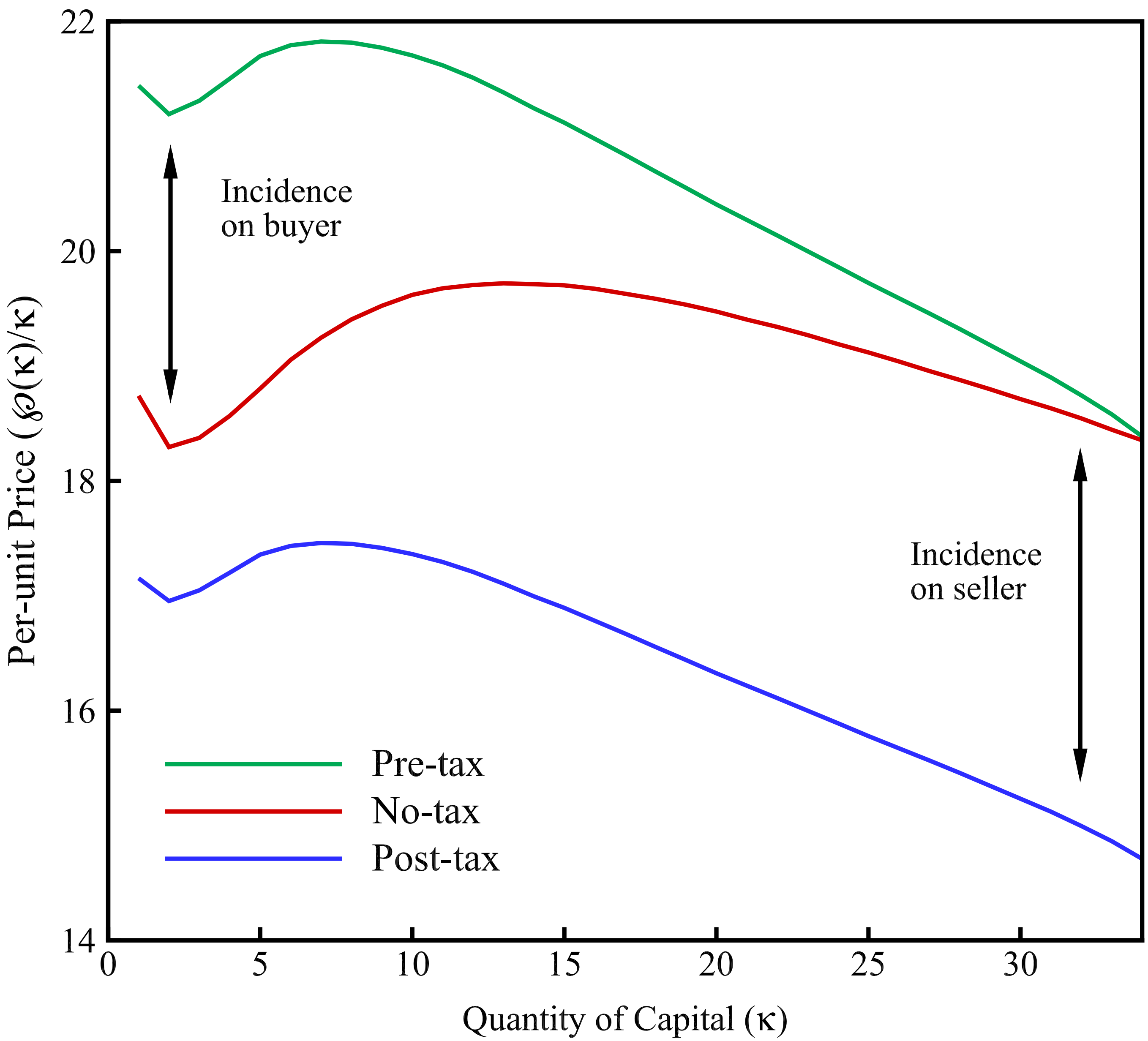
Effects of Tax

- Fewer trades (obvious)
  - Tax eliminates trades where gains are small
- Lower investment and entry (obvious)
  - Tax introduces lock-in effect
- Heterogeneity in tax incidence
  - Larger on buyer if transacted quantity small
  - Larger on seller if transacted quantity large

Heterogeneity in Tax Incidence



Heterogeneity in Tax Incidence



Next Steps

- Theory: add curvature and financing constraints
- Estimation: continue work with IRS data
- Applications: continue work on intangible capital
  - Reallocation
  - Valuation
  - Taxation

Appendix

Dual Planner Problem

$$Q(\phi) = \max_{\mu^a, \mu^b \geq 0} \frac{1}{2} \sum_s (\mu^a(s) + \mu^b(s))\phi(s)$$

$$\text{s.t.} \quad \mu^a(s) + \mu^b(s) \geq X(s, \tilde{s}) \quad \forall s, \tilde{s} \qquad [\pi(s, \tilde{s})]$$

$\Rightarrow$ Multipliers in primal are choice variables in dual

With Non-transferable Utility

- Add extreme value "preference shock" (Galichon et al. 2019)
- Assume all types buy/sell from all others
- Modify slightly the computation of gains to trade W
- Drive preference shock to 0

Galichon-Kominers-Weber Tricks

- After-trade values for buyers (v_b) and sellers (v_s)

$$v_b(s,\tilde{s}) = V([z,\kappa(s)+\kappa(\tilde{s})]) - p^m(s,\tilde{s})$$
$$v_s(s,\tilde{s}) = V(\tilde{s},0) + (1-\tau)p^m(s,\tilde{s})$$

- Matching probability

$$\lambda(s,\tilde{s}) = \exp([v_b(s,\tilde{s}) - W(s)]/\sigma)$$
$$\lambda(\tilde{s},s) = \exp([v_s(\tilde{s},s) - W(s)]/\sigma)$$

- Gains from trade

$$W(s;\lambda) = \sum_{\tilde{s}} \Big\{V([z,\kappa^m(s,\tilde{s})]) - V(s) - p^m(s,\tilde{s})\Big\}\lambda(s,\tilde{s}) - \sigma\lambda(s,\tilde{s})\log\lambda(s,\tilde{s})$$