

THE BREXIT VOTE, PRODUCTIVITY GROWTH AND MACROECONOMIC ADJUSTMENTS IN THE UNITED KINGDOM*

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Abstract

The UK economy experienced significant macroeconomic adjustments following the 2016 referendum on its withdrawal from the European Union. To understand these adjustments, this paper presents empirical facts using novel UK macroeconomic data and estimates a small open economy model with tradable and non-tradable sectors. We demonstrate that the referendum outcome can be interpreted as *news about a future decline in productivity growth in the tradable sector*. While aggregate growth slows, an immediate fall in the relative price of non-tradable goods induces a temporary “sweet spot” for tradable producers. UK interest rates decline and investment growth decelerates.

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1. INTRODUCTION

In the momentous referendum on 23 June 2016, voters decided that the United Kingdom (UK) should leave the European Union (EU). While the final details regarding the UK's withdrawal ('Brexit') are still to be determined, the aftermath of the referendum has been characterized by significant macroeconomic adjustments in the UK economy. UK economic activity has slowed. Growth in the tradable sector has remained resilient in comparison to the non-tradable sector. The British pound sterling has been subject to a pronounced depreciation. At the same time, UK interest rates have declined relative to their world counterpart and investment has slowed down materially, while employment has remained strong. This paper documents these empirical patterns using newly constructed UK macroeconomic data and develops an intuitive model-based narrative to explain them.

Our analysis is motivated by the remarks of [Broadbent \(2017a\)](#), based on which the referendum outcome can be interpreted as a news shock that captures a *future productivity growth slowdown in the tradable sector*. We formalize and assess this idea through the lens of an estimated small open economy model with tradable and non-tradable sectors. Our model allows us to characterize how firms and households respond to news about future productivity growth in the tradable sector by shifting resources across expenditure components, sectors and time. We demonstrate that the dynamics triggered by such news are consistent with the patterns in post-referendum UK data. Most notably, our model captures the somewhat counter-intuitive effect of the referendum on activity in the tradable and non-tradable sectors: although the primary mechanism by which Brexit will eventually affect the UK economy is trade, it was growth in the non-tradable sector that contracted following the referendum result. We also show that other types of news shocks cannot generate the empirical patterns in UK macroeconomic data, and lay out how drivers of growth in tradable sector productivity are indeed linked to specific consequences of Brexit, such as reduced goods, capital and labor mobility.

The paper proceeds in four steps. First, we document a number of stylized facts about the UK economy in the period following the 2016 referendum. This is based on a new quarterly macroeconomic data set in which we construct key variables separately for the tradable and non-tradable sectors.¹ The stylized

¹The construction of the data involves classifying industry data at the 2-digit level into tradable and non-tradable sectors and then constructing separate time series of macroeconomic aggregates for the two sectors. We make the data publicly available at <http://econweb.umd.edu/~drechsel/files/BrexitPaperData.xlsx>.

facts describe growth, exchange rate, interest rate, investment and employment dynamics following the Brexit vote. Second, we introduce a small open economy (SOE) model which is composed of tradable and non-tradable sectors. Our model can encompass differential trend growth rates across these sectors under restrictions on preferences and technology. Introducing these sector-specific trends allows us to conduct the relevant experiments. Third, we estimate the model at business cycle frequency using the newly constructed data set. Estimating the model with information up to the time of the referendum enables us to pin down not only the structural parameters, but also the initial balanced growth path around which we study Brexit news scenarios. Fourth, we use the model to simulate the effects of the referendum outcome. The Brexit vote is a prime example of a news shock: at a well-identified point in time, firms and households receive new information about the future, but no actual changes materialize in the economy upon the announcement. At the heart of our analysis is a benchmark experiment that assesses the economic impact of news that the growth rate of productivity in the tradable sector will be persistently low in the future. This experiment replicates the key empirical patterns we uncover for the UK economy following the Brexit vote and sheds light on the mechanism that generated them.

The mechanism works as follows. The Brexit news – conceptualized as a persistent drop in the growth rate of future productivity in the UK tradable sector – generates a temporary boom in tradable production. This is driven by the response of the relative price of non-tradable output (an ‘internal’ real exchange rate) which jumps down when the news is revealed. Consequently, there is an opportunity to sell tradable output at a temporarily higher relative price before productivity in the sector actually falls, a temporary “sweet spot” for producers of tradable output (Broadbent, 2017a,b). This generates a reallocation of capital and labor towards the tradable sector, a rise in tradable output growth and an increase in net exports, all of which reverse after the productivity growth decline in the tradable sector actually occurs. The Brexit news also moves interest rates. In the model, we compute interest rates that are indexed to tradable and to non-tradable goods, respectively, permitting consideration of relative interest rate developments. In response to the shock, the return on bonds denominated in non-tradable output (the ‘domestic’ interest rate) drops sharply in the short run. Once productivity growth in the tradable sector actually falls, it prompts a reversal of the inter-sectoral resource flows towards the non-tradable sector. This generates a persistent and hump-shaped rise in the real return on non-tradable denominated bonds over the longer term.

In addition, the news about slower productivity growth triggers a material reduction in investment growth, while employment remains resilient.

We explore the robustness of our results along several dimensions, including the timing and profile of the news shock, and show that other types of shocks do not match the dynamics present in the data. One notable insight is that the persistent response of interest rates can plausibly only be related to expectations about growth rate rather than level changes in relative productivities.

While the main contribution of this paper lies in providing new empirical facts and a formal framework to understand the short-run adjustments of the UK economy to the referendum outcome, we also discuss how more specific consequences of Brexit are in fact key drivers of productivity growth in the tradable sector. In particular, we explain how increased trade barriers, reduced capital flows and impediments to labor mobility can all be linked to a looming slowdown in the productivity of tradable goods and services. This discussion of the underlying drivers of productivity growth in the tradable sector lends further support to our choice of analyzing the referendum news in an intuitive macro model. The narrative we develop in this paper not only informs one of the major policy debates in UK history, but also delivers new general insights on the macroeconomic dynamics triggered by economic disintegration.

Our work contributes to several strands of academic research. First, there has been a surge in papers studying the impact of Brexit on the UK economy and beyond, from a variety of angles.² Similar to us, [Born et al. \(2019\)](#) and [Vlieghe \(2019\)](#) focus on the period immediately after the referendum from a macroeconomic perspective. Both of these studies apply a synthetic control method to gauge the effects of Brexit on UK economic growth. While the aggregate effects they find are very similar to ours, the additional insights we provide on sectoral activity and relative prices contributes to understanding the nature of the adjustment mechanism. Other papers on the impact of Brexit focus on long-run trade ([Dhingra et al., 2017](#); [Sampson, 2017](#)), foreign direct investment ([McGrattan and Waddle, 2020](#)), financial market volatility and stock returns ([Davies and Studnicka, 2018](#)), uncertainty ([Steinberg, 2017](#); [Bloom et al., 2018, 2019](#); [Faccini and Palombo, 2020](#); [Hassan et al., 2020](#)), as well as exchange rate pass-through ([Breinlich et al., 2019](#); [Forbes et al., 2018](#)).³ Consistent with the

²There are also various studies that focus on the reasons for the outcome of the referendum rather than its economic impact. See for example [Becker et al. \(2017\)](#), [Fetzer \(2018\)](#).

³In particular the trade literature features many more studies that are helpful to understand Brexit and its effects. See for example [Erceg et al. \(2018\)](#) for an analysis of the short-run macroeconomic effects of specific trade policies such as tariffs, as well as [Caldara et al. \(2020\)](#) and [Graziano et al. \(2018\)](#) for recent papers on the effects of trade policy uncertainty.

notion of a “deglobalization shock” put forward by [Gourinchas and Hale \(2017\)](#), our work provides a narrative of the referendum impact as one fundamental economic shock. We provide a novel interpretation of this shock as news about productivity growth in the tradable sector. The suggested economic mechanism successfully matches the patterns observed in the newly constructed macroeconomic data that we present.

Second, our paper relates to a body of research on the role of news shocks in business cycles, such as the work of [Beaudry and Portier \(2006\)](#), [Jaimovich and Rebelo \(2009\)](#) and [Schmitt–Grohe and Uribe \(2012\)](#). Our paper contributes in particular to the literature that studies the role of news shocks in an open economy setting ([Jaimovich and Rebelo, 2008](#); [Kamber et al., 2017](#); [Siena, 2020](#)) as well as in multi-sector models ([Gortz and Tsoukalas, 2018](#); [Vukotić, 2019](#)). The Brexit vote is perhaps the archetype of a news shock, given that we can precisely pinpoint its time of arrival, and given its clear economy-wide scope. From the perspective of understanding news shocks, the Brexit referendum thus takes the role of large quasi-natural experiment, which we exploit to show that news shocks have important consequences for macroeconomic dynamics in an open economy.

Third, our paper relates to work that has undertaken a serious calibration of models with tradable and non-tradable sectors, such as [De Gregorio et al. \(1994\)](#), [Betts and Kehoe \(2006\)](#) and [Lombardo and Ravenna \(2012\)](#). Similar to this line of research, we allocate 2-digit SIC industry level data into tradable and non-tradable categories. To the best of our knowledge, we are the first to do so for the UK and the first to use such a classification to construct time series aggregates to estimate, rather than calibrate, a structural model.

Finally, we contribute to the broader SOE literature, which follows the classic work of [Mendoza \(1991\)](#). We build on the contribution of [Drechsel and Tenreyro \(2018\)](#) by allowing for productivity growth differentials between sectors.⁴ While [Drechsel and Tenreyro \(2018\)](#) focus on emerging economies, this paper demonstrates that structural shocks to sectoral productivity growth are also a useful modeling device for advanced economies.⁵

⁴[Stockman and Tesar \(1995\)](#) is an earlier example of an open economy model with sectoral productivity differences. Their framework features only stationary shocks. Other contributions to broader the SOE literature include, but are not limited to, [Kose \(2002\)](#), [Schmitt-Grohe and Uribe \(2003\)](#), [Aguiar and Gopinath \(2007\)](#) and [Garcia-Cicco et al. \(2010\)](#). [Gourinchas and Rey \(2014\)](#) survey research on both small open economy and large open economy models.

⁵Modeling different growth rates in technologies across sectors also relates to the literature that studies investment-specific technology alongside TFP, such as [Greenwood et al. \(2000\)](#) and [Justiniano et al. \(2011\)](#). See also [Acemoglu and Guerrieri \(2008\)](#) for a model of differential productivity growth across sectors based on different factor proportions.

The remainder of the paper is structured as follows. Section 2 documents stylized facts about the UK economy following the referendum. Section 3 introduces our two-sector SOE model. Section 4 presents the data and discusses our estimation to pin down the structural parameters and the initial balanced growth path. Section 5 simulates our Brexit scenario and provides a comprehensive description of the results. Section 6 presents a discussion of the economic drivers underlying productivity growth in the tradable sector. Section 7 concludes.

2. UK MACROECONOMIC ADJUSTMENTS TO THE BREXIT VOTE

This section documents stylized facts about the UK economy following the Brexit referendum. Some of these facts are based on a novel quarterly macroeconomic data set, which we build by classifying industry data at the 2-digit level into tradable and non-tradable sectors over time. We are the first to do such an exercise for the UK economy. Details on the data construction are provided in Section 4.1, where we also document the industrial composition of the tradable and non-tradable sectors. Appendix A contains additional empirical facts.

Figure 1 presents key UK macroeconomic time series for the years 2010 to 2018. In each panel, the vertical line indicates the date of the referendum, 23 June 2016, and the shaded area marks the period after the referendum. In this post-referendum period, no actual implementation of the withdrawal from the EU took place. As the figure reveals, however, meaningful adjustments in the economy are visible. Panel A shows the change in aggregate UK growth relative to pre-referendum expectations. Specifically, it reveals how the IMF successively revised its UK GDP growth forecasts following the referendum. A marked decline in these forecasts is visible over time, indicating the significant negative impact of the Brexit vote on UK growth prospects relative to the pre-referendum period.⁶

Panel B presents a decomposition of real gross value added (GVA) into tradable and non-tradable sectors. The two sectors show a similar trend prior to the referendum, after which there is a sharp break in the growth rate for the *non-tradable* sector. With Brexit commonly understood to negatively affect trade, it is notable that growth in the non-tradable sector slows immediately. To explain this, relative price developments need to be accounted for. Panel C

⁶In Appendix A we present the analogous IMF forecasts for the group of advanced economies. While the global economy as a whole was weakening during this period, the downward revision in the outlook were much sharper for the UK.

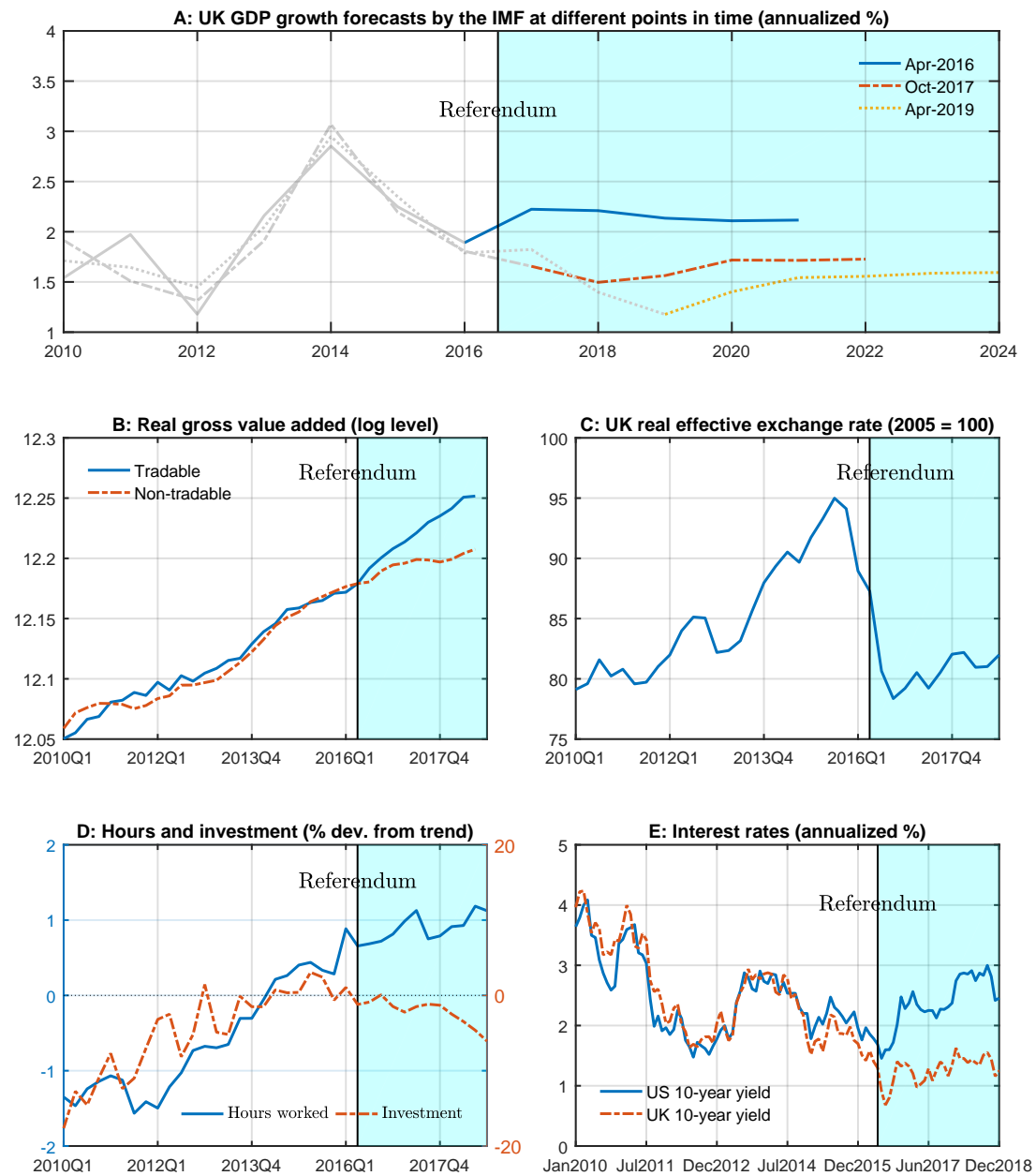


Figure 1: *Adjustments of the UK economy following the Brexit vote*

Notes. Panel A presents the 5-year ahead IMF forecasts of annual UK GDP growth for April 2016, October 2017 and April 2019. The first forecast is before the referendum, the other two are after, and the three forecast dates have equal distance to each other. In each case, observations prior to the forecast date (plotted in gray) reflect actual observations, which differ across vintages due to data revisions (source: World Economic Outlook). Panel B plots the log level of real gross value added separately for the UK tradable and non-tradable sector (source: ONS and own calculations, see details in Section 4.1). Panel C displays the UK real effective exchange rate (source: Bank of England). Panel D plots total hours worked (scaled by size of the labor force) as well as aggregate investment. Both series are in logs and expressed in deviations from a linear trend (source: ONS and own calculations). Panel E shows the 10-year zero coupon yields for the US and the UK (source: Bank of England). In all panels, the vertical line indicates the date of the referendum and the shaded area marks the period from the Brexit referendum until 2018Q4.

presents the UK real effective exchange rate (REER). It is evident that the UK real exchange rate, while fluctuating throughout the whole period shown, drops sharply and persistently around the time of the referendum.⁷ While overall UK growth appears to be negatively affected by the referendum outcome (panel A), the relative price effect (panel C) generates *relatively* beneficial conditions – a “sweet spot” – for the producers of tradable goods and services (panel B).⁸ The fact that relative price developments generate favorable conditions for the tradable sector in the short run will be a cornerstone of our interpretation of the UK economy’s adjustments to the Brexit vote.

Panel D shows the evolution of aggregate factors of production. While total investment weakened following the referendum, it is notable that total labor input has continued to increase, indicating a robust labor market. This difference between investment and employment effects of the Brexit news is consistent with the firm-level survey evidence provided by [Bloom et al. \(2019\)](#).

Panel E shows ten-year zero coupon yields for the UK and the US. These yields closely track each other prior to the Brexit vote but a meaningful spread opens up thereafter, with UK yields remaining persistently below their US counterpart. Omitting inflation risk and term premia considerations, this pattern is already indicative of a mechanism by which market participants may have perceived a fall in productivity in the UK relative to the US. The response of interest rates is particularly important in verifying our interpretation of Brexit relative to alternative explanations.

In summary, while many of the details regarding the UK’s ultimate withdrawal from the EU have been highly uncertain, the aftermath of the referendum has been characterized by significant macroeconomic adjustments in the UK economy. UK economic activity has slowed relative to trend. Growth in the tradable sector has remained resilient relative to the non-tradable sector. The British pound has been subject to a pronounced depreciation. Hours worked have been growing robustly, while investment has slowed down. At the same time, UK interest rates have declined relative to their international counterpart. Our model simulations will jointly explain these facts.

⁷As we will highlight in the exposition of our two-sector model, the real exchange rate is conceptually closely related to the relative price between nontradable and tradable goods and services. Section 4.2 provides a closer comparison between the two in the data.

⁸This interpretation is supported by empirical patterns of UK exports and the UK trade balance, which we present in Appendix A. In the same Appendix, we also show the dynamics of UK consumption growth, which slowed down after the referendum. Our model simulations are consistent with all of these empirical patterns.

3. THE MODEL

The setting is a real small open economy with a tradable (T) and a non-tradable (N) sector. As in [Drechsel and Tenreyro \(2018\)](#), labour-augmenting productivity in each sector grows at its own rate, denoted by g_{Tt} and g_{Nt} . The presence of two stochastic trends implies that the levels of different variables may grow at different rates along a balanced growth path. To aid exposition, we use lower-case letters to denote stationary variables and upper-case letters to denote variables that contain a stochastic trend. The economy is small in the sense that the real interest rate is exogenous and the rest of the world absorbs any trade surplus or deficit fully elastically. To capture both domestic and international interest rate variation, we introduce different bonds, denominated in terms of tradable and non-tradable goods, where the latter is traded by domestic households only. Following [Schmitt-Grohe and Uribe \(2003\)](#), we close the model with a debt elastic premium on external borrowing. After presenting the agents' problems and market clearing conditions, we discuss some key economic forces of the model.

3.1. The firms' problem

Firms in both sectors combine labor and physical capital using a Cobb-Douglas technology to produce final output. Physical capital is sector-specific and previously accumulated capital cannot be reallocated across sectors. Labor is sector-specific.⁹ Formally, sector $M = \{T, N\}$ produces a final good Y_{Mt} by combining capital K_{Mt} and labor n_{Mt} according to

$$Y_{Mt} = a_{Mt} K_{Mt}^{\alpha_M} (X_{Mt} n_{Mt})^{1-\alpha_M}. \quad (1)$$

Here, a_{Mt} denotes a stationary TFP disturbance and X_{Tt} the non-stationary component of labor-augmenting productivity. TFP in sector M follows

$$\ln a_{Mt} = \varrho_M^a \ln a_{Mt-1} + \varepsilon_{Mt}^a, \quad \text{with} \quad \varepsilon_{Mt}^a \sim \mathbb{N}(0, \zeta_M^a). \quad (2)$$

where ϱ_M^a is the persistence (stationary) sectoral TFP and ζ_M^a the variance of the shock. The growth rate of sectoral labor-augmenting productivity is defined as

$$g_{Mt} = \frac{X_{Mt}}{X_{Mt-1}}, \quad (3)$$

⁹Given that we focus on short-run adjustments, assuming that labor and capital are freely mobile would likely generate implausibly rapid inter-sectoral reallocation.

and follows an autoregressive process of the form:

$$\ln(g_{Mt}/\bar{g}_M) = \varrho_M^g \ln(g_{Mt-1}/\bar{g}_M) + \varepsilon_M^g, \quad \text{with } \mathbb{N}(0, \varsigma_{Mt}^g), \quad (4)$$

where ϱ_M^g captures the persistence and ς_{Mt}^g denotes the dispersion of shocks to its process. Transitory shocks to g_{Mt} capture changes to the growth rate of labor-augmenting productivity in sector M , which permanently affect the *level* of productivity. \bar{g}_M denotes the steady state value of the growth rate in sector M . In the remainder of the paper, we refer to the labor-augmenting productivity growth process in each sector sometimes simply as “productivity growth”.

Firms rent capital and labor in competitive factor markets at rental rate r_{Mt}^k and real wage w_{Mt} , respectively. Profits are given by

$$Y_{Tt} - W_{Tt}n_{Tt} - r_{Tt}^k K_{Tt} \quad (5)$$

in the tradable sector and

$$P_t Y_{Nt} - W_{Nt}n_{Nt} - P_t r_{Nt}^k K_{Nt} \quad (6)$$

in the non-tradable sector. Under the assumption of perfect competition, firms make zero profits in equilibrium. The variable P_t denotes the relative price of the non-tradable vis-a-vis tradable goods. This price can be interpreted as an ‘internal’ measure of the real exchange rate. From a conceptual point of view, this interpretation goes back to the work of [Balassa \(1964\)](#) and [Samuelson \(1964\)](#), who have studied international productivity differences and their implications for real exchange rates.¹⁰ Note that the model implies that the relative price P_t exhibits a stochastic trend.¹¹

3.2. The household’s problem

From the perspective of the representative household, while tradable and non-tradable consumption are assumed to be gross complements, the consumption of home tradable goods and their foreign counterpart can be perfectly substi-

¹⁰The *Harrod-Balassa-Samuelson effect* is the empirically observed tendency for countries with stronger productivity in tradable goods relative to non-tradable goods to have higher price levels overall. The mechanics of this effect feature in our model, where a weakness in productivity growth in the tradable sector puts downward pressure on domestic price level. We discuss this and additional key economic forces in our model at the end of this section.

¹¹This is a natural consequence of different growth rates across sectors. We examine the stationarity of the UK real exchange rate empirically in [Appendix D](#).

tuted (the law of one price for tradable goods holds). Following [Drechsel and Tenreyro \(2018\)](#), we specify the period utility function as in [Greenwood et al. \(1988\)](#). We scale the disutility of labor supply by tradable labor-augmenting productivity to ensure that both consumption and labor elements of the utility function grow at the same rate along the balanced growth path. Formally,

$$\mathcal{U}_t(C_t, X_{Tt-1}, X_{Nt-1}, n_{Tt}, n_{Nt}) = \frac{\left[C_t - \frac{X_{Tt-1}}{\omega} (\theta_T n_{Tt}^\omega + \theta_N n_{Nt}^\omega) \right]^{1-\gamma}}{1-\gamma}, \quad (7)$$

where θ_M denotes the disutility of labor in sector M and ω the elasticity of labor supply. The variable C_t is a CES consumption aggregator, expressed in tradable units, that combines tradable and non-tradable consumption (denoted by C_{Tt} and C_{Nt})

$$C_t = \left[\zeta^{1-\sigma} C_{Tt}^\sigma + (1-\zeta)^{1-\sigma} \left(\frac{X_{Tt-1}}{X_{Nt-1}} C_{Nt} \right)^\sigma \right]^{\frac{1}{\sigma}}, \quad (8)$$

where $\gamma > 1$ is the inverse inter-temporal elasticity of substitution (IES) and $\eta = 1/(1-\sigma)$ the elasticity of substitution between tradable and non-tradable consumption.¹² The representative household seeks to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \nu_t \beta^t \frac{\left[C_t - X_{Tt-1} \omega^{-1} (\theta_T n_{Tt}^\omega + \theta_N n_{Nt}^\omega) \right]^{1-\gamma}}{1-\gamma}, \quad (9)$$

subject to the budget constraint (expressed in tradable units)

$$\begin{aligned} & C_{Tt} + P_t C_{Nt} + B_t^* + P_t B_t + I_{Tt} + P_t I_{Nt} + P_t Y_{Nt} \frac{s}{y_N} s_t \\ & + \frac{\phi_T}{2} \left(\frac{K_{Tt+1}}{K_{Tt}} - \bar{g}_T \right)^2 K_{Tt} + P_t \frac{\phi_N}{2} \left(\frac{K_{Nt+1}}{K_{Nt}} - \bar{g}_N \right)^2 K_{Nt} \quad (10) \\ & = r_{Tt}^k K_{Tt} + P_t r_{Nt}^k K_{Nt} + W_{Tt} n_{Tt} + W_{Nt} n_{Nt} + \frac{B_{t+1}^*}{1+r_t^*} + P_t \frac{B_{t+1}}{1+r_t}. \end{aligned}$$

$\beta \in [0, 1)$ denotes the subjective discount factor and the variable ν_t denotes a risk-premium shock given by:

$$\ln \nu_t = \varrho_\nu \ln \nu_{t-1} + \varepsilon_{\nu t} \quad \text{with} \quad \varepsilon_{\nu t} \sim \mathbb{N}(0, \zeta_\nu). \quad (11)$$

¹²Note that X_{Tt-1} and X_{Nt-1} enter the utility function to ensure a balanced growth path. See [Acemoglu and Guerrieri \(2008\)](#) for a model of “unbalanced growth” in the context of differences in factor proportions across sectors. The parameters θ_T and θ_N will allow us to match the relative quantities of labor used in the two sectors.

Sectoral physical capital depreciates at the rate δ_M and ϕ_M captures how costly is to adjust capital in sector M . Physical investment (I_{Mt}) results in the following law of motion:

$$K_{Mt+1} = (1 - \delta_M) K_{Mt} + I_{Mt}. \quad (12)$$

One important aspect of the budget constraint is the presence of two different assets, B_t^* and B_t with corresponding interest rates r_t^* and r_t . These are risk-free bonds that pay one unit of tradable goods and non-tradable goods in the following period, respectively. They can be thought of as bonds that are indexed to different types of inflation rates in practice. While a bond that pays tradable units – a standard ingredient of SOE models – allows the economy to achieve a trade balance that is different from zero, the bond that pays non-tradable units remains in zero net supply. Introducing it allows us to determine its interest rate r_t , which will move differently from r_t^* , shedding light on how “domestic” relative to “world” interest rates can diverge in response to the Brexit news. This is motivated by the different movement of UK and US rates shown in Section 2. The interest rate on the bonds denominated in tradable goods is given by

$$r_t^* = \bar{r}^* + \psi \left(e^{B_{t+1}^*/X_{Tt} - \bar{b}^*} - 1 \right) + (e^{\mu_t - 1} - 1), \quad (13)$$

where \bar{r}^* is the steady state world interest rate, and the term multiplied by ψ captures a country risk premium, which is increasing in the amount of external debt. The latter assumption follows [Schmitt-Grohe and Uribe \(2003\)](#) and ensures a stationary solution of the model after detrending.¹³ Finally, the term $(e^{\mu_t - 1} - 1)$ captures an interest rate shock, which follows

$$\ln \mu_t = \rho_\mu \ln \mu_{t-1} + \varepsilon_{\mu t} \quad \text{with} \quad \varepsilon_{\mu t} \sim \mathbb{N}_t(0, \zeta_\mu). \quad (14)$$

The variable s_t is a government expenditure shock, which can be thought of as a broader aggregate demand shifter, and which follows

$$\ln s_t = \rho_s \ln s_{t-1} + \varepsilon_{st} \quad \text{with} \quad \varepsilon_{st} \sim \mathbb{N}(0, \zeta_s), \quad (15)$$

The ratio s/y_N in (10) is the steady state share of government expenditure to non-tradable output.

Given preferences, the relative price of the aggregate consumption bundle

¹³As we discuss in Section 5.4 and show formally in Appendix G, the conclusions we draw in this paper are robust to alternative assumptions to ensure the model’s stationary solution.

(in terms of tradable units) is

$$P_t^c = \left[\zeta + (1 - \zeta) \left(\frac{X_{Nt-1}}{X_{Tt-1}} P_t \right)^{\frac{\sigma}{\sigma-1}} \right]^{\frac{\sigma-1}{\sigma}}. \quad (16)$$

Note that P_t^c is a stationary variable.

3.3. Market clearing and equilibrium

The market clearing conditions are

$$Y_{Tt} = C_{Tt} + I_{Tt} + \frac{\phi_T}{2} \left(\frac{K_{Tt+1}}{K_{Tt}} - \bar{g}_T \right)^2 K_{Tt} + TB_t \quad (17)$$

in the tradable sector and

$$Y_{Nt} = C_{Nt} + I_{Nt} + \frac{s}{y_N} Y_{Nt} s_t + \frac{\phi_N}{2} \left(\frac{K_{Nt+1}}{K_{Nt}} - \bar{g}_N \right)^2 K_{Nt} \quad (18)$$

in the non-tradable sector. We define the trade balance as

$$TB_t = B_t^* - \frac{B_{t+1}^*}{1 + r_t^*}. \quad (19)$$

The model exhibits two stochastic trends and is de-trended accordingly to characterize a stationary equilibrium. Following [Aguiar and Gopinath \(2007\)](#), [Garcia-Cicco et al. \(2010\)](#) and [Drechsel and Tenreyro \(2018\)](#), we divide the sectoral variables by the corresponding technology level $X_{M,t-1}$. We then calculate the deterministic steady state of this normalized model. All details are given in [Appendix B](#).

3.4. The model's main economic forces

Using the model, we will show that the empirical facts about the UK economy after the referendum can be understood as an adjustment to news about a lower future productivity growth rate in the tradable sector g_T . We do so using careful simulation experiments based on estimated parameters. Before we turn to the estimation and simulation steps, it is useful to highlight some key economic relationships in the model. This helps to guide intuition, and also gives grounds for our choice of a real model, which abstracts from nominal rigidities, but which is able generate the observed macroeconomic adjustments purely through relative price and intertemporal substitution effects.

The most central force behind the mechanism is that the relative price of output across sectors varies inversely with relative productivities. This is a key feature of multi-sector production economies: lower efficiency in making a good implies that it is more expensive relative to other goods. Therefore, a fall in productivity growth in the production of tradables will move the price of nontradable output (the real exchange rate) down. At its core, this is an intratemporal force that would also be present in a static two-sector model.¹⁴

In addition to the effect through productivity differentials, the fall in g_T will make the economy less productive overall and will thus lower permanent income. This in turn will reduce total consumption. When tradable and nontradable goods are complements, and given that productivity in the nontradable sector is unchanged, this fall in consumption demand in both sectors will generate further downward pressure on the relative price of nontradables.¹⁵

Importantly, in a fully dynamic model with forward-looking households and firms, both of these forces – the relative productivity force and the permanent income force – will kick in on impact when news about future productivity developments are received. The economic agents' forward-looking behavior implies that today's consumption-saving and investment allocation decisions depend on the full path of future productivity in both sectors. As a result, news about lower productivity growth in the tradable sector will lead to a reduction in the full path of the relative price of nontradable goods, as well as a fall in consumption that is spread out over the time. Since productivity initially remains unchanged, these forces both generate relatively favorable conditions for the tradable sector until the news actually materialize.

All of the above reasoning applies in a similar fashion with level shocks to relative productivities. Only a shock to growth rates in productivities, however, can generate the very persistent effects on interest rates that were observed empirically after Brexit. We examine this insight in more detail when we turn to the responses and interest rates and when we study alternative types of news shocks for robustness.

¹⁴Indeed, classic papers that have highlighted the relation between relative productivities and real exchange rates have done so in static models (Balassa, 1964; Samuelson, 1964). It is also worth pointing out that relative to these studies, where wages equate across sector, our model features different types of labor and capital across sectors. While this allows for additional margins of adjustment, we show in our simulations that wages in the two sectors move in the same direction in response to the productivity news shock.

¹⁵See Dornbusch (1983) for a simple model that generates similar effects.

4. NEW UK DATA SET AND ESTIMATION STRATEGY

Our strategy of bringing the model to the data and carrying out Brexit simulations consists of two steps. First, we estimate the model up to the quarter of the EU referendum (2016Q2), using our new UK macro data set. By exploiting variation in this data at business cycle frequency, this estimation pins down the structural parameters and balanced growth path of the model, which determine the starting point for the Brexit simulations. Since we interpret Brexit as a unique and unprecedented event, this estimation is based on unanticipated disturbances. Second, we simulate the impact of Brexit from 2016Q3 onwards. We feed an anticipated shock into the model. This section describes the first step. We present the construction of the data, how we select observables for estimation, as well as the estimation algorithm and settings. The second step follows in Section 5, which forms the core of the analysis.

Table 1: *Industries shares in non-tradable and tradable sector (%)*

	Non-tradable	Tradable
Agriculture	0.07	1.35
Mining and Quarrying	0.00	2.29
Manufacturing	0.89	20.99
Electricity, Gas, Steam Air Conditioning	3.19	0.00
Water Supply, Sewage, Waste Mgmt	1.07	0.91
Construction	10.93	0.00
Services	83.85	74.46

Notes. Nominal output shares of each SIC industry broken down by the classification into tradable and non-tradable sectors. This is shown as a snapshot for the year 2016. The supply and use tables are used to calculate the tradability index; nominal GVA data at factor prices are taken from low level aggregates published by the ONS.

4.1. Data and sectoral classification

We construct a new UK macroeconomic data set from 1987Q3 to 2016Q2, the period during which the UK was a full member of the EU. A key contribution of this paper is that we construct time series data for tradable and non-tradable GVA, hours, labor productivity and relative prices. These sectoral time series complement standard macroeconomic variables, such as aggregate GDP, consumption and investment. To the best of our knowledge, we are the first to apply such a classification on industry-level data from the UK and the first to generate sectoral time series aggregates that are used to estimate a structural model. Specifically, we use detailed industry-level GVA data from the UK Office of National statistics (ONS) and treat a given 2-digit SIC industry as tradable

if more than 10% of its final demand is traded, a standard cutoff suggested in the literature, for example in [Lombardo and Ravenna \(2012\)](#). We chain-link the data using the standard national accounts methodology employed by the ONS and also compute series for sectoral total hours by adding up hours data using the same industry classification. The time-series for sectoral labor productivities are then constructed by taking the ratio between sectoral GVA and total hours. Having aggregated detailed GVA data, we calculate the relative price of non-tradable goods by dividing the resulting implicit price deflators.

Our classification into tradable and non-tradable sectors leads to a roughly 50-50 split of total UK hours worked on average over the sample (our model will be calibrated accordingly). The same is approximately the case for the shares of nominal GVA. [Table 1](#) shows that in 2016, 21% of tradable GVA was produced by manufacturing sectors, 74% by services. The corresponding numbers for the non-tradable sector are 1% and 84%. The most important tradable manufacturing industries are motor vehicles, wearing apparel and alcoholic beverages and tobacco products, with food and beverage services, insurance services and financial services, representing key tradable services. For robustness, we also exclude government-related sectors from the non-tradable sector. The resulting dynamics in non-tradable GVA are very similar, with a correlation of 0.93 between non-tradable GVA and non-tradable GVA excluding government. More details on the classification methodology, including a list of all 2-digit industries and their classifications, are provided in [Appendix C](#).

4.2. Mapping the model to observable variables

As observable variables for the estimation, we use the ratios of nominal consumption, investment and trade balance to GDP, (demeaned) total hours (all available from 1987Q3), the quarterly growth rates of sectoral labor productivity (available from 1994Q1), the quarterly growth rate of the relative price of non-tradable goods (only available from 1997Q1), and the quarterly growth rate of the real effective exchange rate. [Table 2](#) presents our full list of observables. We make use of the Kalman filter to handle missing observations in the time-series of the sectoral labor productivities and the relative price of non-tradable goods. In the estimation step, we allow for measurement error for each of the constructed observable variables.¹⁶

In constructing observables to estimate our model we have addressed two

¹⁶We do so to capture measurement error arising from the aggregation of detailed industry level data and because the measures of GVA and GDP are not exactly equivalent.

Table 2: *Observables used for model estimation*

Time series	Transformation	Time period
Consumption / GDP	Nominal ratio	1987Q3 - 2016Q2
Investment / GDP	Nominal ratio	1987Q3 - 2016Q2
Trade balance / GDP	Nominal ratio	1987Q3 - 2016Q2
Total hours (scaled by labor force)	Dev. from mean	1987Q3 - 2016Q2
Labor productivity in T	% QoQ	1994Q1 - 2016Q2
Labor productivity in N	% QoQ	1994Q1 - 2016Q2
Relative price of non-tradable goods	% QoQ	1997Q1 - 2016Q2
Sterling real effective exchange rate	% QoQ	1987Q3 - 2016Q2

Notes. List of time series used to estimate the model. We use the Kalman filter to address the fact that observables become available different starting dates. The estimation period ends in 2016Q2, the quarter of the referendum.

key challenges. The first challenge entails the use of implicit price deflators to derive real quantities. Model consistent consumption and investment can be computed by deflating nominal consumption and investment by the tradable GVA implicit price deflator. However, since the resulting GVA deflators exhibits significant amount of noise (and are only available from 1997Q1), using them to calculate model consistent aggregates would imply discarding useful information (and having to rely on the introduction of additional measurement errors). To circumvent this issue, we use the ratios of nominal aggregates, rather than the growth rate of real quantities, as observable variables following [Christiano et al. \(2015\)](#). To estimate the structural parameters more precisely, our procedures requires that the values of the steady state ratios implied by the model match the averages in the data.

The second challenge is that there are two exchange rates concepts in the model: the relative price of non-tradables vis-a-vis tradables (an ‘internal exchange rate’) and the relative price of aggregate home consumption with respect to its foreign equivalent (an ‘external exchange rate’). We use data on both concepts to estimate the model. The internal exchange rate is calculated using the ratio of the implicit price deflators for non-tradable and tradable output. Mapping the external real exchange rate to the data requires assumptions about the rest of the world. First, preferences in the rest of the world are assumed to be the same as those in the home economy. Second, at business cycle frequencies, we further assume that stochastic trends of the tradable sectors at home and abroad are cointegrated. We define the real effective exchange rate as:

$$Q_t = \frac{\mathcal{E}_t \mathcal{P}_t^c}{\mathcal{P}_t^{c,*}}$$

where \mathcal{E}_t denotes the nominal exchange rate, \mathcal{P}_t^c the nominal price level of the home consumption bundle and $\mathcal{P}_t^{c,*}$ its foreign equivalent. Under the Law of One Price (LOOP), it follows that $\mathcal{P}_t^{c,*}/\mathcal{E}_t = \mathcal{P}_{Tt}$. Using this relation, and dividing the numerator and denominator in the definition of the real effective exchange rate by \mathcal{P}_{Tt} to obtain a ratio of real consumption bundles, we get

$$\mathcal{Q}_t = \frac{\mathcal{P}_t^c}{\mathcal{P}_t^{c,*}} = \frac{\mathcal{P}_t^c}{\bar{\zeta}_t}.$$

where $\bar{\zeta}_t$ captures exogenous movements in foreign prices ($\mathcal{P}_t^{c,*}$) and is governed by the following stochastic process

$$\ln \bar{\zeta}_t = \rho_{\bar{\zeta}} \ln \bar{\zeta}_{t-1} + \varepsilon_{\bar{\zeta}t} \quad \text{with} \quad \varepsilon_{\bar{\zeta}t} \sim \text{N}(0, \zeta_{\bar{\zeta}}). \quad (20)$$

This shock captures variation in the exchange rate that arises in the rest of the world. We emphasize that the exchange rate is an endogenous object and the shock will play a minor role. It allows us to use more information in the estimation to pin down structural parameters more precisely and can be interpreted as a persistent measurement error.

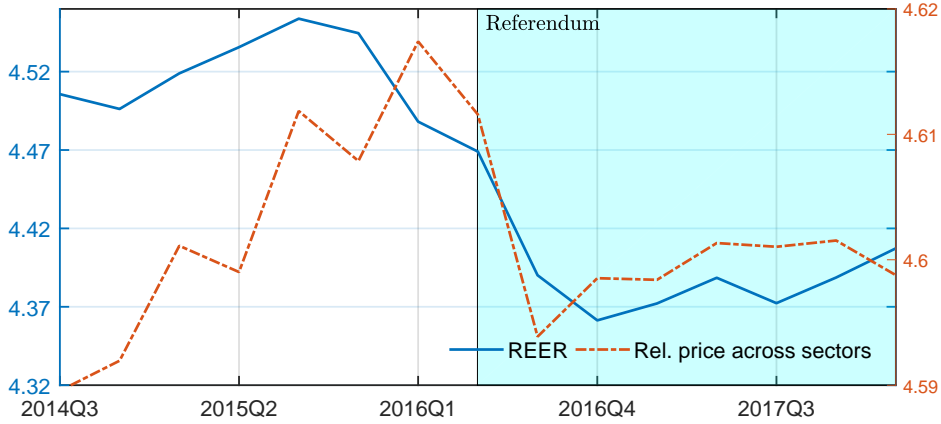


Figure 2: UK real effective exchange rate and relative price of nontradables

Notes. Time series of the (log) UK real effective exchange rate (left scale, source: Bank of England) as well as the (log) relative price of nontradable goods, calculated using the relative deflators of nontradable and tradable goods and services from our newly constructed data set (right scale). The shaded area represents the post-referendum period.

Figure 2 plots the behavior of the relative price of non-tradables and the UK real effective exchange rate around the referendum date. Both measures drop sharply and persistently around the time of the Brexit vote, which confirms the notion that the internal and external exchange rate concepts are closely linked.

As a final remark on mapping model exchange rates to their data counterpart, note that it is a prediction of our two-sector model that the real exchange

rate exhibits a stochastic trend in levels. While we use growth rates to estimate the model, Appendix D presents a discussion of this implication as well as some formal stationarity tests. There is indeed empirical evidence that the UK real exchange rate exhibits nonstationary behavior, consistent with the model.

4.3. Estimation procedure, calibration and priors

The model is estimated with Bayesian techniques using the observables shown in Table 2. The variation in UK macroeconomic time series from 1987 to 2016 is assumed to be driven by the collection of structural shocks present in the model in Section 3. The structural parameters and initial balanced growth path (starting point for our Brexit simulations experiments) are estimated based on the information coming from this variation prior to the Brexit vote.

Table 3: *Calibrated parameter values*

	Description	Source	Period	Value or target
γ	inverse of the IES	Drechsel and Tenreyro (2018)		2
σ	elast. of subst.	mid-range estimate		-0.5
δ_M	depreciation in M	ONS	1987 – 2016	$i/y = 0.181$
ϕ_N	capital adjustment cost (N)			4
θ_T	disutil. of labor (T)	ONS/own calc.	1994 – 2016	$n_T/n = 0.5$
θ_N	disutil. of labor (N)	ONS/own calc.	1994 – 2016	$n_N/n = 0.5$
$\frac{s}{y}$	govt exp./GDP	own calculations	1994 – 2016	0.184
$\frac{tb}{y}$	trade balance/GDP	own calculations	1994 – 2016	-0.015
\bar{g}_T	trend growth rate of productivity (T)	ONS/own calc.	1990 – 2016	1.83% ann.
\bar{g}_N	trend growth rate of productivity (N)	ONS/own calc.	1990 – 2016	1.02% ann.
β	discount factor			$r^* = 0.01$
ψ	debt-elasticity of premium			5×10^{-6}

The model has a relatively small number of parameters due to its parsimonious structure. We calibrate some of the key parameters based on empirical targets, and estimate the remaining ones, including those governing the dynamics of the shock processes. We first comment on the set of calibrated parameters, summarized in Table 3, and then turn to the estimated ones. Given that we do not have information on sectoral consumption to estimate σ we fix this parameter to -0.5. This corresponds to an elasticity of substitution equal to $\eta = \frac{1}{1-\sigma} = 0.67$, which is within the range of estimates in the literature, and which gives rise to a gross complementarity across consumption aggregates that helps generating unconditional output co-movement across sectors. The depreciation rates are assumed to be equal across sectors and match the sample

average of the ratio of nominal investment to GDP (18.12%). The fact that we only observe aggregate investment allows us to identify only the relative adjustment costs between sectors, so we fix ϕ_N and estimate ϕ_T . We choose θ_N and θ_T to target the empirically observed equal distribution of hours worked across sectors in our data on tradable and non-tradable sectors. We calculate the ratios $\frac{s}{y} = 0.184$ and $\frac{tb}{y} = -0.015$ using ONS data. We also compute \bar{g}_T and \bar{g}_N directly from the data. The discount factor (β) is set to match a quarterly foreign real interest rate of 1%. Finally, the elasticity of the foreign interest rate with respect to debt (ψ) is set to a very small number (5×10^{-6}). We do so to exclude the debt-elastic premium as part of the core mechanism, that is, Brexit does not make the UK more default prone in our model.

Table 4: Priors and posteriors for estimated parameters

Description	Prior			Posterior				
	Prior Distr.	Mean	Std	Mode	Mean	90% HPDI		
Structural parameters								
c_T/c	share T consumption	Gaussian	0.60	0.01	0.59	0.59	0.57	0.61
ω	elasticity of labor supply	Gaussian	1.6	0.125	1.99	1.99	1.85	2.13
α_T	capital share in T	Gaussian	0.316	0.0075	0.31	0.31	0.30	0.32
α_N	capital share in N	Gaussian	0.245	0.0075	0.25	0.25	0.24	0.26
ϕ_T	capital adjustment cost in T	Gaussian	4	1	9.65	9.65	8.45	10.85
Shocks								
ζ_N^s	st.dev. of prod. growth shock in N	Inv. Gamma	0.1	2	0.014	0.014	0.012	0.016
ζ_T^s	st.dev. of prod. growth shock in T	Inv. Gamma	0.1	2	0.014	0.014	0.012	0.016
ζ_s	st.dev. of expenditure shock	Inv. Gamma	0.1	2	0.036	0.036	0.031	0.04
ζ_μ	st.dev. of foreign interest rate shock	Inv. Gamma	0.1	2	0.01	0.01	0.009	0.011
ζ_ν	st.dev. of risk-premium shock	Inv. Gamma	0.1	2	0.035	0.036	0.03	0.042
ζ_T^a	st.dev. of TFP level shock in T	Inv. Gamma	0.1	2	0.013	0.013	0.011	0.015
ζ_N^a	st.dev. of TFP level shock in N	Inv. Gamma	0.1	2	0.013	0.013	0.011	0.012
ζ_ξ	st.dev. of exchange rate shock	Inv. Gamma	0.1	2	0.026	0.026	0.022	0.029
ρ_N^s	persistence of prod. growth shock in N	Beta	0.5	0.2	0.23	0.25	0.07	0.43
ρ_T^s	persistence of prod. growth shock in T	Beta	0.5	0.2	0.12	0.15	0.04	0.25
ρ_s	persistence of expenditure shock	Beta	0.5	0.2	0.88	0.86	0.79	0.94
ρ_μ	persistence of foreign interest rate shock	Beta	0.5	0.2	0.03	0.04	0.01	0.08
ρ_ν	persistence of risk-premium shock	Beta	0.5	0.2	0.94	0.93	0.88	0.98
ρ_N^a	persistence of TFP level shock in N	Beta	0.5	0.2	0.80	0.75	0.58	0.93
ρ_T^a	persistence of TFP level shock in T	Beta	0.5	0.2	0.97	0.97	0.95	0.99
ρ_ξ	persistence of exchange rate shock	Beta	0.5	0.2	0.95	0.94	0.91	0.99
Measurement errors								
l_N	labor productivity in N	Inv. Gamma	0.1	1	0.013	0.013	0.011	0.015
l_T	labor productivity in T	Inv. Gamma	0.1	1	0.014	0.014	0.012	0.016
l_P	relative price	Inv. Gamma	0.1	1	0.014	0.015	0.012	0.017

Table 4 shows the specification of our priors for the set of parameters that we estimate. Using the ONS supply-and-use tables for the period 1997-2015, we compute the annual shares of tradables into aggregate consumption and then set the prior mean of ζ to target the sample average of 0.6. We calculate the sample means of the sectoral capital shares to be $\alpha_T = 0.316$ and $\alpha_N = 0.245$ in the tradable and non-tradable sectors. In principle, we could calibrate these parameters, but we introduce some estimation uncertainty given that the

calculated values come out to be somewhat smaller than in existing studies. We center the prior means relatively tightly around the sample averages and then compute their posterior distributions. We set the prior mean for ϕ_T equal to the same value as ϕ_N .

Our posterior estimates are also presented in Table 4. The posterior mean of the elasticity of labor supply ω is estimated to be 1.99, in line with standard values in the literature. The mean estimate of the investment adjustment cost in the tradable sector (9.65) is somewhat higher than in related studies, but plausible given the relatively low depreciation rates. The discount factor, the temporary sectoral TFP, expenditure and foreign price are estimated to be highly persistent stochastic processes ($\rho_\nu = 0.93$, $\rho_N^a = 0.75$, $\rho_T^a = 0.97$, $\rho_s = 0.86$ and $\rho_\xi = 0.94$ respectively). A common finding in most models featuring stochastic trends in labour-augmenting productivity is that the estimated persistence of the growth shocks tends to be relatively low ($\rho_N^g = 0.25$ and $\rho_T^g = 0.15$). The foreign interest rate shock displays very little persistence ($\rho_\mu = 0.04$). The posterior standard deviation of measurement errors (denoted by ι) for sectoral labor productivities and the relative price of non-tradable goods are similar and statistically different from zero.

5. THE BREXIT SIMULATION

In this section we present our Brexit simulation, which shows that the economy's response to news about a reduction in tradable sector productivity growth matches the empirical facts for the post-referendum UK economy. Naturally, the interpretation of Brexit as productivity news abstracts from a wide range of specific implications of Brexit. We dedicate Section 6 to a detailed discussion on how these relate to a reduction in tradable sector productivity growth.

5.1. Settings of the Brexit simulation

We model the news shock to g_{Tt} as persistent, but temporary. As a result, there is a permanent effect on the *level* of future tradable sector productivity, but the growth rate eventually fully recovers. In the short-run, a fully permanent reduction in the growth rate delivers qualitatively similar results. In contrast, as we will show in alternative simulations, a temporary news shock about only the level of productivity does not suffice to explain the empirical patterns in UK macroeconomic data. To implement the persistent reduction, we modify the process for g_{Tt} in the estimated version of the model. While that estimated

process captures business cycle movements during the period of EU membership, it is less suitable for analyzing a structural change of the type we are investigating. Specifically, in the simulation g_{Tt} is determined by

$$\begin{aligned}\ln(g_{Tt}) &= \varrho_T^g \ln(g_{Tt-1}) + (1 - \varrho_T^g) \ln(\tilde{g}_{Tt}), \\ \ln(\tilde{g}_{T,t}) &= \tilde{\varrho}_T^g \ln(\tilde{g}_{Tt-1}) + (1 - \tilde{\varrho}_T^g) \ln(\tilde{g}_T) + \varepsilon_{Tt}^g,\end{aligned}$$

where we set $\tilde{\varrho}_T^g = 0.95$ and $\varrho_T^g = 0.8$. This implies that the initial fall in tradable sector productivity growth is gradual and that the total reduction in the *level* of productivity in the tradable sector is complete after about 30 years.

Table 5: *Estimates of long-run effects of WTO rules on UK trade and GDP*

Study	Estimated reduction in	
	trade (%)	GDP (%)
Ebell and Warren (2016)	21–29	2.7–3.7
IMF (2018)		5.2–7.8
Kierzenkowski et al. (2016)	10–20	2.7–7.5
UK Government (2018)	13–18	6.3–10.7

The experiment is configured so that the future reduction in productivity growth is fully anticipated. The economy starts on a balanced growth path in quarter 0. In quarter 1, it is revealed that there will be a persistent reduction in productivity growth in the tradable sector from quarter 11 onward. This anticipation horizon broadly mimics the planned timeline for EU exit following the referendum.¹⁷ We calibrate the scale of the shock with reference to existing studies of the potential effects of Brexit on trade. We use those that focus on the move to trading arrangements governed by World Trade Organization (WTO) rules. This is not necessarily because we believe this is the most likely outcome, but because the underlying assumptions about the eventual trading arrangements are more consistent across studies. Table 5 summarizes recent estimates. We calibrate our experiment so that trade falls by 10% in the long run, in line with the smaller estimates of the effects of moving to WTO rules. Our results could therefore be regarded either as a lower bound estimate of a transition to WTO rules or as a simulation of a transition to a relatively closer trading relationship with the EU.

¹⁷The referendum was held on 23 June 2016. The UK government triggered Article 50 of the Lisbon treaty on 30 March 2017, with the United Kingdom’s membership of the European Union to end within two years of that date. The end date of the UK’s EU membership was subsequently postponed as the negotiation process developed.

Our simulation abstracts from *uncertainty* about the news shock and its timing. Of course, the event of the Brexit vote is likely to have affected both first and second moments. Our simulations demonstrate that first moment effects go a long way in rationalizing the adjustments in the economy that we document in the data. We refer to the work of [Bloom et al. \(2019\)](#) for a firm-level analysis that focuses explicitly on uncertainty. Using a structural model [Steinberg \(2017\)](#) finds that uncertainty plays a relatively small role in the context of Brexit. [Caldara et al. \(2020\)](#) provide a model for US economy that features both news and uncertainty about trade. We will present various robustness checks of our simulation with respect to the profile and the timing of the news shock.

5.2. Main results of the Brexit simulation

Figures 3 and 4 present our simulation results. The main variables of interest, for which we have presented empirical facts in Section 2, are examined in Figure 3. Figure 4 contains the dynamics of additional model variables which will help explain the economic mechanisms at play in our Brexit experiment. In each panel, the shaded area (quarters 1 to 10) marks the phase after the news about the shock have been revealed, but before it materializes (in quarter 11). This mimics the shaded area after the referendum but before Brexit implementation in Figure 1. The black dashed line represents the counterfactual balanced growth path along which the Brexit shock does not occur.

Panel A of Figure 3 shows the trajectory of tradable sector labor-augmenting productivity growth that we feed into the model. In quarter 11, productivity growth falls for several quarters before starting to recover gradually. The cumulative effect of the shock is a permanent reduction in the level of tradable sector productivity of around 10%. During the anticipation phase, agents know about the future change, but tradable sector productivity growth is unchanged from the baseline balanced growth path. The news about this future productivity growth trajectory leads to an immediate fall in the relative price of *non-tradable* output, which will become relatively more efficient to produce in the future. This is one of the core economic forces of the model fleshed out in Section 3.4. Indeed Panel E shows that the price of non-tradable output falls immediately, well before productivity growth has changed. During the anticipation phase, tradable goods are relatively attractive to produce because productivity growth has not yet begun to fall (the “sweet spot” effect). As shown in panels C and D, this effect encourages production of tradable goods.

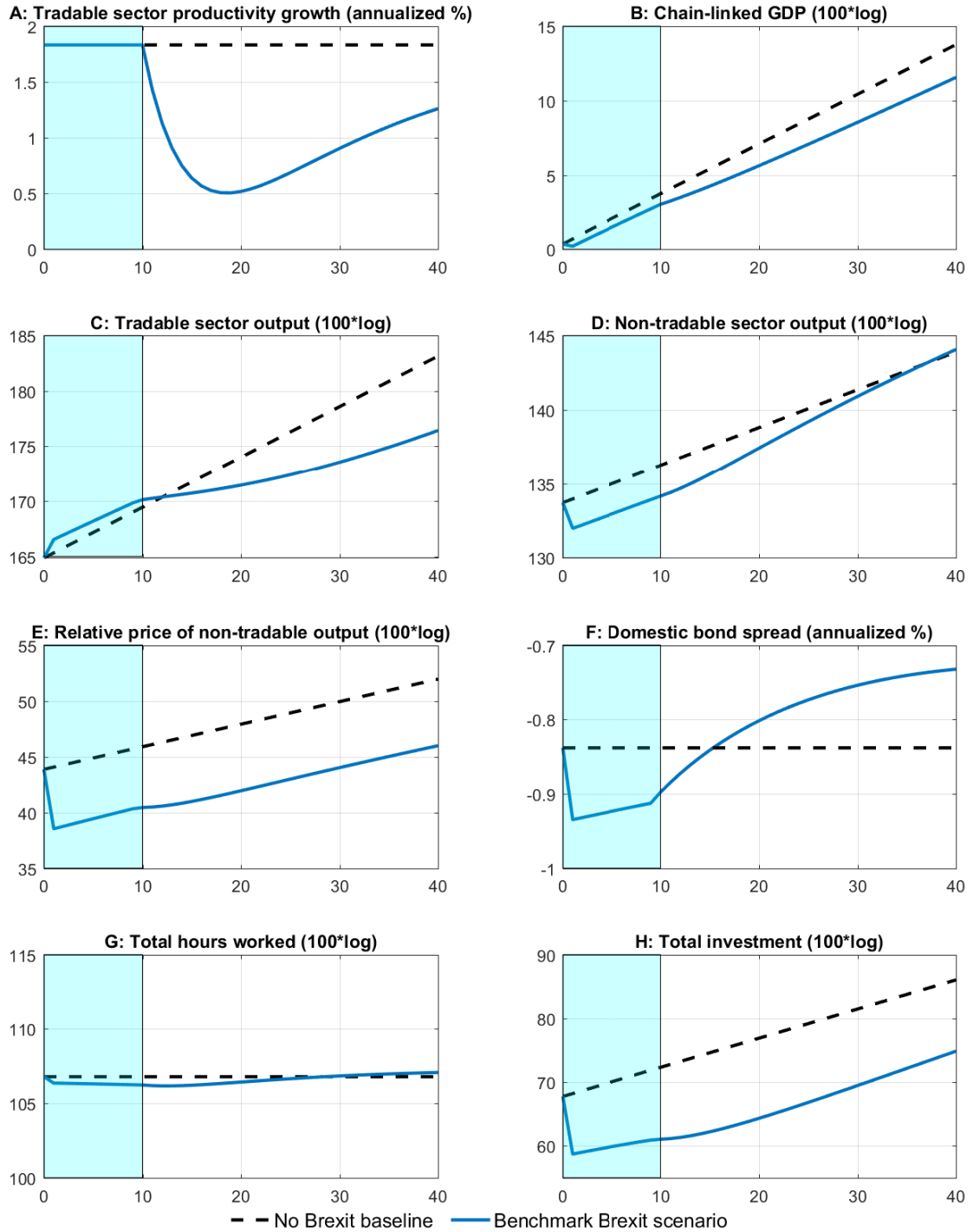


Figure 3: Main model responses in benchmark Brexit scenario

Once tradable sector productivity falls, however, the incentives to produce tradable goods decline and output and the trade balance fall in the longer term. The profile of non-tradable output is the mirror image of tradable output. During the anticipation phase, producing non-tradable output is relatively unattractive and output declines. Once tradable sector productivity falls, it is now optimal to produce more non-tradables and output increases in the longer term. Eventually non-tradable output converges back to the pre-shock trajectory, since the balanced growth path for the non-tradable sector is unaffected by the shock.

The net effect of the opposing forces on the tradable and non-tradable sectors gives rise to a negative but relatively muted initial response of GDP (panel B).¹⁸ The persistent negative effect on GDP builds over time and its long-run level is around 3% lower relative to the no-Brexit balanced growth path. This is towards the smaller end of the range of estimates in Table 5, consistent with the fact that the scale of the shock we study generates a relatively small reduction in trade compared to the studies cited. In any case, it is clear that aggregate activity starts decelerating relative to what agents would have expected in the absence of the news shock, in line with the data.

Panel F examines the effect on UK interest rates, by plotting the return on bonds denominated in nontradable goods as a spread over the return on bonds paying tradable goods (expressed in tradable units). This is the notion of domestic interest rates in our model. Note that the bond rate denominated in tradable goods is exogenously given and only changes minimally due to the debt elastic premium, so this plot is driven almost entirely by the interest rate on bonds paying nontradables.¹⁹ The plot shows that the domestic UK interest rate falls persistently during the anticipation phase, before eventually rising above the steady-state level. These dynamics reflect the behavior of the marginal product of capital, which falls in the near term because returns to production in the non-tradable sector are temporarily lower. As we discuss further below, only a news shock to the growth rate (rather than levels) generates an interest rate response as persistent as the one observed in UK data after the Brexit vote.

¹⁸The chain-linked GDP growth rate is computed as:

$$g_t^{GDP} = \omega_{T,t} \frac{y_{Tt}}{y_{Tt-1}} g_{Tt-1} + (1 - \omega_{T,t}) \frac{y_{Nt}}{y_{Nt-1}} g_{Nt-1},$$

where $\omega_{T,t}$ is computed as a one-year rolling average of the expenditure share on tradable goods, $\frac{y_{Tt}}{y_{Tt} + p_t y_{Nt}}$. This approximates a national accounts treatment.

¹⁹The near term rise in exports reduces foreign debt and hence the interest rate. By calibrating ψ to be small, the effect on r_t^* is restricted to a few basis points.

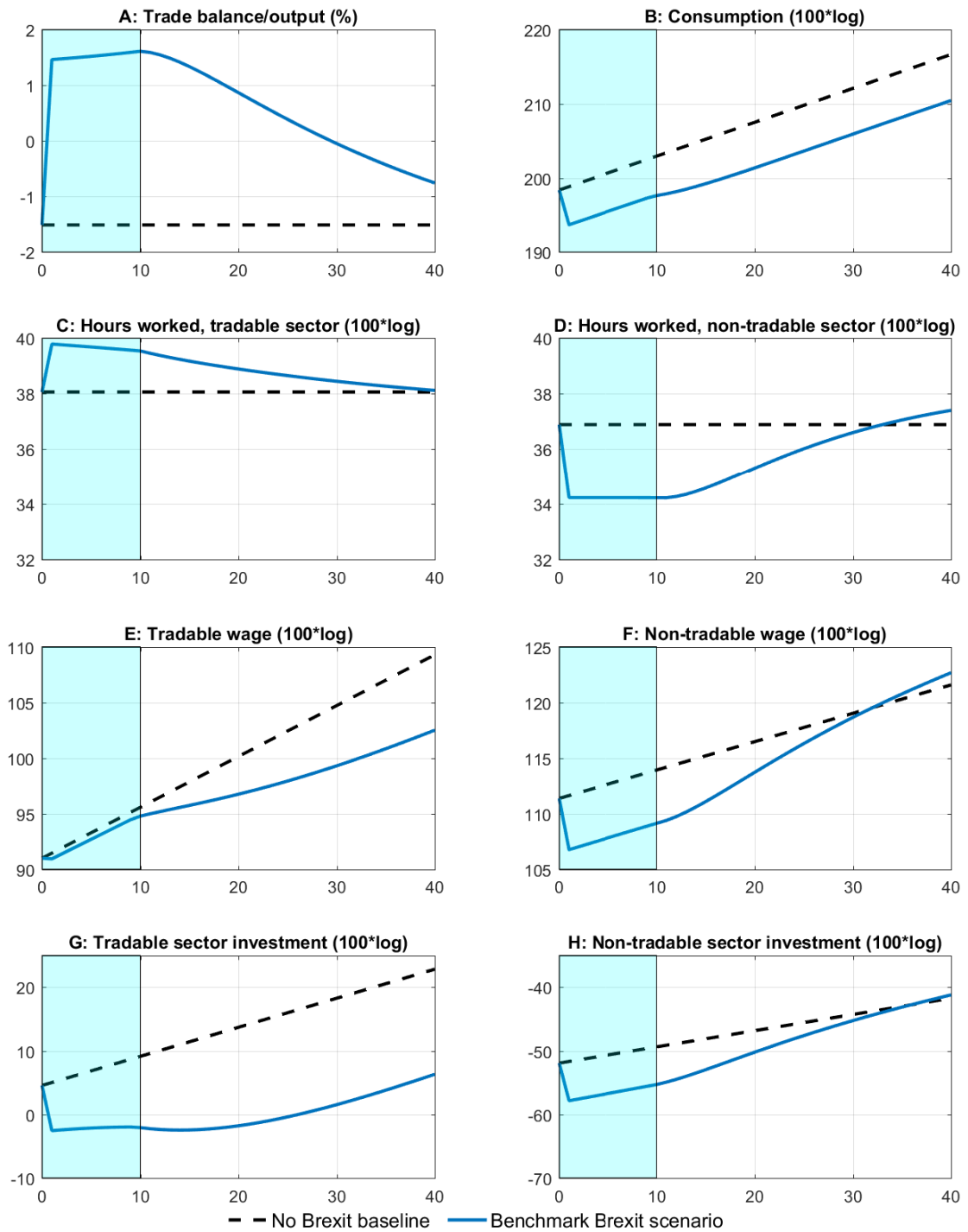


Figure 4: Additional model responses in benchmark Brexit scenario

Panels G and H of Figure 3 turn to factors of production. In line with the dynamics we document in post referendum UK data, a gradual slowdown in investment starts taking place after the Brexit news arrives. Aggregate hours, however, remain relatively stable. The former response is related to the fact that the economy as a whole is now known to be permanently less productive in the future, so agents reduce the extent to which they move resources to the future. The latter response reflects the “sweet spot” effect: since activity today is still relatively strong – driven by the devaluation and robust activity in the tradable sector – aggregate labor input does not fall materially. We additionally provide a sectoral breakdown of the hours and investment responses below.

Figure 4 presents additional model responses, which go beyond the variables examined Section 2. These responses allow us to further unpack the mechanism behind the simulations, by highlighting how the economy adjusts to the productivity news by shifting resources across expenditure components, sectors and time. Panel A shows the trade balance scaled by output, which rises significantly upon announcement of the Brexit news. This is a direct reflection of the dynamics of relative prices and higher activity in the tradable sector, with the UK temporarily exporting more goods and services.²⁰

Panel B shows that consumption growth slows down in response to the Brexit news. While our discussion in Section 2 did not focus on consumption dynamics, we show in Appendix A that this consumption slowdown is visible in UK data, albeit with a delay. In the model, the slowdown in consumption growth is a manifestation of the permanent income force brought about by the future productivity reduction, as explained in Section 3.4. While in our narrative for the Brexit news, we mainly emphasize the effect of productivity differentials on relative prices, this permanent income effect puts additional downward pressure on the exchange rate. When tradable and nontradable goods are complements, and given that productivity in the nontradable sector remains unchanged, reduced consumption demand in an economy with less permanent income reduces the relative price of nontradables by even more.

Panels C and D examine the sectoral labor responses. The apparent inter-sectoral reallocation is consistent with the main mechanism underpinning our results: during the anticipation phase, the tradable sector becomes relatively attractive, but this effect is reversed once tradable sector productivity actually falls. Labor input is reduced in the non-tradable sector and rises in the tradable sector during the anticipation period, to support increased production of

²⁰The increase in the trade balance is in line with the data, as shown in Appendix A.

tradable goods. Overall, total employment is reduced minimally during the anticipation phase. This pattern starts to reverse once productivity growth in the tradable sector actually falls. As shown in panels E and F, wages fall in both sectors.

Panel G shows that investment in the tradable sector falls abruptly before slowly converging to a new, lower, level. Investment prospects in the tradable sector are dominated by the longer-term outlook for productivity. In contrast, panel H shows that, while non-tradable investment initially falls, it subsequently rises above the baseline path. As discussed above, these dynamics add up to a deceleration in aggregate investment following the news shock, consistent with the patterns observed in the data, and owing to lower overall productivity prospects in the economy.

5.3. Comparison with empirical patterns

Despite the simplicity of our two-sector SOE model, our simulation is broadly consistent with the macroeconomic dynamics of the UK economy since presented in Section 2. The simulation predicts a long-run reduction of GDP of around 3%, relative to the baseline path. As shown in Figure 1, the IMF forecasts of UK GDP were reduced by roughly 0.5% per year following the referendum, amounting to a reduction in the level of UK GDP (relative to the pre-referendum forecast) of around 2.5% over the five-year forecast horizon.²¹ The simulated path of quantities and prices in the tradable and non-tradable sectors also match the data fairly well, with a sharp decline in both the relative price of non-tradable output and the real effective exchange rate around the referendum date, as well as a marked slowdown in GVA growth in the non-tradable relative to the tradable sector. The simulation implies that the news of Brexit triggers a fall in interest rates, consistent with the decline in UK long-term government bond yields following the referendum shown in Figure 1, though the fall in the data is larger and more protracted.²² The simulation predicts a temporary boom in exports, mirroring the UK's relatively strong export performance following the referendum. While the movement in the trade balance in the data is relatively modest, the pickup in the export to GDP ratio is around 2 percentage points, similar to the response of the trade balance

²¹Born et al. (2019) provide an estimate of 2% based on constructing a no-Brexit counterfactual.

²²The comparison with the data is complicated by the range of factors affecting government bond yields that are omitted from the model. The model abstracts entirely from nominal prices and risk.

to GDP ratio in our simulation.²³ The volatility in the trade data makes it difficult to draw strong conclusions, but according to the latest vintage of data, calendar-year export growth in 2017 was 5.6%, which is substantially above the Bank of England’s (pre-referendum) May 2016 *Inflation Report* forecast of 1.25% (see [Bank of England, 2016](#), Table 5.E, page 34). Moreover, as our simulation predicts, there was a substantial fall in UK investment following the referendum result in the data. Estimates presented by [Carney \(2019\)](#) suggest that the effects of the referendum result may have reduced UK business investment by around 25%. At the same time, employment has been strong in the data, consistent with the pick-up in total hours in our simulation. Our model simulations suggest an economy-wide shift from capital towards labor, a phenomenon directly in line with firm-level survey evidence provided by [Bloom et al. \(2019\)](#).²⁴

Our overall conclusion is that the model performs well in matching the broad contours of UK macroeconomic performance since the referendum. The interpretation of the referendum outcome as news about a reduction future productivity growth in the tradable sector is consistent with the UK’s adjustment visible in the data.

5.4. Robustness

Appendix E shows that our main results are robust with respect to three important assumptions. First, the results are qualitatively similar for a range of plausible variation in the timing of the decline in tradable sector productivity growth. Appendix E.1 reports results for cases in which the shock is anticipated to occur 5 and 15 quarters in the future, alongside the baseline assumption of 11 quarters. The timing of reversals in inter-sectoral allocation changes, but the dominant force underpinning the scenario is the long-run decline in the level of tradable sector productivity. Since the long-run decline is independent of the timing of the productivity growth reduction, the results from the variants considered are very similar. Second, the responses are robust to the assumption

²³The model abstracts from gross trade flows (differentiated imports and exports) making it less straightforward to map from model concepts to the data. Mechanically, the fact that the trade balance increases by less than exports suggests that imports rose following the referendum. In the absence of strong expenditure switching effects, the value of imports may increase because of the higher price of imports associated with the depreciation of sterling.

²⁴These observations suggest that the source of the shock matters for the pattern of sectoral reallocations. Though a formal comparison is beyond the scope of this paper, it is instructive to compare the simulation with the behavior of the UK economy following the depreciation of sterling associated with the UK’s exit from the Exchange Rate Mechanism in 1992. The period following that depreciation saw a significant investment boom, more apparent in tradables than non-tradables. Our simulation does not have these properties, because the depreciation is the result of the anticipation of a negative shock that depresses the returns on investment.

that productivity falls more sharply than the benchmark case. Again, this reflects the fact that the dominant force is the effect on the long-run level of tradable sector productivity. Holding the scale of this effect constant, a faster decline in tradable sector productivity has relatively little effect on the dynamic responses, even in the near term. Appendix E.2 provides the details. Third, the responses are not sensitive to the assumption used to close the model, that is, to ensure a determinate return to the steady-state net foreign asset position (see the discussion below equation (13)). Our model achieves this through the presence of a debt-elastic premium on foreign borrowing. Appendix E.3 demonstrates that the simulations are almost identical in a variant of the model in which r_t^* is fixed and a determinate net foreign asset position is achieved by the assumption that there is constant growth in the population of households.

5.5. Comparison with other structural shocks

Simulations using other shocks are inconsistent with the empirical patterns in UK data. We demonstrate this by repeating our simulation for two different news shocks. The first is news about a reduction in the *level* (rather than the growth rate) of productivity in the tradable sector. The second generates news about an acceleration in the growth rate of productivity growth in the non-tradable sector (rather than a deceleration in the tradable sector). Note that in principle, we can repeat the simulation for any of the various shocks present in our model. We chose to focus on these two, as they enter as a very similar *wedge* as our benchmark shock in the equations of the model. They are likely “competitors” in terms of generating comparable dynamics.²⁵

The first alternative shock we study reduces a_{Tt} in the future. Again, we focus on a shock that arrives with certainty 11 quarters ahead. We calibrate the shock to generate the same reduction in trade as in our benchmark simulation in Section 5.2. This experiment is obviously a relevant comparison, as it works through the same economic margin but reduces the level rather than the growth of productivity in the production of traded output. The full simulation results are provided in Appendix F. The simulation shows that the level shock indeed generates dynamics in activity across sectors that are qualitatively similar. We do see a boom in tradable and a reduction in non-tradable production, accompanied by a fall in the relative price. However, what stands out as the important difference is the response of relative returns. A level shock

²⁵Simulations based on other shocks, such as expenditure, labor supply, etc., are available upon request.

generates a short-lived interest rate differential at the exact time when the shock materializes. This is clearly inconsistent with the persistent decoupling of UK from world interest rates in the data starting immediately after the referendum outcome. The Euler equations of the model need variation in the technology *growth rate* to imply *persistent* changes in returns. This tells us that the adjustment of the UK economy is more plausibly related to expectations about growth rates than about levels.

The motivation for studying the second alternative shock, an acceleration the growth rate of non-tradable productivity, is to verify whether our mechanism is symmetric in the sense that it only requires a variation in the ratio g_{Tt}/g_{Nt} but does depend on whether numerator or denominator are changed. The corresponding results in Appendix F show that for a calibration of the experiment that generates the same on-impact reduction in the relative price of non-tradable goods, a very (perhaps implausibly) large increase in productivity growth in the non-tradable sector is required. More importantly, the longer run projection of this simulation is vastly different from our benchmark. The acceleration in future g_{Nt} implies a large long-run expansion in GDP as well as a large increase in the real rate on bonds denominated in non-tradable goods. While the unobserved longer run outcomes cannot be used to reject the explanation with certainty, we are not aware of any theoretical argument which would link Brexit with a productivity growth improvement in the non-tradable sector. On the other hand, a number of mechanisms link Brexit to a reduction in tradable productivity growth, as we discuss in the next section.

6. DRIVERS OF TRADABLE SECTOR PRODUCTIVITY GROWTH

We conceptualize the 2016 referendum outcome as news about a future slow-down in productivity growth in the UK's tradable sector. Our simulations show that forward-looking decisions of households and firms in a general equilibrium environment lead to aggregate and sectoral economic dynamics that closely resemble the behavior of the UK economy immediately after the Brexit vote. The appeal of our exercise is that it provides a simple framework to understand the short-run adjustments to the referendum news: when nothing fundamental has actually materialized, but when the economy is already responding strongly.

Naturally, the drawback of our analysis is that it cannot speak to the more specific changes in the economy that Brexit is expected to entail. We now consider more detailed structural adjustments that firms and households may expect, and explain how these affect productivity growth in the tradable sector.

We focus on three expected consequences of Brexit: barriers to trade in goods and services, reduced capital flows and declining labor mobility. All three have featured prominently in the public debate around the referendum.²⁶ Importantly, a large-scale survey of UK firms by Bloom et al. (2019) reveals that these potential consequences feature prominently as self-reported sources of Brexit uncertainty of economic decision-makers. For each aspect, we explain how they link to a fall in $g_{T,t}$, by highlighting relevant research.

6.1. Barriers to trade in goods and services

Leaving the single market is commonly understood to bring about impediments to trade, most notably tariffs. There is a large strand of research that has provided integrated theories of international trade and economic growth, for example the seminal work of Grossman and Helpman (1989, 1991). While we simulate an exogenous shock to the productivity growth rate, these authors provide a theory of how trade barriers endogenously determine the rate of growth of an economy.²⁷ This line of research highlights distortions in the allocation of resources towards technical change as central to the mechanism by which trade in goods and services endogenously affects growth. For example, if importing human-capital intensive goods becomes more difficult, substituting these imports with domestic production will absorb factor inputs that would otherwise be used for innovative activities. Therefore Brexit can be expected to harm trade in a way that growth-enhancing resources in the UK's productive tradable industries have to be diverted to import substitution.

In addition to this flow of resources from productive to less productive industries, it is possible that trade barriers also lower import competition within given industries, which in turn reduces the incentive to innovate in those industries. Indeed Bloom et al. (2015) find empirically that trade liberalization

²⁶For an up-to-date description of the specifics of the EU Withdrawal Agreement, see <https://www.bankofengland.co.uk/eu-withdrawal/eu-withdrawal-agreement-act>.

²⁷More recent contribution include Sampson (2016) and Impullitti and Licandro (2018), who show that trade liberalization increases growth via firm selection. Note that trade barriers are frequently studied in an influential class of trade models sometimes referred to as *New Trade Theory (NNTT)*. Prominent examples include Eaton and Kortum (2002) and Melitz (2003). The effect of tariffs in these models is typically studied by means of comparison between different steady states, so do not allow us to directly think about the impact on the economy's (expected) growth dynamics.

fosters technical change across and within firms.²⁸ These effects of looming barriers to trade in goods and services constitute plausible mechanisms through which decision makers indeed expect productivity growth in the tradable sector to fall as a consequence of Brexit.

6.2. Reduced capital flows

Trade barriers and regulations may reduce cross-border capital mobility and hinder in particular inflows of foreign direct investment (FDI) by multinational corporations that operate in the UK and the EU. The longer run effects of this are explicitly studied in a recent paper by [McGrattan and Waddle \(2020\)](#). Using a multi-country dynamic equilibrium model, these authors present a Brexit simulation in which the economy converges to a new balanced growth path with lower aggregate growth. This is largely driven by a reduction in investment in technology capital, which in their baseline experiment is shown to fall by almost one third. While the model of [McGrattan and Waddle \(2020\)](#) does not feature separate sectors for tradable and non-tradable output, it is conceivable that this technology capital channel operates largely through investments made by tradable producers.²⁹ Along these lines, [Benigno et al. \(2020\)](#) provide a recent discussion as well as a host of references on the role of technological spillovers through capital flows. The prospect of diminished FDI as part of leaving the EU therefore provides a rationale for diminished productivity growth expectations in the tradable sector.

6.3. Lower labor mobility

[Portes and Forte \(2017\)](#) assess the potential restrictions on movement of workers after Brexit, and conclude that they will likely have a significant negative impact on UK growth and productivity. [Bloom et al. \(2019\)](#) find that firms with a larger share of EU migrants in their workforce are significantly more concerned about the uncertain prospects of the withdrawal process. To the

²⁸While we focus on increased trade barriers and how they reduce productivity growth, [Bloom et al. \(2015\)](#) consider a reduction in trade barriers and positive effects on productivity growth. It is plausible to think that the mechanism and the resulting macroeconomic dynamics we highlight in this paper operate (with opposite sign) for trade liberalizations. One interesting example is the Canada-US trade agreement. After it was announced, but before it was signed, the Canadian Dollar experienced a strong appreciation.

²⁹A reduction in capital flows may also reduce productivity through broader aggregate demand forces. [Anzoategui et al. \(2019\)](#) show in a closed economy setting how endogenous technology adoption leads to a persistent slowdown in productivity growth after a large demand contraction.

extent that Brexit may reduce the UK's ability to attract labor that is employed in productivity-enhancing activities in the tradable sector, the news about Brexit generates expectations of a lower trajectory for productivity growth in tradable production. This is again akin to the factor allocation aspect at the heart of frameworks linking trade and growth such as [Grossman and Helpman \(1991\)](#).

In brief, expected impediments to the free flow of goods and services, capital and labor provide a range of mechanisms that could reduce future productivity growth in the tradable sector. As phrased by [Gourinchas and Hale \(2017\)](#), the common component of these fundamental channels is a “deglobalization shock” that reduces specialization and efficiency. While each of the channels deserves greater attention in their own right, the contribution of our work lies in summarizing the mechanism in a concise formal framework that sheds light on the short-run macroeconomic response of the UK economy to Brexit. The direct link between the setup of our experiments and the underlying drivers of growth in the tradable sector makes this a credible and powerful exercise.

7. CONCLUSION

While Brexit encompasses a variety of economic forces, this paper provides a concise narrative to explain its macroeconomic impact. This narrative is of both academic and policy relevance. On the academic front, our analysis contributes to the news shock literature by studying a large quasi-natural policy experiment, the announcement of Brexit. Negative news to productivity growth in the tradable sector – our way to conceptualize Brexit – results in a large adjustment in the real exchange rate. This devaluation almost paradoxically ends up benefiting the tradable sector itself, creating a sweet spot for exporters. This pattern is temporary and we predict will eventually reverse. Our mechanism is in line with patterns observed in macroeconomic data, and helps to rationalize even some of the surprising survey evidence, in which exporters report no (first-order) effects from Brexit ([Hassan et al., 2020](#)). On the policy front, the paper contributes to our understanding of the impact and propagation of a shock that has governed the macroeconomic and political dynamics of the UK for several years and might find parallels in nations that decide to increase trade barriers in future. As such, the mechanism we identify can contribute as a key input into more complex macroeconomic models used for forecasting and policy analysis.

REFERENCES

- Acemoglu, D. and Guerrieri, V. (2008), 'Capital deepening and nonbalanced economic growth', *Journal of Political Economy* **116**(3), 467–498.
- Aguiar, M. and Gopinath, G. (2007), 'Emerging Market Business Cycles: The Cycle Is the Trend', *Journal of Political Economy* **115**, 69–102.
- Anzoategui, D., Comin, D., Gertler, M. and Martinez, J. (2019), 'Endogenous technology adoption and r&d as sources of business cycle persistence', *American Economic Journal: Macroeconomics* **11**(3), 67–110.
- Balassa, B. (1964), 'The purchasing-power parity doctrine: a reappraisal', *Journal of political Economy* **72**(6), 584–596.
- Bank of England (2016), 'Inflation report', *May 2016* .
- Beaudry, P. and Portier, F. (2006), 'Stock Prices, News, and Economic Fluctuations', *American Economic Review* **96**(4), 1293–1307.
- Becker, S. O., Fetzer, T. and Novy, D. (2017), 'Who voted for Brexit? A comprehensive district-level analysis', *Economic Policy* **32**(92), 601–650.
- Benigno, G., Fornaro, L. and Wolf, M. (2020), 'The global financial resource curse', *Working paper* .
- Betts, C. M. and Kehoe, T. J. (2006), 'U.S. real exchange rate fluctuations and relative price fluctuations', *Journal of Monetary Economics* **53**(7), 1297–1326.
- Bloom, N., Bunn, P., Chen, S., Mizen, P., Smietanka, P. and Thwaites, G. (2019), 'The Impact of Brexit on UK Firms', NBER Working Papers 26218, National Bureau of Economic Research, Inc.
- Bloom, N., Bunn, P., Chen, S., Mizen, P., Smietanka, P., Thwaites, G. and Young, G. (2018), 'Brexit and uncertainty: Insights from the decision maker panel', *Fiscal Studies* **39**(4), 555–580.
- Bloom, N., Draca, M. and Van Reenen, J. (2015), 'Trade Induced Technical Change? The Impact of Chinese Imports on Innovation, IT and Productivity', *The Review of Economic Studies* **83**(1), 87–117.
- Born, B., Müller, G. J., Schularick, M. and Sedláček, P. (2019), 'The Costs of Economic Nationalism: Evidence from the Brexit Experiment', *Economic Journal* **129**(623), 2722–2744.

- Breinlich, H., Leromain, E., Novy, D. and Sampson, T. (2019), Exchange Rates and Consumer Prices: Evidence from Brexit, CEP Discussion Papers dp1667, Centre for Economic Performance, LSE.
- Broadbent, B. (2017a), 'Brexit and interest rates', speech given at the London School of Economics, Bank of England.
- Broadbent, B. (2017b), 'Brexit and the pound', speech given at Imperial College, Bank of England.
- Caldara, D., Iacoviello, M., Molligo, P., Prestipino, A. and Raffo, A. (2020), 'The economic effects of trade policy uncertainty', *Journal of Monetary Economics* **109**(C), 38–59.
- Carney, M. (2019), 'The global outlook', *Speech given at Frobisher's Hall, Barbican Centre, London, UK* .
- Christiano, L. J., Eichenbaum, M. S. and Trabandt, M. (2015), 'Understanding the Great Recession', *American Economic Journal: Macroeconomics* **7**(1), 110–167.
- Davies, R. B. and Studnicka, Z. (2018), 'The heterogeneous impact of brexit: Early indications from the ftse', *European Economic Review* **110**, 1 – 17.
- De Gregorio, J., Giovannini, A. and Wolf, H. C. (1994), 'International evidence on tradables and nontradables inflation', *European Economic Review* **38**(6), 1225–1244.
- Dhingra, S., Huang, H., Ottaviano, G., Paulo Pessoa, J., Sampson, T. and Van Reenen, J. (2017), 'The costs and benefits of leaving the eu: trade effects', *Economic Policy* **32**(92), 651–705.
- Dornbusch, R. (1983), 'Real interest rates, home goods, and optimal external borrowing', *Journal of Political Economy* **91**(1), 141–153.
- Drechsel, T. and Tenreyro, S. (2018), 'Commodity booms and busts in emerging economies', *Journal of International Economics* **112**(C), 200–218.
- Eaton, J. and Kortum, S. (2002), 'Technology, geography, and trade', *Econometrica* **70**(5), 1741–1779.
- Ebell, M. and Warren, J. (2016), 'The long-term economic impact of leaving the EU', *National Institute Economic Review* **236**(1), 121–138.

- Erceg, C., Prestipino, A. and Raffo, A. (2018), The Macroeconomic Effect of Trade Policy, 2018 Meeting Papers 221, SED.
- Faccini, R. and Palombo, E. (2020), 'Policy uncertainty in brexit u.k.', *American Economic Review: Insights (forthcoming)* .
- Fetzer, T. (2018), Did Austerity Cause Brexit?, CESifo Working Paper Series 7159, CESifo Group Munich.
- Forbes, K., Hjortsoe, I. and Nenova, T. (2018), 'The shocks matter: Improving our estimates of exchange rate pass-through', *Journal of International Economics* **114**(C), 255–275.
- Garcia-Cicco, J., Pancrazi, R. and Uribe, M. (2010), 'Real Business Cycles in Emerging Countries?', *American Economic Review* **100**(5), 2510–2531.
- Gortz, C. and Tsoukalas, J. D. (2018), 'Sectoral TFP news shocks', *Economics Letters* **168**(C), 31–36.
- Gourinchas, P.-O. and Hale, G. (2017), 'Brexit: Whither the pound?', *FRBSF Economic Letter* **11**.
- Gourinchas, P.-O. and Rey, H. (2014), 'External adjustment, global imbalances, valuation effects', *Handbook of International Economics* **4**, 585 – 645.
- Graziano, A., Handley, K. and Limão, N. (2018), Brexit uncertainty and trade disintegration, Technical report, NBER.
- Greenwood, J., Hercowitz, Z. and Huffman, G. W. (1988), 'Investment, capacity utilization, and the real business cycle', *The American Economic Review* **78**(3), 402–417.
- Greenwood, J., Hercowitz, Z. and Krusell, P. (2000), 'The role of investment-specific technological change in the business cycle', *European Economic Review* **44**(1), 91–115.
- Grossman, G. M. and Helpman, E. (1989), Growth and welfare in a small open economy, Technical report, NBER.
- Grossman, G. M. and Helpman, E. (1991), 'Trade, knowledge spillovers, and growth', *European economic review* **35**(2-3), 517–526.

- Hassan, T. A., Hollander, S., van Lent, L. and Tahoun, A. (2020), The global impact of brexit uncertainty, Technical report, National Bureau of Economic Research.
- IMF (2018), 'United Kingdom: Selected issues', *IMF Country Report No. 18/317* .
- Impullitti, G. and Licandro, O. (2018), 'Trade, firm selection and innovation: The competition channel', *The Economic Journal* **128**(608), 189–229.
- Jaimovich, N. and Rebelo, S. (2008), 'News and business cycles in open economies', *Journal of Money, Credit and Banking* **40**(8), 1699–1711.
- Jaimovich, N. and Rebelo, S. (2009), 'Can News about the Future Drive the Business Cycle?', *American Economic Review* **99**(4), 1097–1118.
- Justiniano, A., Primiceri, G. and Tambalotti, A. (2011), 'Investment Shocks and the Relative Price of Investment', *Review of Economic Dynamics* **14**(1), 101–121.
- Kamber, G., Theodoridis, K. and Thoenissen, C. (2017), 'News-driven business cycles in small open economies', *Journal of International Economics* **105**(C), 77–89.
- Kierzenkowski, R., Pain, N., Rusticelli, E. and Zwart, S. (2016), 'The economic consequences of Brexit', *OECD Economic Policy Paper No. 16* .
- Kose, M. A. (2002), 'Explaining business cycles in small open economies: 'How much do world prices matter?'' , *Journal of International Economics* **56**(2), 299–327.
- Lombardo, G. and Ravenna, F. (2012), 'The size of the tradable and non-tradable sectors: Evidence from input–output tables for 25 countries', *Economics Letters* **116**(3), 558–561.
- McGrattan, E. R. and Waddle, A. (2020), 'The impact of brexit on foreign investment and production', *American Economic Journal: Macroeconomics* **12**(1), 76–103.
- Melitz, M. (2003), 'The impact of trade on aggregate industry productivity and intra-industry reallocations', *Econometrica* **71**(6), 1695–1725.
- Mendoza, E. G. (1991), 'Real business cycles in a small open economy', *The American Economic Review* **81**(4), 797–818.

- Portes, J. and Forte, G. (2017), 'The economic impact of Brexit-induced reductions in migration', *Oxford Review of Economic Policy* 33, S31–S44.
- Rabanal, P. and Rubio-Ramirez, J. F. (2015), 'Can international macroeconomic models explain low-frequency movements of real exchange rates?', *Journal of International Economics* 96(1), 199–211.
- Sampson, T. (2016), 'Dynamic selection: an idea flows theory of entry, trade, and growth', *The Quarterly Journal of Economics* 131(1), 315–380.
- Sampson, T. (2017), 'Brexit: the economics of international disintegration', *Journal of Economic Perspectives* 31(4), 163–84.
- Samuelson, P. A. (1964), 'Theoretical notes on trade problems', *The Review of Economics and Statistics* 46(2), 145–154.
- Schmitt-Grohe, S. and Uribe, M. (2003), 'Closing small open economy models', *Journal of International Economics* 61(1), 163–185.
- Schmitt-Grohe, S. and Uribe, M. (2012), 'What's News in Business Cycles', *Econometrica* 80(6), 2733–2764.
- Siena, D. (2020), 'The Euro Area Periphery and Imbalances: Is it an Anticipation Story?', *Review of Economic Dynamics* .
- Steinberg, J. (2017), Brexit and the Macroeconomic Impact of Trade Policy Uncertainty, 2017 Meeting Papers 216, Society for Economic Dynamics.
- Stockman, A. C. and Tesar, L. L. (1995), 'Tastes and technology in a two-country model of the business cycle: Explaining international comovements', *The American Economic Review* 85(1), 168–185.
- UK Government (2018), 'EU Exit: Long-Term Economic Analysis', *Parliamentary Report prepared by Her Majesty's Government* .
- Vlieghe, G. (2019), 'The economic outlook: fading global tailwinds, intensifying Brexit headwinds', *Speech at Resolution Foundation, UK* .
- Vukotić, M. (2019), 'Sectoral Effects of News Shocks', *Oxford Bulletin of Economics and Statistics* 81(2), 215–249.

APPENDIX FOR ONLINE PUBLICATION

Appendix [A](#) provides some additional empirical facts about the period following the Brexit vote. Appendix [B](#) contains details about the model. It presents the optimality conditions ([B.1](#)), describes the de-trending and stationary equilibrium of the model ([B.2](#)), and determines its steady state ([B.3](#)). Appendix [C](#) contains details on the data construction. Appendix [D](#) examines properties of the different measures of the UK real exchange rate in the data. Appendix [E](#) examines the sensitivity of the Brexit simulations to alternative assumptions about the timing of the shock and the speed of the fall in productivity growth. Appendix [F](#) presents simulations based on alternative news shocks. Appendix [G](#) lays out the alternative model with population growth.

A. ADDITIONAL FACTS ON MACROECONOMIC ADJUSTMENTS

A.1. The advanced economy outlook before and after the Brexit vote

Figure A.1 repeats Panel A of Figure 1 in the main text for the group of all *advanced economies* rather than for the UK alone. We again present the IMF's annual growth forecasts at different points in time before and after the time of the Brexit referendum (April 2016, October 2017, April 2019). It is clear that the growth outlook for advanced economies as a whole also became slightly gloomier during the period. This can likely be attributed to heightened uncertainty around global trade disputes, in particular between the US and China. Importantly, however, the downward revisions in growth forecasts were much sharper for the UK economy after the Brexit vote. For example, as shown by Figure 1 in the main text, between 2016 and 2019 the forecast for the year 2021 for the UK were revised downward from 2.1% to 1.5%, while those for the group of advanced economies were cut from 1.9% to 1.7%. The IMF revised its outlook for the UK downward roughly three times as sharply as for all advanced economies taken together. This highlights that the expected slowdown of UK growth does not appear to be driven primarily by a weakening global economy.

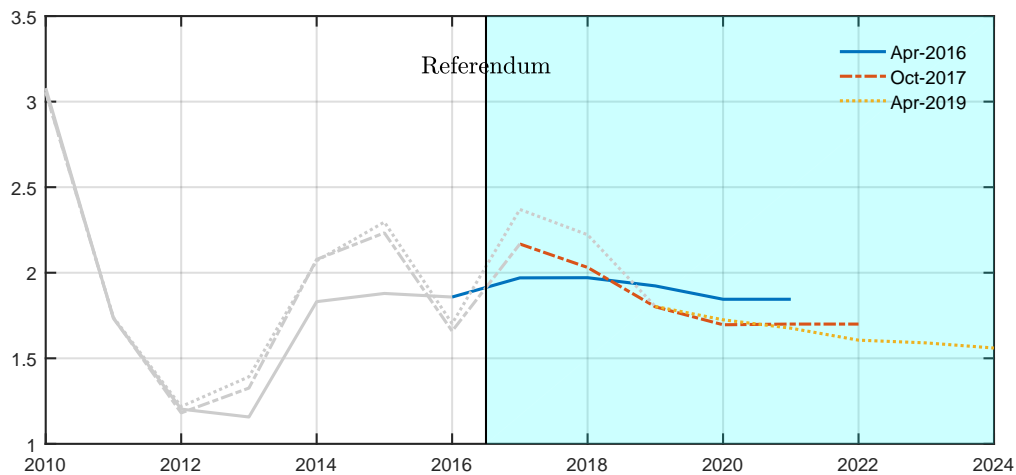


Figure A.1: Annual growth forecasts for all advanced economies by the IMF (%)

Notes. 5-year ahead IMF forecasts of annual GDP growth for the group of advanced economies for April 2016, October 2017 and April 2019. The first forecast is before the referendum, the other two are after, and the three forecast dates have equal distance to each other. In each case, observations prior to the forecast date (plotted in gray) reflect actual observations, which differ across vintages due to data revisions (source: World Economic Outlook).

A.2. Exports and trade balance

In Section 2 of the main text we show that the activity in the tradable sector grew robustly after the Brexit vote, while the nontradable sector experienced a slowdown. The “sweet spot” interpretation we provide for this pattern is supported with further evidence in Figure A.2. Here we plot exports and the trade balance, both measured as a percentage of GDP. While there are some meaningful swings in the pre-referendum period, the figure shows that UK trade developed relatively robustly following the Brexit vote.

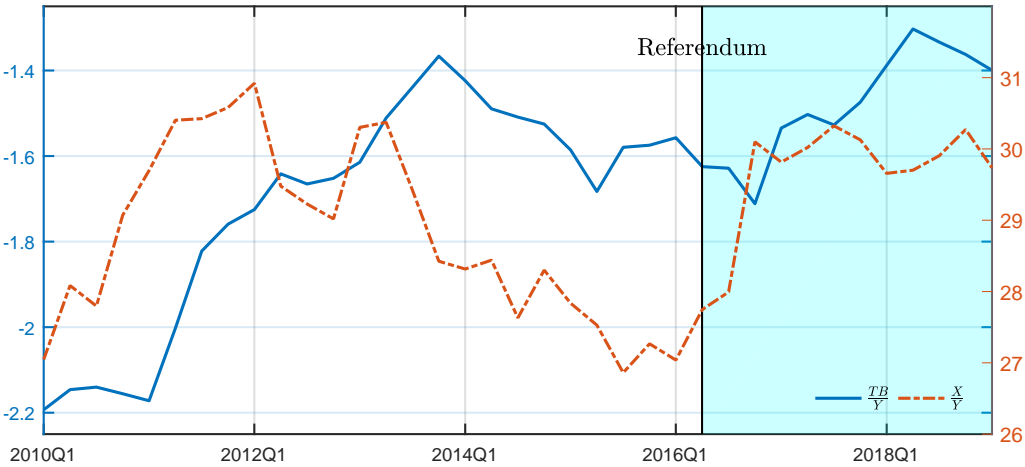


Figure A.2: UK exports and trade balance pre and post referendum

Notes. Time series for the 3-year moving average of the ratio between the trade balance and GDP (left scale) and the ratio between exports and GDP (right scale) (source: ONS and own calculations).

A.3. Consumption

Figure A.3 shows the rate of annual growth of aggregate consumption in the UK before and after the June 2016 referendum. It is visible that consumption did not fall for about two quarters after the referendum, but displays a sharp drop thereafter. This is consistent with the idea that, upon arrival of the Brexit news, households foresee a reduction in permanent income. In Section 5 we explain that the fall in permanent income is one of the mechanisms at work in the model that can generate adjustments in consumption that are in line with this observation.

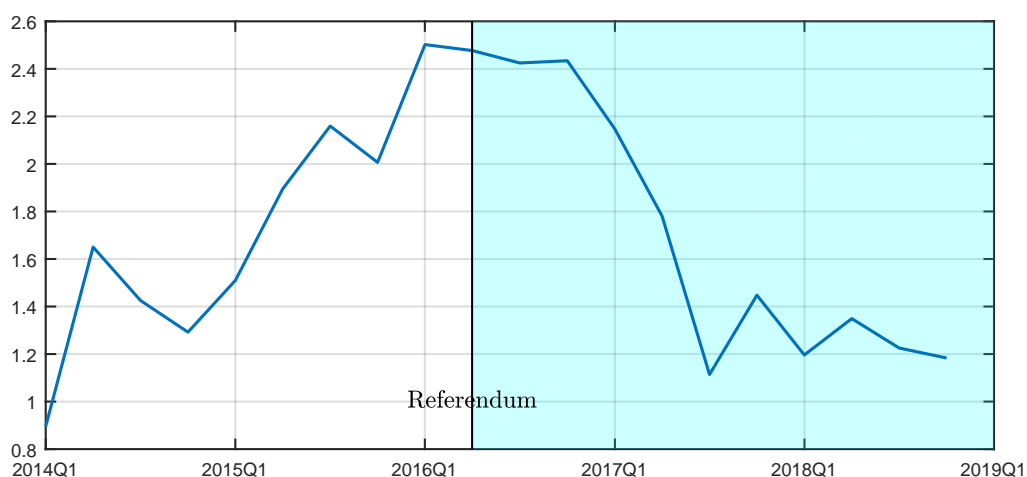


Figure A.3: *Year-on-Year Consumption growth pre and post referendum*

Notes. Yearly annual growth rate (in %) of aggregate UK consumption expenditures (source: ONS and own calculations)

B. MODEL DETAILS

B.1. First order conditions

The optimality conditions of firms are:

$$r_{Tt}^k = \alpha_T a_{Tt} K_{Tt}^{\alpha_T - 1} (X_{Tt} n_{Tt})^{1 - \alpha_T}, \quad (21)$$

$$W_{Tt} = (1 - \alpha_T) a_{Tt} K_{Tt}^{\alpha_T} (X_{Tt} n_{Tt})^{-\alpha_T} X_{Tt}, \quad (22)$$

$$r_{Nt}^k = \alpha_N a_{Nt} K_{Nt}^{\alpha_N - 1} (X_{Nt} n_{Nt})^{1 - \alpha_N}, \quad (23)$$

and

$$W_{Nt} = P_t (1 - \alpha_N) a_{Nt} K_{Nt}^{\alpha_N} (X_{Nt} n_{Nt})^{-\alpha_N} X_{Nt}. \quad (24)$$

These conditions state that sectoral factors of productions are paid their marginal products.

The household's optimality conditions with respect to C_{Tt} , C_{Nt} , n_{Tt} , n_{Nt} , K_{Tt+1} , K_{Nt+1} , B_{t+1}^* , B_{t+1} are:

$$\left[C_t - X_{Tt-1} \omega^{-1} (\theta_T n_{Tt}^\omega + \theta_N n_{Nt}^\omega) \right]^{-\gamma} \left(\frac{C_{Tt}}{\zeta C_t} \right)^{\sigma-1} = X_{Tt-1}^{-\gamma} \lambda_t, \quad (25)$$

$$\left[C_t - X_{Tt-1} \omega^{-1} (\theta_T n_{Tt}^\omega + \theta_N n_{Nt}^\omega) \right]^{-\gamma} \left[\frac{C_{Nt}}{(1 - \zeta) C_t} \frac{X_{Tt-1}}{X_{Nt-1}} \right]^{\sigma-1} \frac{X_{Tt-1}}{X_{Nt-1}} = X_{Tt-1}^{-\gamma} \lambda_t P_t, \quad (26)$$

$$\left[C_t - X_{Tt-1} \omega^{-1} (\theta_T n_{Tt}^\omega + \theta_N n_{Nt}^\omega) \right]^{-\gamma} \theta_T X_{Tt-1} n_{Tt}^{\omega-1} = X_{Tt-1}^{-\gamma} \lambda_t W_{Tt}, \quad (27)$$

$$\left[C_t - X_{Tt-1} \omega^{-1} (\theta_T n_{Tt}^\omega + \theta_N n_{Nt}^\omega) \right]^{-\gamma} \theta_N X_{Tt-1} n_{Nt}^{\omega-1} = X_{Tt-1}^{-\gamma} \lambda_t W_{Nt}, \quad (28)$$

$$X_{Tt-1}^{-\gamma} \lambda_t v_t \left[1 + \phi_T \left(\frac{K_{Tt+1}}{K_{Tt}} - \bar{g}_T \right) \right] = X_{Tt}^{-\gamma} \beta \mathbb{E}_t \lambda_{t+1} v_{t+1} \left[r_{Tt+1}^k + (1 - \delta) + \phi_T \left(\frac{K_{Tt+2}}{K_{Tt+1}} - \bar{g}_T \right) \frac{K_{Tt+2}}{K_{Tt+1}} - \frac{\phi_T}{2} \left(\frac{K_{Tt+2}}{K_{Tt+1}} - \bar{g}_T \right)^2 \right], \quad (29)$$

$$X_{Tt-1}^{-\gamma} \lambda_t \nu_t P_t \left[1 + \phi_N \left(\frac{K_{Nt+1}}{K_{Nt}} - \bar{g}_N \right) \right] = X_{Tt}^{-\gamma} \beta \mathbb{E}_t \lambda_{t+1} \nu_{t+1} P_{t+1} \left[r_{Nt+1}^k + (1 - \delta) + \phi_N \left(\frac{K_{Nt+2}}{K_{Nt+1}} - \bar{g}_N \right) \frac{K_{Nt+2}}{K_{Nt+1}} - \frac{\phi_N}{2} \left(\frac{K_{Nt+2}}{K_{Nt+1}} - \bar{g}_N \right)^2 \right], \quad (30)$$

$$X_{Tt-1}^{-\gamma} \lambda_t \nu_t = X_{Tt}^{-\gamma} \beta (1 + r_t^*) \mathbb{E}_t \lambda_{t+1} \nu_{t+1}, \quad (31)$$

and

$$X_{Tt-1}^{-\gamma} \lambda_t \nu_t P_t = X_{Tt}^{-\gamma} \beta (1 + r_t) \mathbb{E}_t \lambda_{t+1} \nu_{t+1} P_{t+1}. \quad (32)$$

Equations (25)-(26) pin down the optimal tradable and non-tradable consumption choices, equations (27)-(28) state the labor supply choices as increasing functions of sectoral wages, equations (29)-(30) denote the Euler equations associated to sectoral physical capital and equations (31)-(32) the Euler equations for bonds. Note that, by taking the ratio between (26) and (25), we can determine the relative price of non-tradable output (the internal exchange rate).

B.2. Stationary equilibrium

We now proceed to characterize the stationary equilibrium by introducing "lower-case" variables, denoting the detrended counterparts of non-stationary variables. Define $c_t = \frac{C_t}{X_{Tt-1}}$, $c_{Tt} = \frac{C_{Tt}}{X_{Tt-1}}$, $c_{Nt} = \frac{C_{Nt}}{X_{Nt-1}}$, $K_{Tt} = \frac{K_{Tt}}{X_{Tt-1}}$, $K_{Nt} = \frac{K_{Nt}}{X_{Nt-1}}$, $p_t = P_t \frac{X_{Nt-1}}{X_{Tt-1}}$.

The household first order conditions in normalized forms become

$$c_t = \left[\zeta^{1-\sigma} c_{Tt}^\sigma + (1 - \zeta)^{1-\sigma} (c_{Nt})^\sigma \right]^{\frac{1}{\sigma}}, \quad (33)$$

$$\left(c_t - \left(\frac{\theta_T}{\omega} n_{Tt}^\omega + \frac{\theta_N}{\omega} n_{Nt}^\omega \right) \right)^{-\gamma} \left(\frac{c_{Tt}}{\zeta c_t} \right)^{\sigma-1} = \lambda_t, \quad (34)$$

$$\left(c_t - \left(\frac{\theta_T}{\omega} n_{Tt}^\omega + \frac{\theta_N}{\omega} n_{Nt}^\omega \right) \right)^{-\gamma} \left(\frac{c_{Nt}}{(1 - \zeta) c_t} \right)^{\sigma-1} = p_t \lambda_t, \quad (35)$$

$$\left(c_t - \left(\frac{\theta_T}{\omega} n_{Tt}^\omega + \frac{\theta_N}{\omega} n_{Nt}^\omega \right) \right)^{-\gamma} \theta_T n_{Tt}^{\omega-1} = \lambda_t w_{Tt}, \quad (36)$$

$$\left(c_t - \left(\frac{\theta_T}{\omega} n_{Tt}^\omega + \frac{\theta_N}{\omega} n_{Nt}^\omega \right) \right)^{-\gamma} \theta_N n_{Nt}^{\omega-1} = \lambda_t w_{Nt}, \quad (37)$$

$$\begin{aligned} \lambda_t v_t \left[1 + \phi_T \left(\frac{k_{Tt+1}}{k_{Tt}} g_{Tt} - \bar{g}_T \right) \right] &= \beta g_{Tt}^{-\gamma} \mathbb{E}_t \left\{ \lambda_{t+1} v_{t+1} \left[r_{Tt+1}^k + (1 - \delta) \right. \right. \\ &\left. \left. + \phi_T \left(\frac{k_{Tt+2}}{k_{Tt+1}} g_{Tt+1} - \bar{g}_T \right) \frac{k_{Tt+2}}{k_{Tt+1}} - \frac{\phi_T}{2} \left(\frac{k_{Tt+2}}{k_{Tt+1}} g_{Tt+1} - \bar{g}_T \right)^2 \right] \right\}, \end{aligned} \quad (38)$$

$$\begin{aligned} \lambda_t v_t p_t \left[1 + \phi_N \left(\frac{k_{Nt+1}}{k_{Nt}} g_{Nt} - \bar{g}_N \right) \right] &= \beta \frac{g_{Tt}^{1-\gamma}}{g_{Nt}} \mathbb{E}_t \left\{ \lambda_{t+1} v_{t+1} p_{t+1} \left[r_{Nt+1}^k + (1 - \delta) \right. \right. \\ &\left. \left. + \phi_N \left(\frac{k_{Nt+2}}{k_{Nt+1}} g_{Nt+1} - \bar{g}_N \right) \frac{k_{Nt+2}}{k_{Nt+1}} - \frac{\phi_N}{2} \left(\frac{k_{Nt+2}}{k_{Nt+1}} g_{Nt+1} - \bar{g}_N \right)^2 \right] \right\}, \end{aligned} \quad (39)$$

$$\lambda_t v_t = \beta (1 + r_t^*) g_{Tt}^{-\gamma} \mathbb{E}_t \lambda_{t+1} v_{t+1}, \quad (40)$$

and

$$\lambda_t v_t p_t = \beta (1 + r_t) \frac{g_{Tt}^{1-\gamma}}{g_{Nt}} \mathbb{E}_t p_{t+1} \lambda_{t+1} v_{t+1}. \quad (41)$$

The firms' first order conditions become

$$r_{Tt}^k = \alpha_T a_{Tt} k_{Tt}^{\alpha_T - 1} (n_{Tt} g_{Tt})^{1 - \alpha_T}, \quad (42)$$

$$w_{Tt} = (1 - \alpha_T) a_{Tt} k_{Tt}^{\alpha_T} (n_{Tt})^{-\alpha_T} g_{Tt}^{1 - \alpha_T}, \quad (43)$$

$$r_{Nt}^k = \alpha_N a_{Nt} k_{Nt}^{\alpha_N - 1} (n_{Nt} g_{Nt})^{1 - \alpha_N} \quad (44)$$

and

$$w_{Nt} = p_t (1 - \alpha_N) a_{Nt} k_{Nt}^{\alpha_N} n_{Nt}^{-\alpha_N} g_{Nt}^{1 - \alpha_N}. \quad (45)$$

The normalized constraints are

$$y_{Tt} = c_{Tt} + i_{Tt} + \frac{\phi_T}{2} \left(\frac{k_{Tt+1}}{k_{Tt}} g_{Tt} - \bar{g}_T \right)^2 + t b_t, \quad (46)$$

$$y_{Nt} \left(1 - \frac{s}{y_N} s_t \right) = c_{Nt} + i_{Nt} + \frac{\phi_N}{2} \left(\frac{k_{Nt+1}}{k_{Nt}} g_{Nt} - \bar{g}_N \right)^2, \quad (47)$$

$$i_{Tt} = k_{Tt+1} g_{Tt} - (1 - \delta) k_{Tt}, \quad (48)$$

$$i_{Nt} = k_{Nt+1} g_{Nt} - (1 - \delta) k_{Nt}, \quad (49)$$

$$y_{Tt} = a_{Tt} k_{Tt}^{\alpha_T} g_{Tt}^{1 - \alpha_T} n_{Tt}^{1 - \alpha_T}, \quad (50)$$

$$y_{Nt} = a_{Nt} k_{Nt}^{\alpha_N} g_{Nt}^{1-\alpha_N} n_{Nt}^{1-\alpha_N}, \quad (51)$$

and

$$tb_t = b_t^* - \frac{b_{t+1}^*}{1+r_t^*} g_{Tt}. \quad (52)$$

B.3. Steady state

B.3.1 Analytical derivation of the steady state (for Brexit simulation purposes)

We remove time subscripts to compute the steady state values. From equations (38)-(41), it follows that

$$\beta = \frac{1}{(1+r^*) \bar{g}_T^{-\gamma}}, \quad (53)$$

$$r = \frac{g_N}{\beta \bar{g}_T^{1-\gamma}} - 1, \quad (54)$$

$$r_T^k = \frac{1}{\beta \bar{g}_T^{-\gamma}} - (1-\delta), \quad (55)$$

$$r_N^k = \frac{\bar{g}_N}{\beta \bar{g}_T^{1-\gamma}} - (1-\delta). \quad (56)$$

From the rental rates of capital in (42) and (44) we recover the sectoral capital to labor ratios

$$\frac{k_T}{n_T} = \left(\frac{r_T^k}{\alpha_T} \right)^{\frac{1}{\alpha_T-1}} \bar{g}_T, \quad (57)$$

$$\frac{k_N}{n_N} = \left(\frac{r_N^k}{\alpha_N} \right)^{\frac{1}{\alpha_N-1}} \bar{g}_N. \quad (58)$$

Steady state wages can be calculated from equations (43) and (45) as

$$w_T = (1-\alpha_T) \bar{g}_T^{1-\alpha_T} \left(\frac{k_T}{n_T} \right)^{\alpha_T} \quad (59)$$

and

$$\frac{w_N}{p} = (1-\alpha_N) \bar{g}_N^{1-\alpha_N} \left(\frac{k_N}{n_N} \right)^{\alpha_N}. \quad (60)$$

By calibrating $\frac{s}{y_N}$, and using equations (47) and (51), we can express non-tradable consumption as:

$$c_N = \left\{ \left(\frac{k_N}{n_N} \right)^{\alpha_N} \bar{g}_N^{1-\alpha_N} \left(1 - \frac{s}{y_N} \right) - [\bar{g}_N - (1-\delta)] \frac{k_N}{n_N} \right\} n_N = A_N n_N. \quad (61)$$

By calibrating the ratio $\frac{tb}{y}$ and using equation (46) and (50), we can express consumption in T in terms of n_N ,

$$c_T = \left\{ \left(\frac{k_T}{n_T} \right)^{\alpha_T} \bar{g}_T^{1-\alpha_T} \left(1 - \frac{tb}{y} \right) - [\bar{g}_T - (1-\delta)] \frac{k_T}{n_T} \right\} n_T = A_T n_T. \quad (62)$$

Dividing equations (35) by (34), we can get an expression for p . We use equations (61) and (62) to recover the ratio of sectoral hours,

$$p = \left[\frac{\zeta c_N}{(1-\zeta) c_T} \right]^{\sigma-1} = \left[\frac{\zeta A_N n_N}{(1-\zeta) A_T n_T} \right]^{\sigma-1} \Rightarrow \frac{n_N}{n_T} = p^{\frac{1}{\sigma-1}} \frac{(1-\zeta) A_T}{\zeta A_N}. \quad (63)$$

We divide equation (37) by equation (36) and substitute for n_N/n_T as above to get

$$\frac{w_N}{p w_T} = \frac{1}{p} \frac{\theta_N}{\theta_T} \left(\frac{n_N}{n_T} \right)^{\omega-1} = \frac{1}{p} \frac{\theta_N}{\theta_T} \left(p^{\frac{1}{\sigma-1}} \frac{(1-\zeta) A_T}{\zeta A_N} \right)^{\omega-1} \Rightarrow p = \left[\frac{p w_T \theta_N}{w_N \theta_T} \left(\frac{(1-\zeta) A_T}{\zeta A_N} \right)^{\omega-1} \right]^{\frac{1}{\omega-\sigma}}. \quad (64)$$

We take the ratio between equation (33) and c_T to obtain the ratio

$$\frac{c}{c_T} = \left(\zeta^{1-\sigma} + (1-\zeta)^{1-\sigma} \left(\frac{c_N}{c_T} \right)^{\sigma} \right)^{\frac{1}{\sigma}}. \quad (65)$$

Finally, we divide equations (36) and (34) and then substitute for $\frac{c}{c_T}$ to obtain n_T

$$\left[\frac{w_T}{\theta_T} \left(\frac{c_T}{\zeta c} \right)^{\sigma-1} \right]^{\frac{1}{\omega-1}} = n_T. \quad (66)$$

Once the value of n_T is pinned down, the remaining algebra is simple.

B.3.2 Numerical computation of steady state (for estimation purposes)

The steady state values of β , r , r_T^k , r_N^k , $\frac{k_T}{n_T}$ and $\frac{k_N}{n_N}$ are given by equations (53)-(58). Given the values of sectoral hours (n_N and n_T), we can compute the steady state values of sectoral physical capital

$$k_N = \frac{k_N}{n_N} n_N, \quad (67)$$

and

$$k_T = \frac{k_T}{n_T} n_T. \quad (68)$$

Sectoral outputs are therefore

$$y_N = k_N^{\alpha_N} (n_N \bar{g}_N)^{1-\alpha_N} \quad (69)$$

and

$$y_T = k_T^{\alpha_N} (n_T \bar{g}_T)^{1-\alpha_T}. \quad (70)$$

Given the ratios s/y , tb/y and c_T/c , we solve for p , c_N , c_T and y .

$$c_N + \frac{s}{y} y = y_N \left\{ 1 - [\bar{g}_N - (1 - \delta_N)] \frac{\alpha_N}{r_N^k} \right\} \quad (71)$$

$$c_T + \frac{tb}{y} y = y_T \left\{ 1 - [\bar{g}_T - (1 - \delta_T)] \frac{\alpha_T}{r_T^k} \right\} \quad (72)$$

$$p c_N = c_T \frac{1 - \frac{c_T}{c}}{\frac{c_T}{c}} \quad (73)$$

$$y = y_T + p y_N. \quad (74)$$

We then compute the constants which allow us to exactly match the analytic steady state in the previous section as follows:

$$\frac{s}{y_N} = \frac{\frac{s}{y}}{p y_N}, \quad (75)$$

$$\frac{tb}{y_T} = \frac{tb}{y} \frac{y}{y_T}, \quad (76)$$

$$\zeta = \frac{p^{\frac{1}{\sigma-1}}}{p^{\frac{1}{\sigma-1}} + \frac{c_N}{c_T}}, \quad (77)$$

$$\theta_N = \frac{\frac{w_N}{p} (1 - \zeta)^{1-\sigma} \left(\frac{c_N}{c}\right)^{\sigma-1}}{n_N^{\omega-1}} \quad (78)$$

and

$$\theta_T = \frac{w_T \zeta^{1-\sigma} \left(\frac{c_T}{c}\right)^{\sigma-1}}{n_T^{\omega-1}}, \quad (79)$$

where w_N and w_T are given by (59) and (60).

C. DATA CONSTRUCTION

We use ONS supply and use tables for 1997 – 2015 to calculate, for each 2-digit SIC industry, a tradability index at basic prices (the ratio of exports plus imports to final demand). Exports denote exports of domestic output only (i.e. excluding re-exports of imported goods). Following [Lombardo and Ravenna \(2012\)](#), we define a sector as ‘tradable’ if more than 10% of its total demand is traded using the 2-digit SIC industry level classification. This threshold is arbitrary, but it coincides with those suggested by [De Gregorio et al. \(1994\)](#) and [Betts and Kehoe \(2006\)](#).¹ Table C.1 at the end of the section shows the full list of industries corresponding to the tradable and non-tradable categories. Figure C.1 shows the industry classification that yields from using the 10% cut-off. In particular, around 0.54 of aggregate GVA is classified as non-tradable and the remaining as tradable. As is evident in Table 1 in the main text, service industries tend to have lower ratios (although there are many exceptions), and manufacturing industries higher ratios.

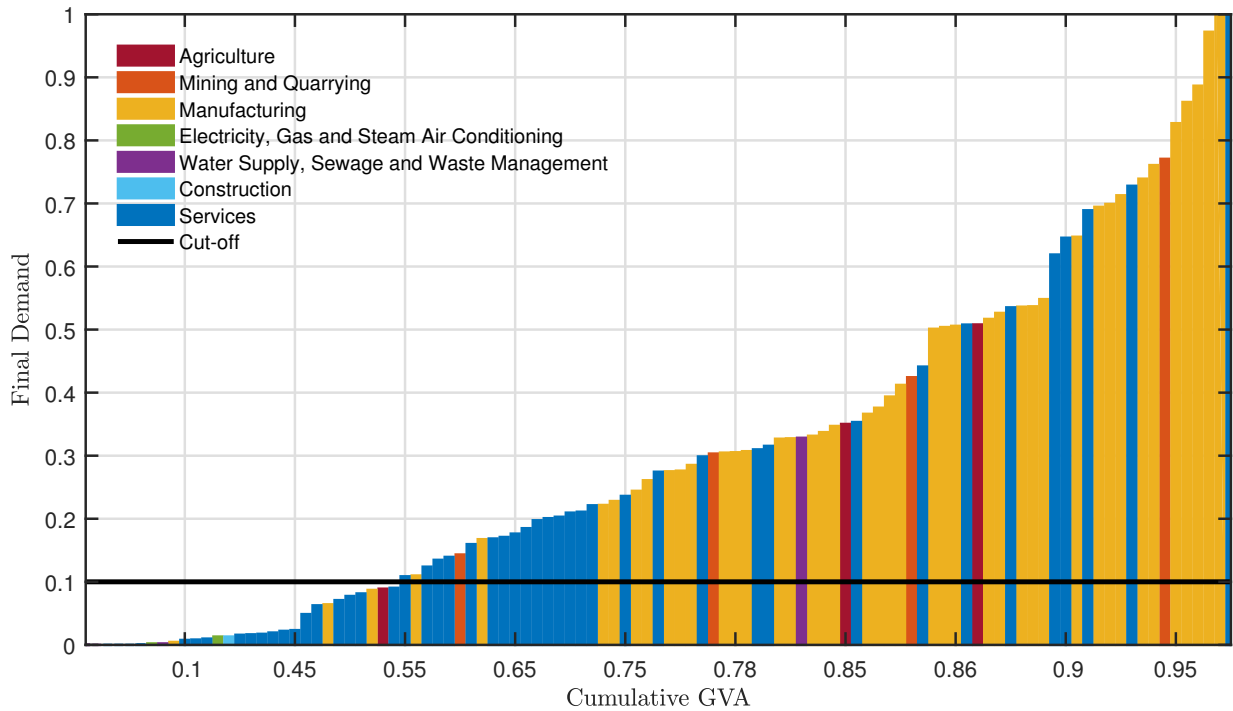


Figure C.1: Industry classification using 2016 Supply and Use Tables

After classifying each of the 114 industries into the tradable and non-tradable categories, we add the consumption expenditure of households and non-profit institutions serving households. We then divide sectoral expenditure by aggregate consumption for the years 1997 to 2016 to calculate the share of tradable consumption into total consumption. We then compute the sample mean and retrieve a value of 0.59. As shown

¹An alternative definition is proposed by [De Gregorio et al. \(1994\)](#) that classify a sector as ‘tradable’ if 10% of its total supply is exported.

in the Figure C.2, this share is rather constant over time. This value is in line with the estimate for the UK in [Lombardo and Ravenna \(2012\)](#), who calculated a value of 0.64 (based on 2000 – 2005 data) and with a previous internal Bank of England’s estimate of 0.5 – 0.6. Using the same threshold, we also find that around half of the economy by GVA can be classified as tradable. It is worth noting that the share of tradable output in aggregate output is lower than the tradable share in aggregate consumption because non-tradable services, such as construction, public administration and defense and compulsory social security services, have a much higher weight in output than in household consumption.

The factor shares are computed using the supply and use tables from 1997 to 2016. In line with [Goodridge et al. \(2018\)](#), we use partial appropriation of self-employed income to labor income. This assumes a fraction of self-employed income accruing to labor income. The labor share in sector i in year t is then defined as the sum of compensation of employees and the fraction of self-employed income accrues to labor divided by total GVA (at basic prices). In computing the labor share in the N sector, we exclude imputed rents as they tend to bias the estimates. The capital share is residually determined as one minus the labor share. The sample means of the capital shares are 0.315 and 0.245 in the T and N sectors sector respectively. Note that assigning self-employed income to labor income tends to increase the values of the labor shares. Figure C.2 shows the evolution of the consumption share of T goods into aggregate consumption and the labor shares in the T and N sectors respectively.

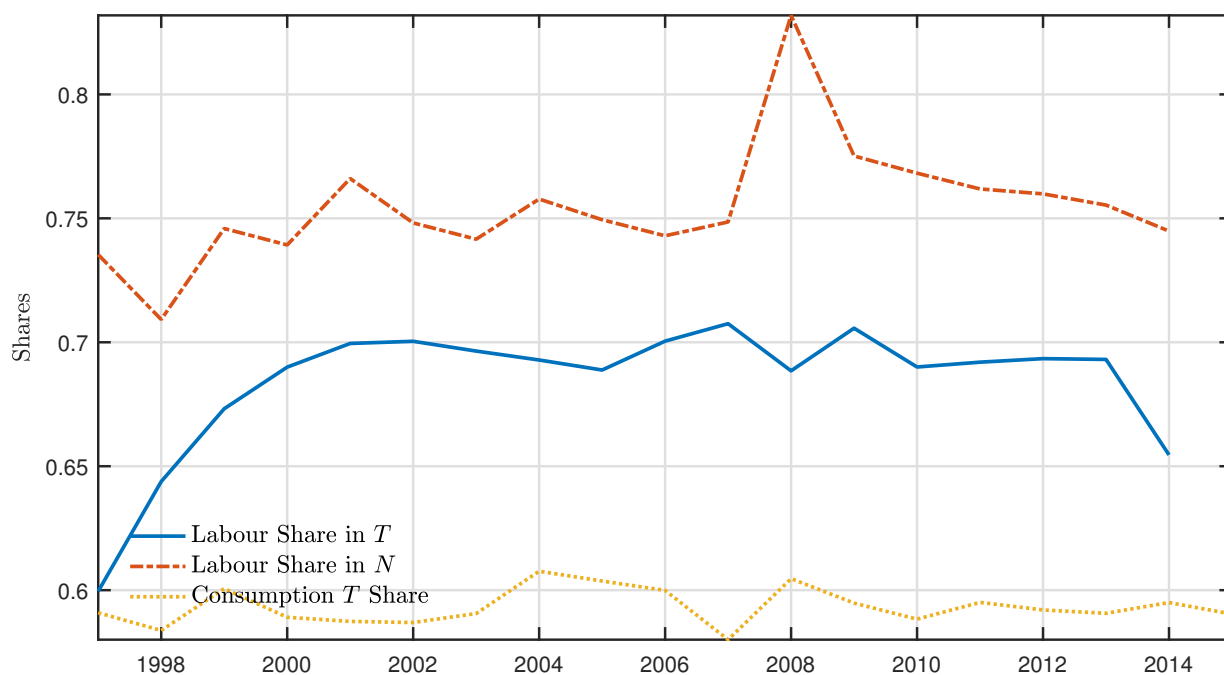


Figure C.2: Labor Shares in T and N and Consumption T Shares

Once we have classified each 2-digit industries into the tradable or the non-tradable category, we use the associated ONS detailed industry-level GVA data to construct a

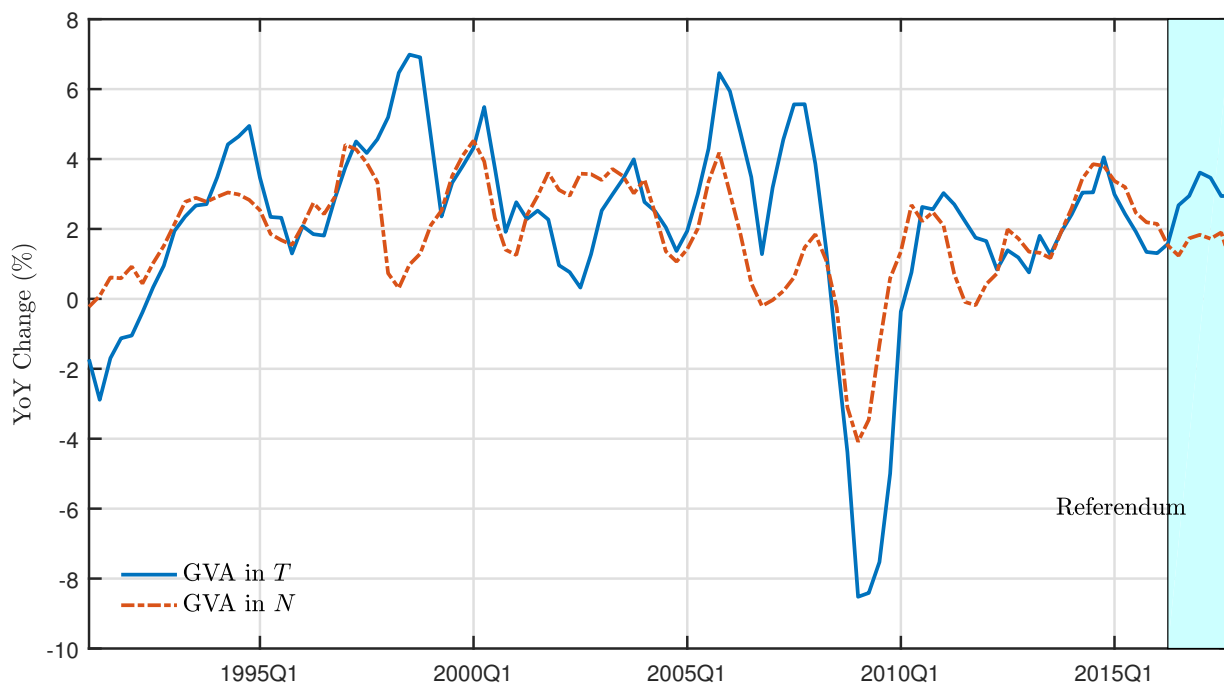


Figure C.3: *Gross Value Added in T and N*

time-series for tradable output consistent with aggregate GVA, by aggregating GVA over the set of industries in each category, using ONS’s standard national accounts chain-linking methodology. The resulting time-series for GVA growth are shown in Figure C.3. The growth rates of GVA output do not display significant differences across the two categories since total hours in the non-tradable sector display an upward trend.

We also construct tradable and non-tradable total hours, using the published industry hours data underlying ONS labor productivity estimates, together with our classification of industries.² Hours worked by sector are measured following Tenreyro (2018). We then compute the average labor productivity growth rate in each sector from 1994-2017. Over this period labor productivity growth in the tradable sector averaged about 1.8% on an annual basis.

²The data on hours are available at a slightly higher level of aggregation than 2-digit level; therefore, we need to make a judgement about the tradability of each grouping of 2-digit industries in the hours data, based on the tradability of the underlying 2-digit industries.

Table C.1: Detailed industry classification

Tradable Industries	Non-Tradable Industries
Employment services	Sewerage services, sewage sludge
Repair and maintenance of ships and boats	Remediation serv. and other waste management serv.
Computer programming, consultancy and related services	Retail trade serv., except of motor vehicles
Services of head offices, management consulting services	Veterinary services
Gambling and betting services	Services furnished by membership organisations
Food and beverage serving services	Residential Care & Social Work Activities
Accounting, bookkeeping and auditing services, tax consulting services	Natural water, water treatment and supply services
Warehousing and support services for transportation	Gas, distr. of fuels, steam and air cond. supply
Rail transport services	Printing and recording services
Legal services	Households as employers of domestic personnel
Libraries, archives, museums and other cultural services	Construction
Telecommunications services	Public admin. and defence, compulsory soc. security
Mining support services	Wholesale & retail trade & repair of motor vehicles
Prepared animal feeds	Electricity, transmission and distribution
Bakery and farinaceous products	Services to buildings and landscape
Scientific research and development services	Other personal services
Soft drinks	Owner-Occupiers' Housing Services
Advertising and market research serv.	Travel agency, tour operator and other rel. serv.
Architectural and engineering serv.; technical testing and analysis serv.	Real estate serv.& imputed rent
Insurance, except compulsory social security & Pension funding	Human health serv.
Repair and maintenance of aircraft and spacecraft	Security and investigation services
Grain mill products, starches and starch products	Real estate activities on a fee or contract basis
Financial services, except insurance and pension funding	Rest of repair, Installation
Motion Picture, Video & TV & Music & Programming And Broadcasting	Education services
Products of forestry, logging and related services	Rental and leasing services
Dairy products	Manufacture of cement, lime, plaster (and articles of)
Information services	Land transport and transport via pipelines, excl. rail
Weapons and ammunition	Sports serv. and amusement and recreation serv.
Publishing services	Postal and courier serv.
Furniture	
Alcoholic beverages & Tobacco products	
Repair services of computers and personal and household goods	
Preserved meat and meat products	
Financial Services (and Auxiliary) And Insurance Activities	
Fabricated metal products, excl. machinery& ammunition	
Waste collection, treatment and disposal, materials recovery serv.	
Other food products	
Products of agriculture, hunting and related services	
Processed and preserved fish, crustaceans, molluscs, fruit and vegetables	
Soap and detergents, cleaning and polishing, perfumes and toilet	
Paper and paper products	
Coke and refined petroleum products	
Wood and (products of), except furniture	
Textiles	
Glass, refractory, clay, other porcelain & ceramic, stone and abrasive	
Paints, varnishes and similar coatings, printing ink and mastics	
Other transport equipment	
Accommodation services	
Rubber and plastic products	
Vegetable and animal oils and fats	
Fish and other fishing products	
Other manufactured goods	
Wearing apparel	
Ships and boats	
Industrial gases, inorganics and fertilisers (all inorganic chemicals)	
Creative, arts and entertainment services	
Coal and lignite	
Air transport services	
Basic iron and steel	
Leather and related products	
Office administrative, office support and other business support services	
Services auxiliary to financial services and insurance services	
Electrical equipment	
Motor vehicles, trailers and semi-trailers	
Crude Petroleum And Natural Gas & Metal Ores	
Dyestuffs, agro-chemicals	
Basic pharmaceutical products and pharmaceutical preparations	
Other professional, scientific and technical services	
Water transport services	
Other basic metals and casting	
Petrochemicals	
Computer, electronic and optical products	
Other chemical products	
Other mining and quarrying products	
Machinery and equipment n.e.c.	
Air and spacecraft and related machinery	
Wholesale trade services, except of motor vehicles and motorcycles	

Notes. Industry categorization computed using the supply used tables from 1997-2016. Tradability index is calculated as the ratio between the trade and final demand and then averaged over 1997-2016. Cut-off is set to 10%.

D. PROPERTIES OF THE REAL EXCHANGE RATE

Our two-sector SOE model has two important implications for the real exchange rate. The first implication is that the real exchange rate and the relative price of nontraded goods are conceptually related. This is discussed in the exposition of the model in Section 3 (see in particular the reference made to the *Harrod-Balassa-Samuelson effect*), as well as in the discussion of mapping model variables to the data in Section 4.2.

The second implication is that the model’s real exchange rate concept (P_t) displays a stochastic trend. While we use the growth rate of P_t (together with the growth rate of the REER, Q_t) as observables in the model estimation, this Appendix explores whether this second implication of our model – non-stationarity in the level of the real exchange rate – is borne out by the data.

Table D.1: RESULTS OF UNIT ROOT TESTS ON REER AND RELATIVE PRICE

	Test statistic	5% critical value	Reject?
REER (levels)	-2.5945	-2.8877	No
REER (first differences)	-4.3478	-2.8878	Yes
 P (levels)	 -3.5708	 -2.8968	 Yes
P (first differences)	-4.7763	-2.8972	Yes

Notes. Unit root test on alternative exchange rate concepts. The table reports the relevant t -statistics for the null hypothesis of a unit root in level and first difference of each time series, based on an augmented Dicker-Fuller (ADF) test with 4 lags, intercept and time trend. The time period is 1990Q1 - 2018Q4 for the REER and 1997Q1 - 2018Q4 for the relative price of nontradables.

To this end, Table D.1 reports the results from an augmented Dicker-Fuller (ADF) test on the UK real effective exchange rate and the relative price of nontradable goods. The model under the null has a unit root, the alternative is the same model with drift and deterministic trend. The lag order is 4. As the table shows, the test indeed fails to reject a unit root in the level, but rejects a unit root in after first-differencing for the real effective exchange rate. For the relative price measure (which is available for a shorter sample), the null is rejected for both level and first-difference. We note that this measure is available for a shorter sample, which gives the test lower power for the relative price variable.

We conclude that there is at least some, albeit weak, evidence for nonstationarity in the UK real exchange rate, given that for one of the two concepts we fail to reject a unit root. See also Rabanal and Rubio-Ramirez (2015) for a systematic analysis of the importance of low frequency movements in real exchange rates.

E. SENSITIVITY EXPERIMENTS

This appendix presents results from variations of our benchmark Brexit scenario of Section 5. Appendix E.1 shows that the results are similar for alternative assumptions about the timing of the decline in tradable sector productivity growth. Appendix E.2 presents simulations that assume that productivity falls more sharply than the benchmark case. Appendix E.3 demonstrates that the responses are not sensitive to the assumption used to close the model.

E.1. The anticipation horizon

Figures E.1 and E.2 show the results of the baseline experiment (solid blue lines) and a variant in which the decline in tradable sector productivity growth starts after five quarters (dot-dashed red lines). The long-run effects of the shock are the same, but alternative timing assumptions are likely to affect short-term dynamics.

Unsurprisingly, the dynamics of the alternative assumption are slightly different when the decline in productivity growth occurs earlier. The initial export boom is less long-lived and requires a larger reallocation of labor to deliver higher tradable sector output. The switch in factor flows (from the tradable sector to the non-tradable sector) occurs earlier, commensurate with the earlier reduction in tradable sector productivity growth.

Figures E.3 and E.4 show the results of the baseline experiment (solid blue lines) and a variant in which the decline in tradable sector productivity growth is delayed for fifteen quarters (dot-dashed red lines).

The relative effect of a longer anticipation horizon is, unsurprisingly, the opposite of the previous case of a shorter anticipation horizon. The adjustment dynamics are more protracted and the near-term reallocation of labor during the anticipation horizon is more muted relative to the baseline case. With a longer anticipation horizon, tradable sector investment falls by less, as the reduction in tradable sector productivity occurs in the more distant future.

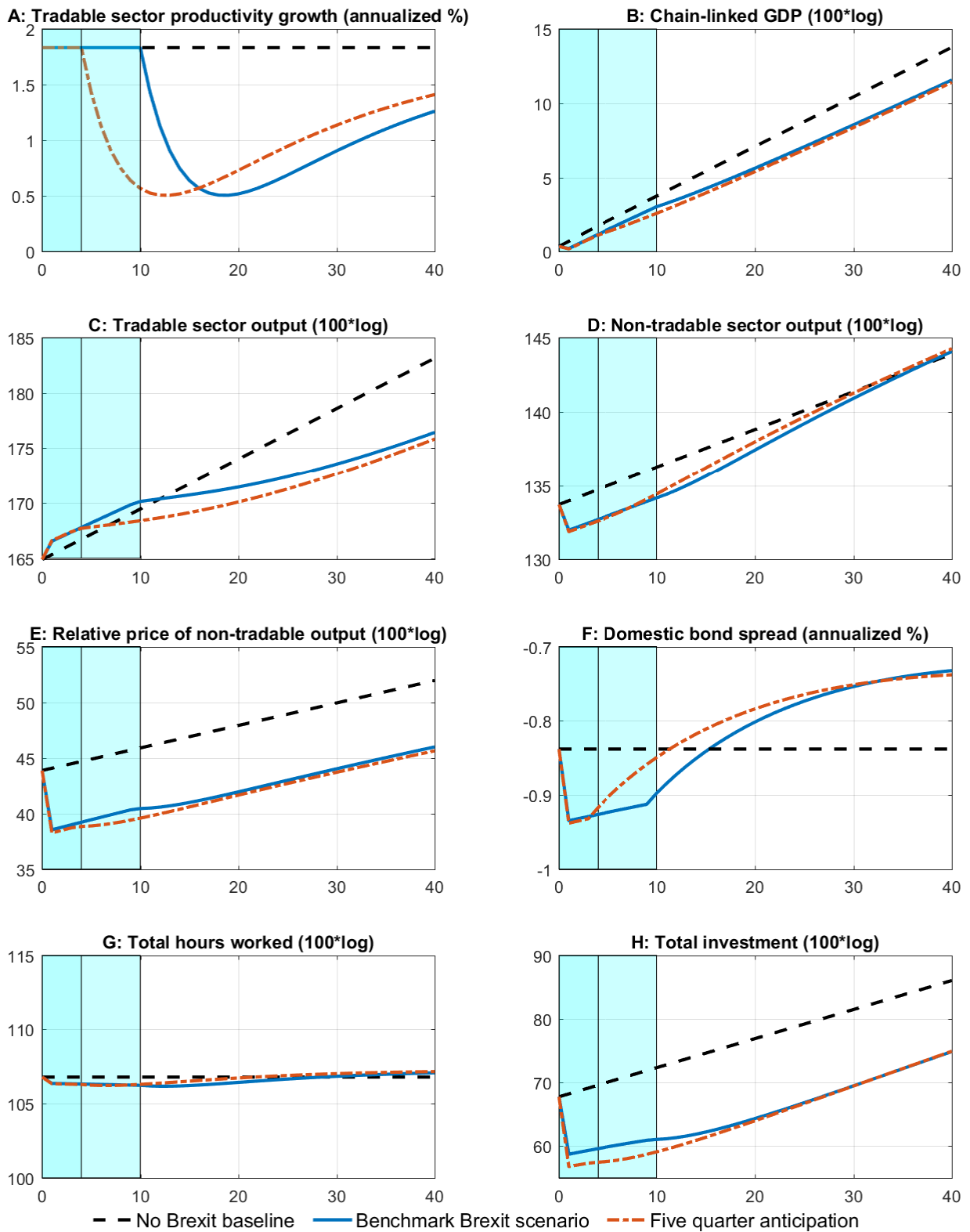


Figure E.1: Main model responses in Brexit scenario with alternative timing

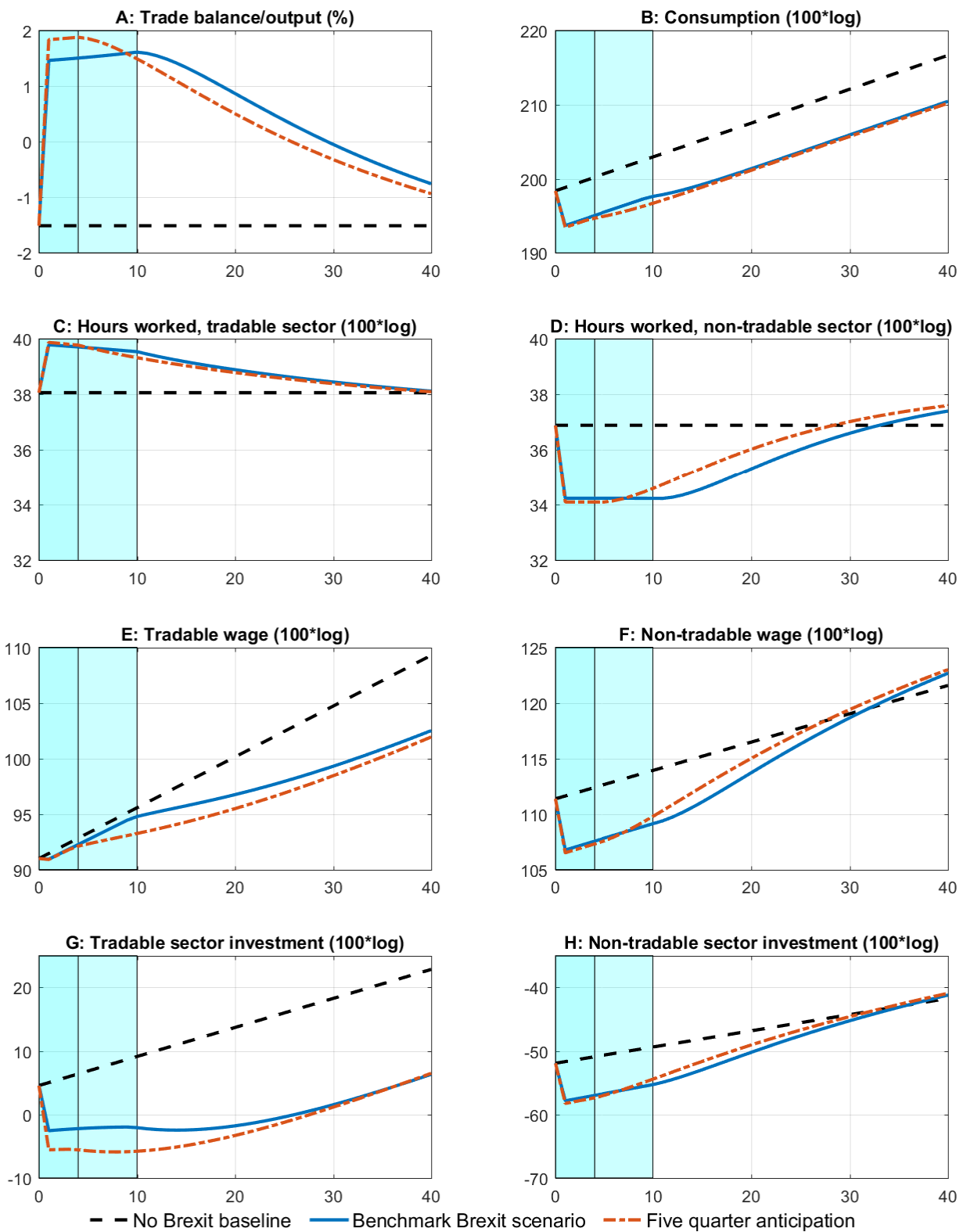


Figure E.2: Additional model responses in Brexit scenario with alternative timing

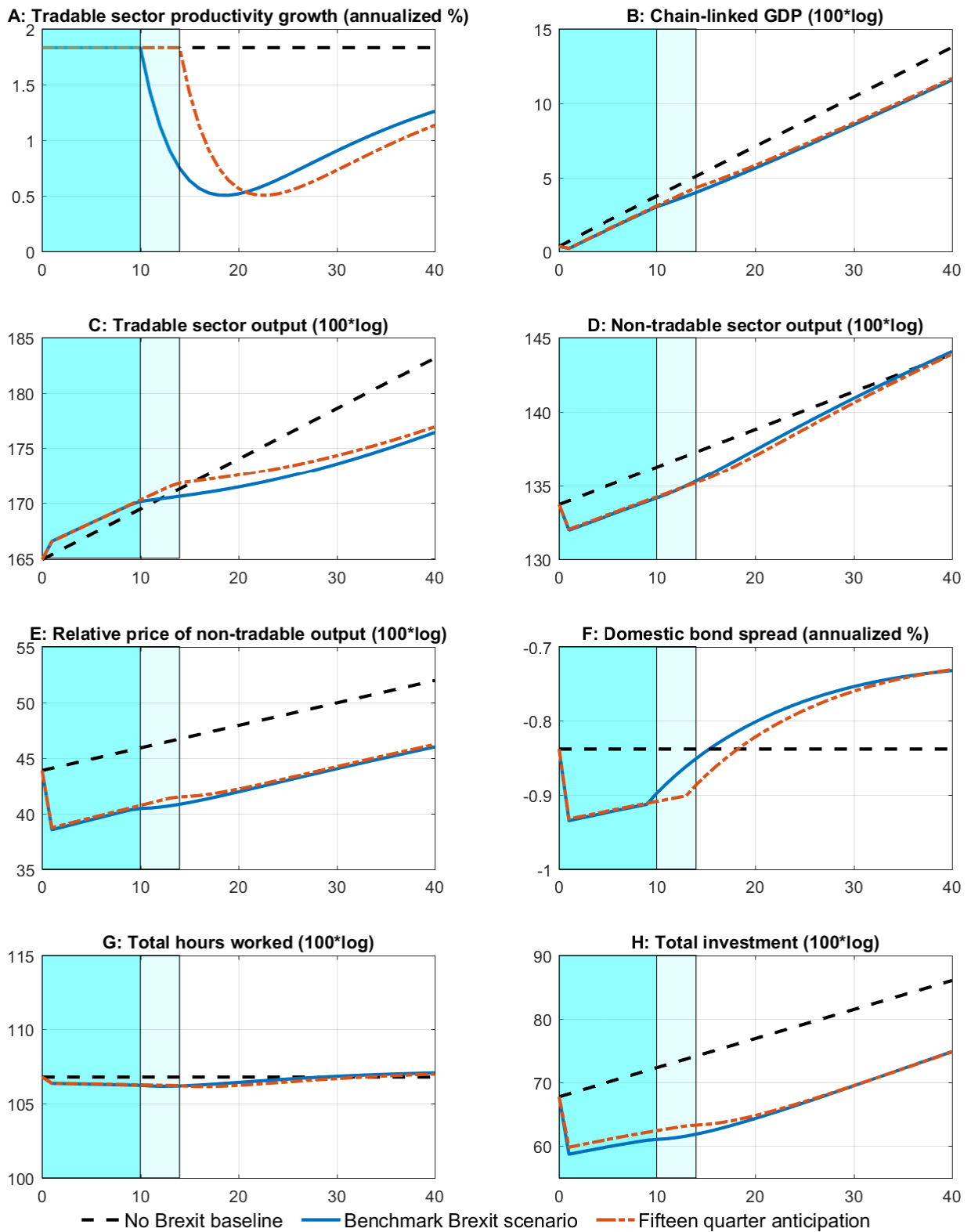


Figure E.3: Main model responses in Brexit scenario with alternative timing

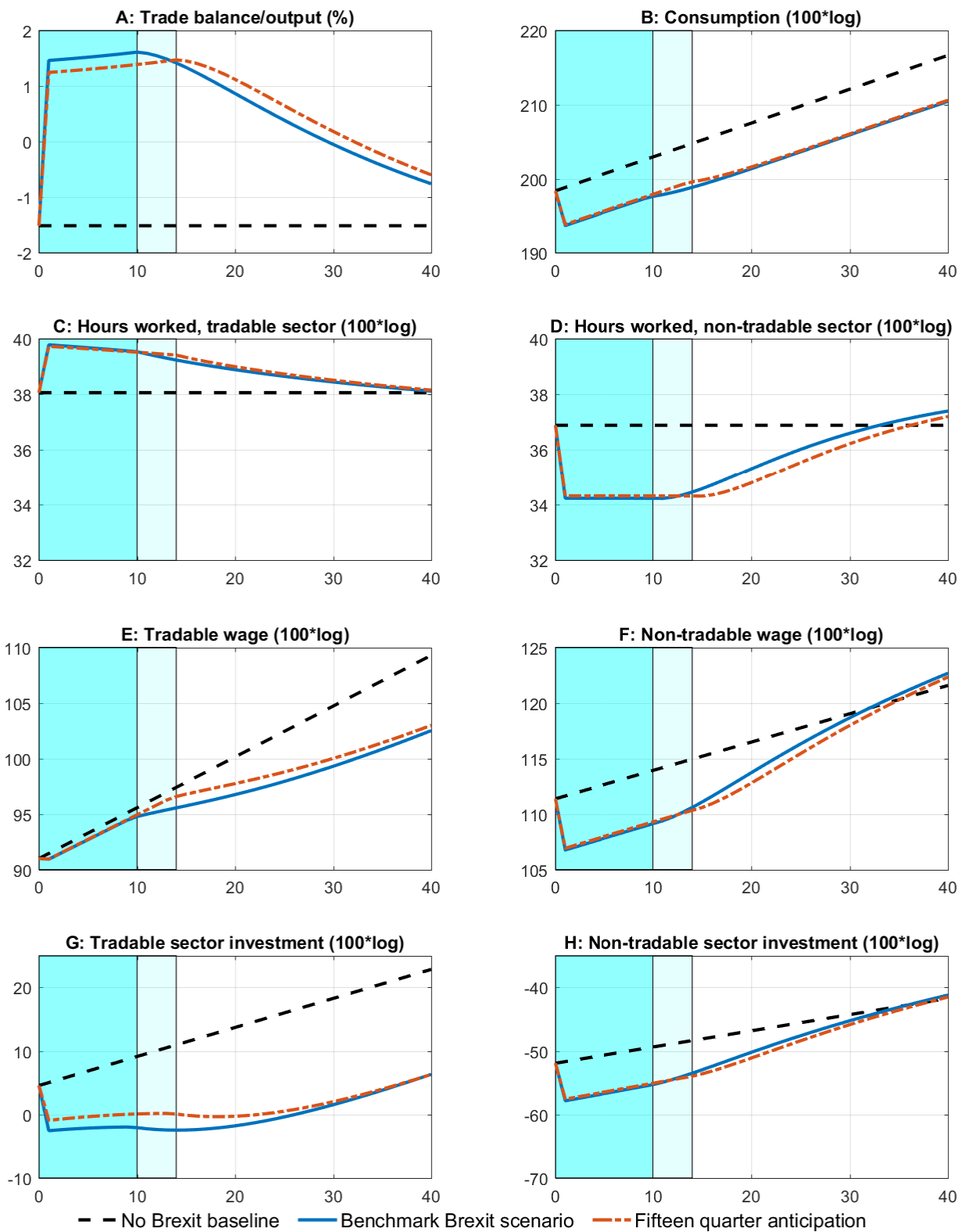


Figure E.4: Additional model responses in Brexit scenario with alternative timing

E.2. Faster fall in tradable sector productivity

Figures E.5 and E.6 show the results of the baseline scenario (solid blue lines) alongside a case in which the decline in tradable sector productivity growth occurs more rapidly (red dot-dashed lines). The alternative scenario is constructed by assuming that the parameter controlling the persistent component of tradable sector productivity growth is set to $\tilde{\varrho}_T^g = 0.9$ (compared with the baseline assumption of 0.95).

The alternative scenario implies that tradable sector productivity reaches its new, lower, level in roughly half the time of the baseline scenario. The scale of the productivity growth shock is roughly doubled to ensure that the long-run effect on tradable sector productivity is identical to the baseline scenario.

Unsurprisingly, the dynamic responses to the more rapid productivity growth shock variant are somewhat faster in some cases. However, the broad contours of the macroeconomic responses are very similar in both cases. This demonstrates that the dominant effect is the anticipation of permanently lower tradable sector productivity in the *long run*. This effect drives the key relative price in the model: the impact effect on the relative price of non-tradable output is very similar (Figure E.5).

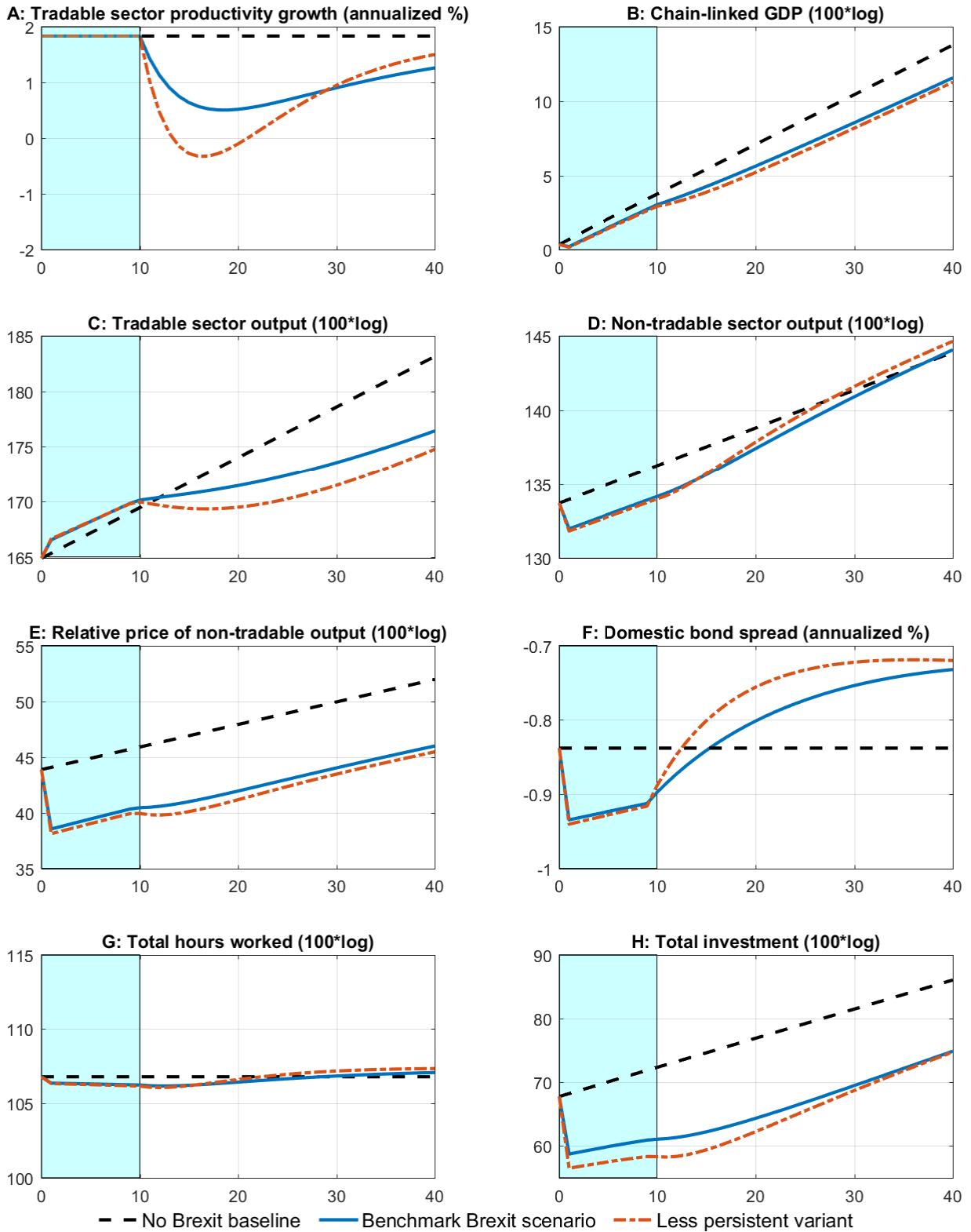


Figure E.5: Main model responses in Brexit scenario with alternative persistence

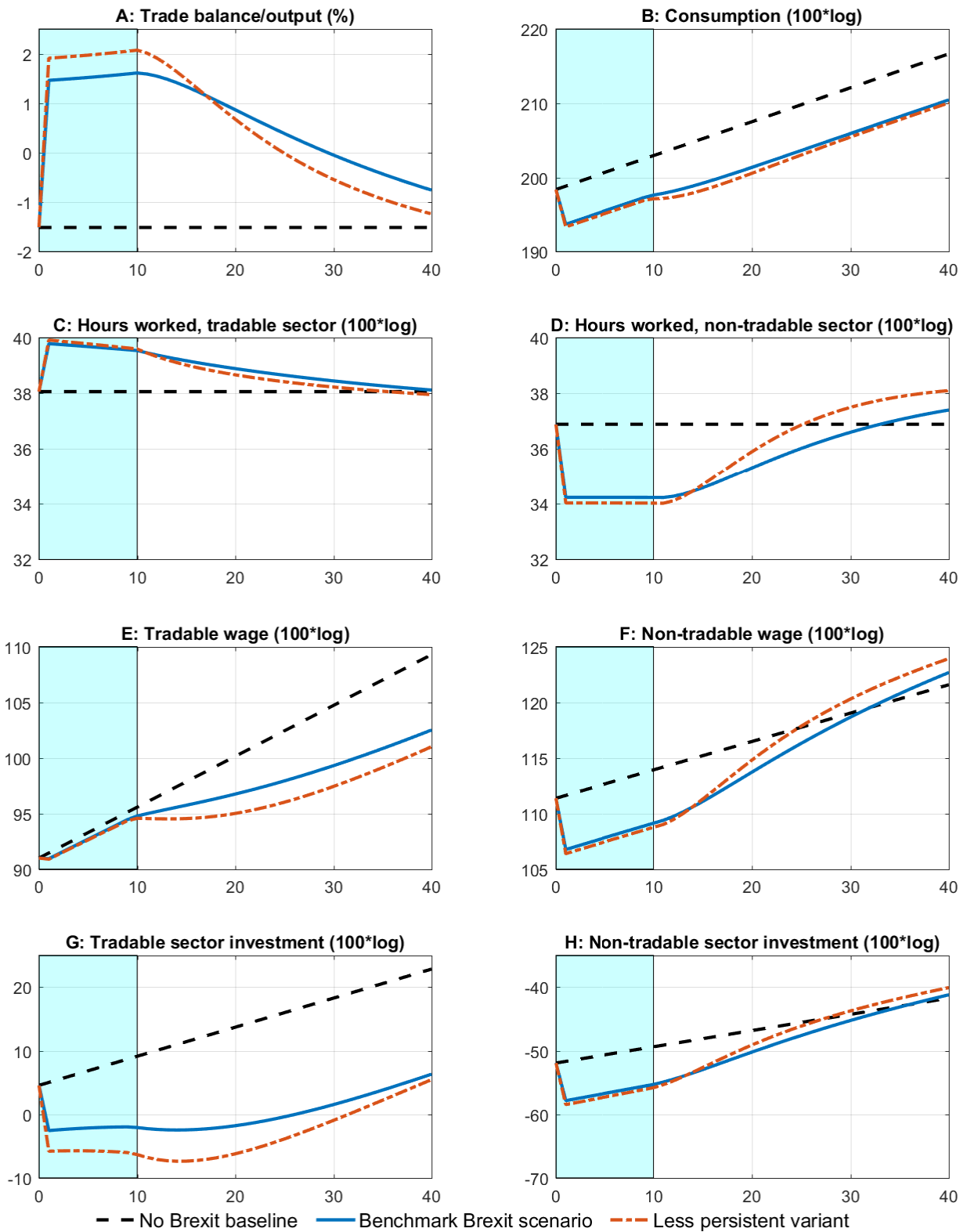


Figure E.6: Additional model responses in Brexit scenario with alternative persistence

E.3. Population growth variant

Figures E.7 and E.8 show results for a variant of the model that incorporates population growth. A derivation of this variant is presented in Appendix G. However, the innovation compared with the baseline model is straightforward. In the variant, we assume that households are infinitely lived, but that new households are born each period. The population growth rate is constant. Individual households have identical preferences to those that we have assumed in previous versions of the model. Therefore their first order conditions identical to the baseline model.

However, population growth means that aggregate consumption is not characterized by the same Euler equation as the one that holds for each individual household. This is because new households are born with no financial wealth. Accounting for the heterogeneity in financial wealth delivers an aggregate consumption equation that depends on the distribution of wealth. The simple population structure implies that the distribution of wealth can be summarized by aggregate stocks of wealth (ultimately, the stock of foreign debt).

The dependence of the aggregate consumption Euler equation on wealth means that the steady state net foreign asset position is pinned down, even if the economy may freely borrow and lend at a fixed world interest rate. The steady state NFA position is pinned down by the (im)patience of domestic agents relative to the (growth adjusted) world real interest rate.

While population growth is just a device to close the model, rather than a plausible model of demographics, we set the constant population growth rate to be consistent with 0.5% annual population growth (broadly consistent with 1997-2016 UK data).

The results show that the dynamics are virtually identical in the two variants of the model, despite the fact that the tradable bond rate remains fixed in the population growth variant. The slight differences in relative bond rates generate small differences in the returns to capital across the two variants, with minor implications for the dynamic responses of hours and investment. However, the broad contours of the simulation are very similar in the two variants.

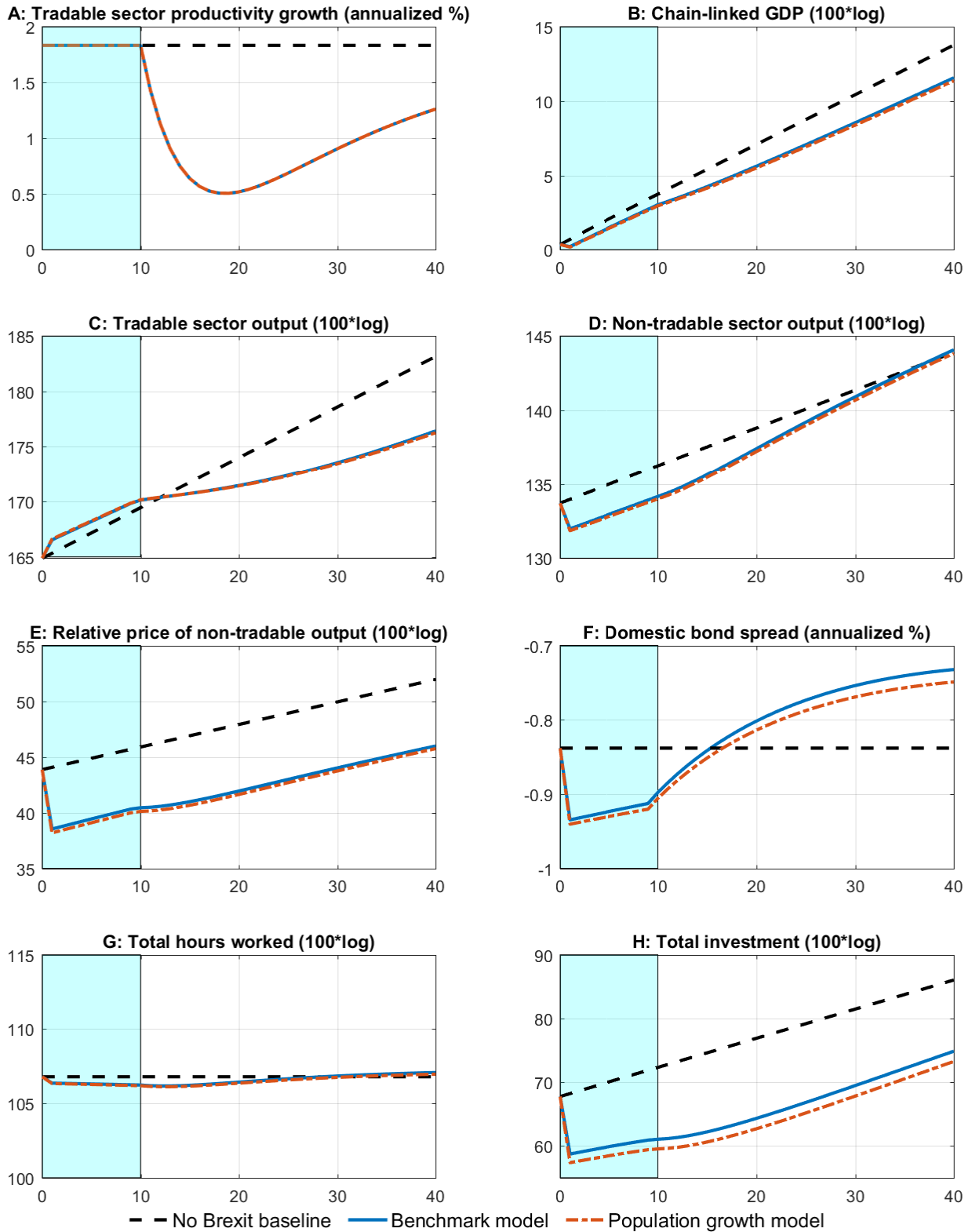


Figure E.7: Main model responses in Brexit scenario in alternative model version

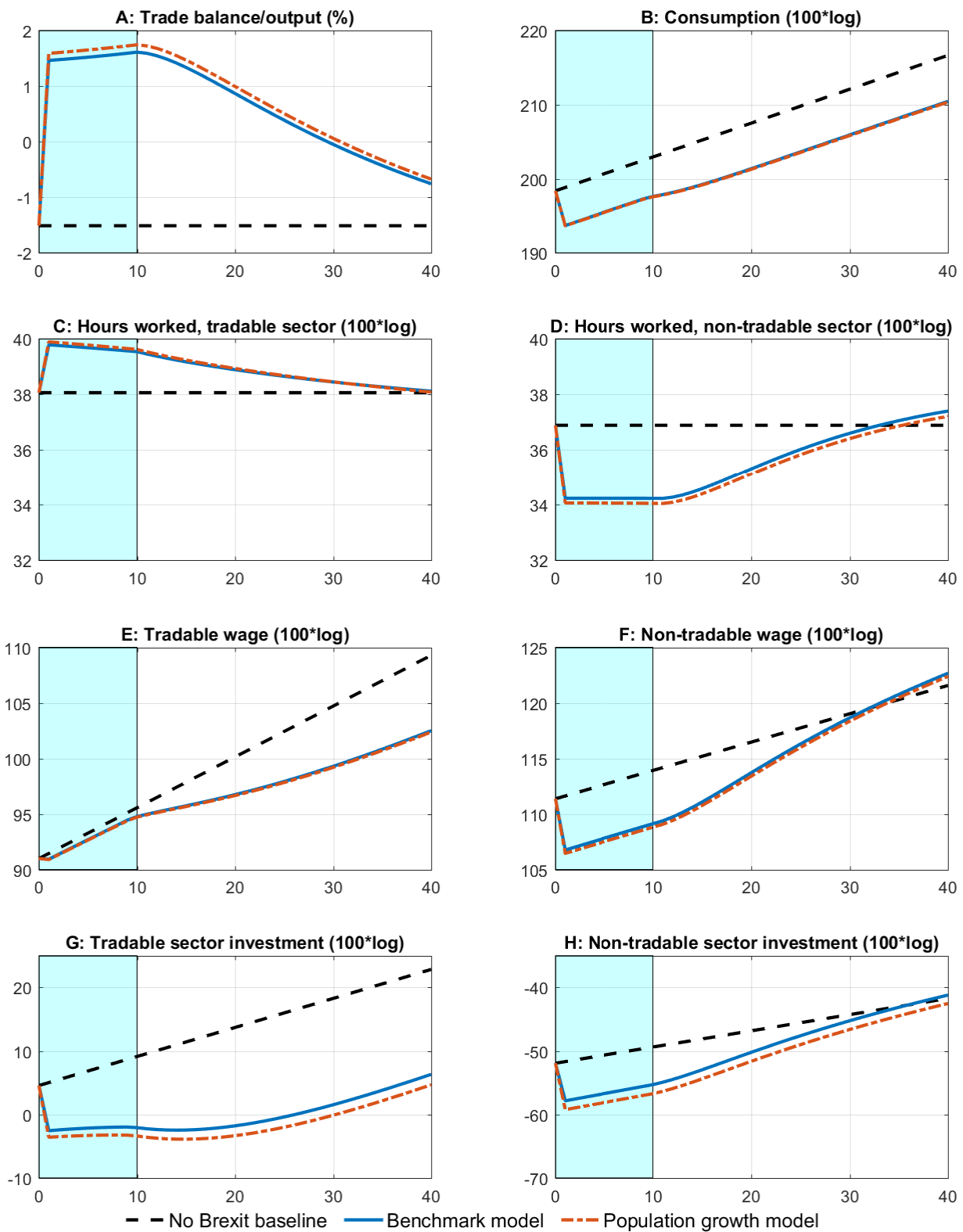


Figure E.8: Additional model responses in Brexit scenario in alternative model version

F. SIMULATIONS BASED ON OTHER STRUCTURAL SHOCKS

This appendix presents simulation results for two other structural shocks. In both cases, the simulations are carried out following the methodology described in the text. Figures [F.1](#) and [F.2](#) show the results for a news shock about a reduction in the *level* (rather than the growth rate) of productivity in the tradable sector.

Figure [F.3](#) and [F.4](#) present the results for a shock containing news about acceleration in the growth rate of productivity growth in the non-tradable sector (rather than a deceleration in the tradable sector). This is calibrated to match the response of the relative price of non-tradable goods on impact in the baseline Brexit scenario.

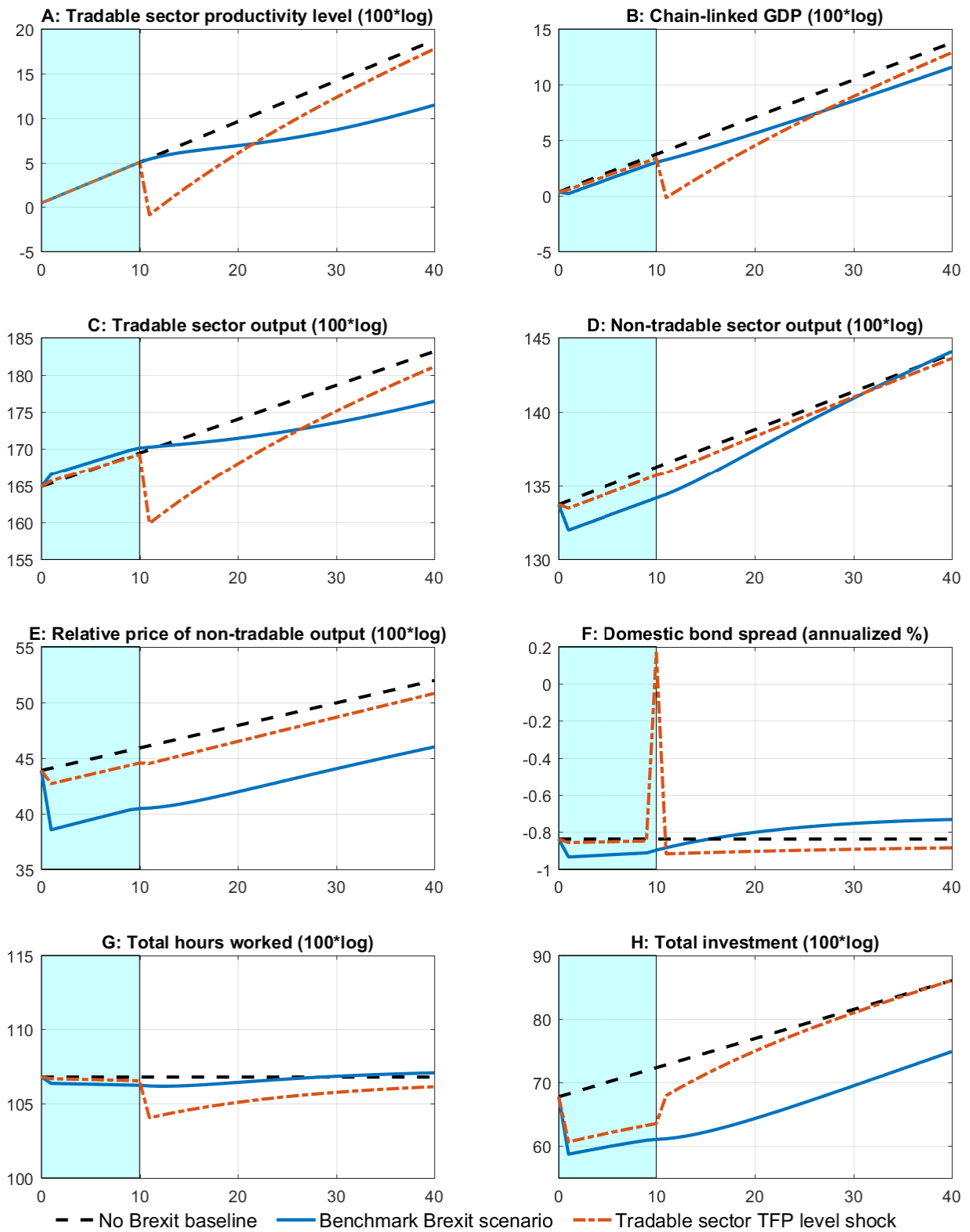


Figure F.1: Main model responses to a shock to the level of tradable sector productivity

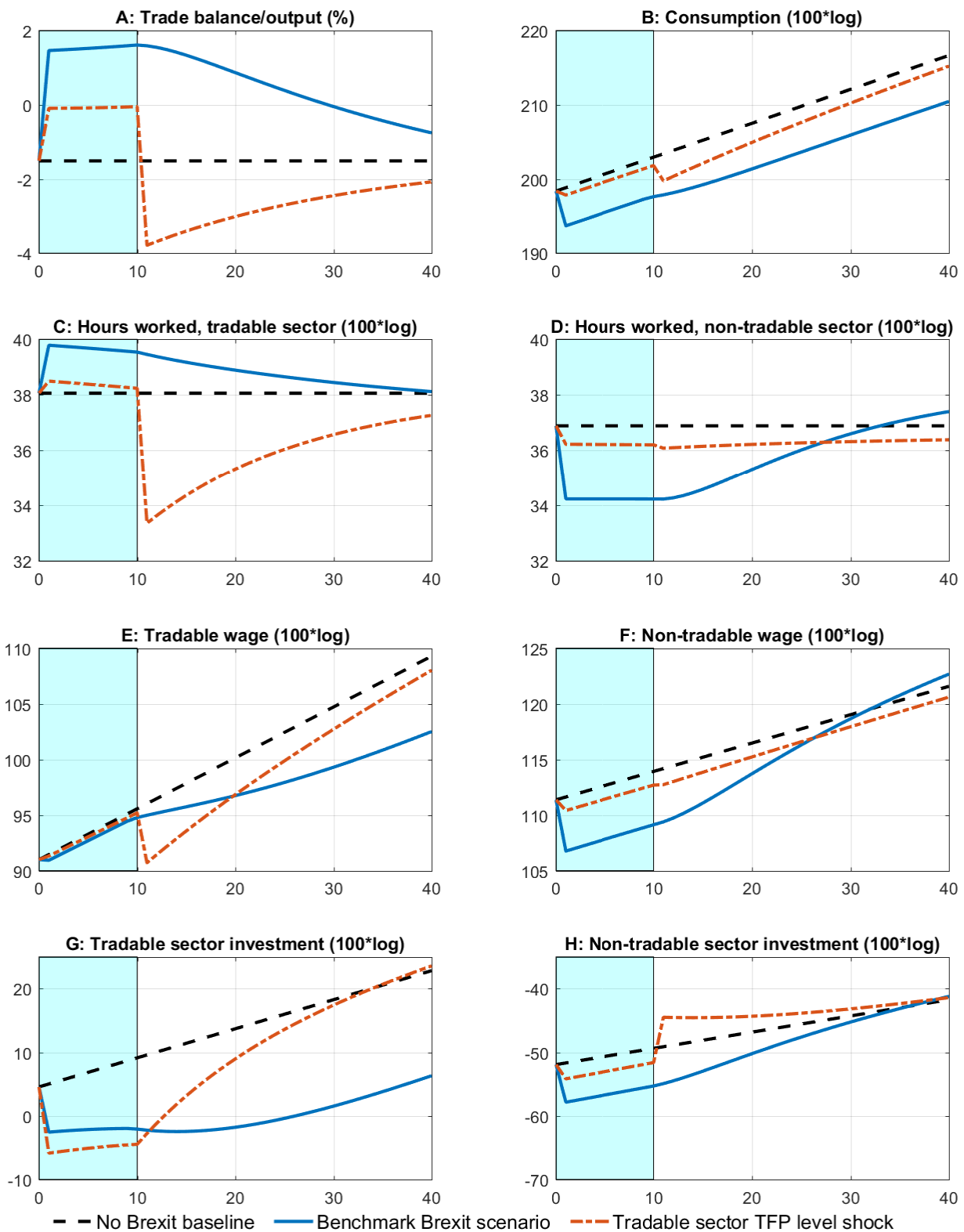


Figure F.2: Additional model responses to a shock to the level of tradable sector productivity

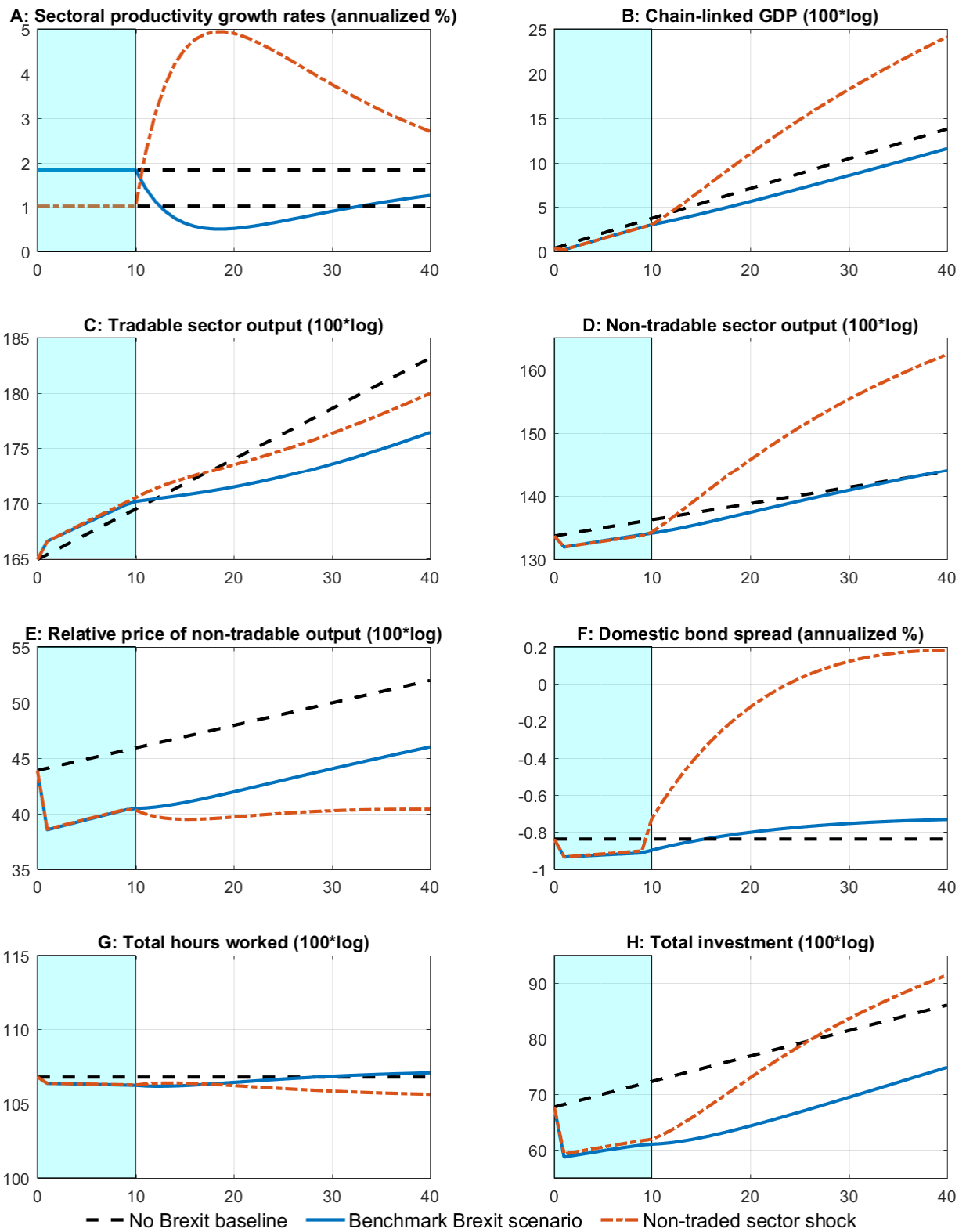


Figure E.3: Main model responses to a shock to non-tradable sector productivity growth

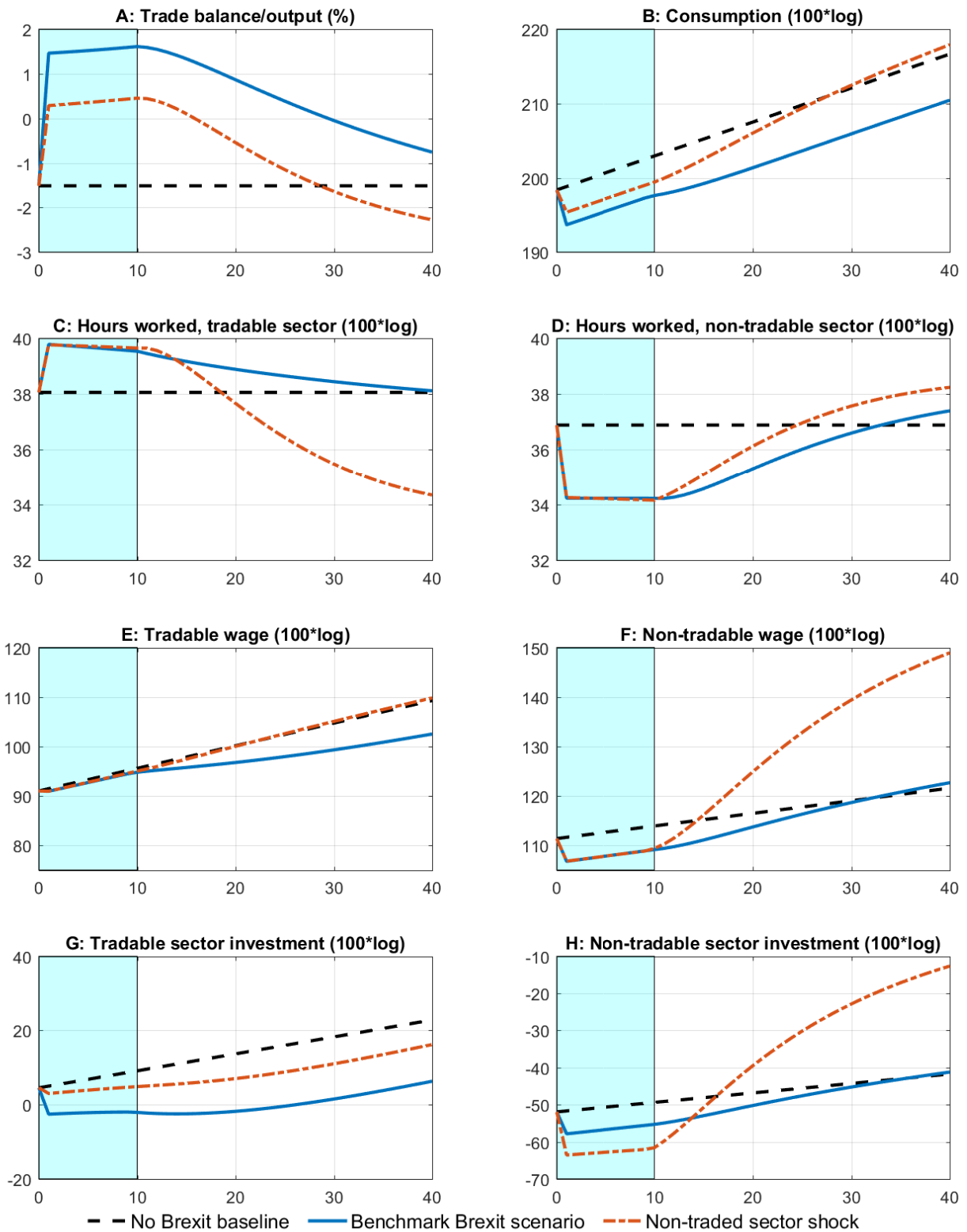


Figure F.4: Additional model responses to a shock to non-tradable sector productivity growth

G. MODEL WITH POPULATION GROWTH

G.1. Overview

This variant of the model involves a small adjustment to the household sector. The model is essentially an open economy variant of the [Weil \(1989\)](#), [Weil \(1991\)](#) model with GHH preferences.³

In this variant, we assume that households are infinitely lived, but that new households are born each period. The population growth rate is constant. Individual households have identical preferences to those in the baseline model, so their first order conditions identical to the ones derived in the main text. However, population growth means that aggregate consumption is no-longer characterized by the same Euler equation as the one that holds for each individual household. This is because new households are born with no financial wealth and accounting for the heterogeneity in financial wealth delivers an aggregate consumption equation that depends on the distribution of wealth. The simple population structure implies that the distribution of wealth can be summarized by aggregate stocks of wealth (ultimately, in our model, the stock of foreign debt).

The dependence of the aggregate consumption Euler equation on wealth means that the steady state net foreign asset position is pinned down, even if the economy may freely borrow and lend at a fixed (tradable-good denominated) interest rate. The steady-state net foreign asset position is pinned down by the (im)patience of domestic agents relative to the (growth adjusted) world real interest rate.

G.2. Households

The number of households alive in period t is Z_t . Population evolves according to:

$$Z_{t+1} = (1 + \vartheta) Z_t \quad (80)$$

As before, to convert the model into stationary units, aggregate quantities must be detrended by sectoral growth rates. However, the introduction of population growth means that the quantities of interest in this variant are measured in *per capita* terms. This means that the detrending factors for quantities (though not prices must also account for population growth). With this in mind, sectoral growth rates for this variant are defined as:

$$g_{Mt} = (1 + \vartheta) \frac{X_{Mt}}{X_{Mt-1}}, \quad (81)$$

for $M = \{N, T\}$.

³A closed economy version of the Weil model with GHH preferences is analyzed by [Ireland \(2005\)](#), which also forms a guide for our approach.

A household born in period s maximizes the following utility function:

$$\sum_{t=s}^{\infty} \beta^{t-s} \frac{\left[C_t^s - X_{Tt-1} \omega^{-1} \left(\theta_T (n_{Tt}^s)^\omega + \theta_N (n_{Nt}^s)^\omega \right) \right]^{1-\gamma}}{1-\gamma}$$

so that the preferences of an individual household are identical to those in the baseline model. The s superscript indexes the date of birth of the household.

The household budget constraint, denominated in traded goods, is given by:

$$P_t^c C_t^s + B_t^{*,s} + P_t B_t^s = W_{Tt} n_{Tt}^s + W_{Nt} n_{Nt}^s + \Pi_t + \frac{B_{t+1}^{*,s}}{1+r_t^*} + P_t \frac{B_{t+1}^s}{1+r_t} \quad (82)$$

where it is assumed that households are born with no financial wealth or debt, so that $B_s^s = B^{*,s} = 0$. As in previous derivations, positive values of B and B^* represent *debt*.

Relative to the baseline model, two adjustments are made to facilitate the subsequent derivation. First, the budget constraint is written in terms of the total consumption bundle, incorporating the price of consumption in terms of tradable output, P_t^c . Second, households are assumed to receive lump sum profits (allocated from both tradable and non-tradable firms) denoted by Π . These profits are distributed equally to all households (including newborns). This means that households do not own the capital stock in this model. Instead, firms are assumed to own the capital stock, discussed below.

Writing the budget constraint in terms of the aggregate consumption bundle simplifies the derivations of the consumption function considerably. The total expenditure on consumption satisfies:

$$P_t^c C_t = P_t C_{Nt} + C_{Tt} \quad (83)$$

and the consumption bundle (as in the baseline model) is given by

$$C_t = \left[\zeta^{1-\sigma} C_{Tt}^\sigma + (1-\zeta)^{1-\sigma} \left(\frac{X_{Tt-1}}{X_{Nt-1}} C_{Nt} \right)^\sigma \right]^{\frac{1}{\sigma}} \quad (84)$$

The allocation of consumption between tradable and non-tradable consumption is a static problem and the optimality conditions imply:

$$\left[\frac{C_{Nt}}{C_{Tt}} \frac{\zeta}{1-\zeta} \frac{X_{Tt-1}}{X_{Nt-1}} \right]^{\sigma-1} \frac{X_{Tt-1}}{X_{Nt-1}} = P_t \quad (85)$$

which is the ratio of the first two first order conditions for the household in the previous derivation.

These equations provide solutions for C_{Nt} , C_{Tt} , P_t^c given a solution for C_t . The rest of the subsection derives a representation of the aggregate consumption function (and

hence a solution for C_t).

The first order conditions for the household can be written as:

$$\left[C_t^s - X_{Tt-1} \omega^{-1} (\theta_T (n_{Tt}^s)^\omega + \theta_N (n_{Nt}^s)^\omega) \right]^{-\gamma} = \frac{\beta (1 + r_t^*) P_t^c}{P_{t+1}^c} \left[C_{t+1}^s - X_{Tt} \omega^{-1} (\theta_T (n_{Tt+1}^s)^\omega + \theta_N (n_{Nt+1}^s)^\omega) \right]^{-\gamma}$$

$$1 + r_t = (1 + r_t^*) \frac{P_t}{P_{t+1}}$$

$$X_{Tt-1} \theta_T (n_{Tt}^s)^{\omega-1} = \frac{W_{Tt}}{P_t^c}$$

$$X_{Tt-1} \theta_N (n_{Nt}^s)^{\omega-1} = \frac{W_{Nt}}{P_t^c}$$

The household's inter-temporal budget constraint is:

$$\begin{aligned} \sum_{j=0}^{\infty} \mathcal{D}_{t+j} P_{t+j}^c C_{t+j} &= \sum_{j=0}^{\infty} \mathcal{D}_{t+j} \left(W_{Tt+j} n_{Tt+j}^s + W_{Nt+j} n_{Nt+j}^s + \Pi_{t+j} \right) \\ &\quad - \sum_{j=0}^{\infty} \mathcal{D}_{t+j} \left(\frac{P_{t+j}}{P_{t+j+1} (1 + r_{t+j})} - \frac{1}{1 + r_{t+j}^*} \right) P_{t+j+1} B_{t+j+1}^s - B_t^{*,s} - P_t B_t^s \end{aligned}$$

where the discount factor satisfies

$$\mathcal{D}_{t+j} \equiv \begin{cases} \frac{\mathcal{D}_{t+j-1}}{1 + r_{t+j-1}^*} & \text{for } j \geq 1 \\ 1 & \text{for } j = 0 \end{cases}$$

and the usual transversality condition

$$\lim_{j \rightarrow \infty} \mathcal{D}_{t+j+1} \frac{B_{t+j+1}^{*,s}}{1 + r_{t+j}^*} = 0$$

has been applied.

The inter-temporal budget constraint says that the present value of consumption expenditures equals non-financial wealth (the top line on the right hand side), net of expected debt revaluation effects and existing debts (second line).⁴

The first order condition for asset allocations implies that expected debt revaluations

⁴The debt revaluation effects measure the difference between the expected returns on non-tradable and tradable bonds, given that the tradable bond rate is chosen to value the intertemporal resource constraint.

are zero in all future periods, so that the inter-temporal budget constraint is:

$$\sum_{j=0}^{\infty} \mathcal{D}_{t+j} P_{t+j}^c C_{t+j} = \sum_{j=0}^{\infty} \mathcal{D}_{t+j} \left(W_{Tt+j} n_{Tt+j}^s + W_{Nt+j} n_{Nt+j}^s + \Pi_{t+j} \right) - B_t^{*s} - P_t B_t^s$$

A household's non-financial wealth is given by:

$$\begin{aligned} \Omega_t^s &\equiv \sum_{j=0}^{\infty} \mathcal{D}_{t+j} \left(W_{Tt+j} n_{Tt+j}^s + W_{Nt+j} n_{Nt+j}^s + \Pi_{t+j} \right) \\ &= W_{Tt} n_{Tt}^s + W_{Nt} n_{Nt}^s + \Pi_t + \frac{1}{1+r_t^*} \Omega_{t+1}^s \end{aligned}$$

where the first line is a definition and the second line exploits the properties of the discount factor and employs a transversality condition.⁵

The household's first order conditions for labor supply demonstrate an important result: labor supply is determined entirely by aggregate conditions (productivity, wages and prices). This means that all households will supply the same labor, independently of their consumption. As a result the non-financial wealth of all households is identical and given by:

$$\Omega_t = W_{Tt} n_{Tt} + W_{Nt} n_{Nt} + \Pi_t + \frac{1}{1+r_t^*} \Omega_{t+1} \quad (86)$$

where

$$n_{Tt} = \left(\frac{W_{Tt}}{\theta_T X_{T,t-1} P_t^c} \right)^{\frac{1}{\omega-1}} \quad (87)$$

$$n_{Nt} = \left(\frac{W_{Nt}}{\theta_N X_{T,t-1} P_t^c} \right)^{\frac{1}{\omega-1}} \quad (88)$$

To simplify the Euler equation, define the disutility of labor supply as:

$$\begin{aligned} \mathcal{N}_t^s &\equiv X_{Tt-1} \omega^{-1} \left(\theta_T (n_{Tt}^s)^\omega + \theta_N (n_{Nt}^s)^\omega \right) \\ &= X_{Tt-1} \omega^{-1} \left(\theta_T \left(\frac{W_{Tt}}{\theta_T X_{T,t-1} P_t^c} \right)^{\frac{\omega}{\omega-1}} + \theta_N \left(\frac{W_{Nt}}{\theta_N X_{T,t-1} P_t^c} \right)^{\frac{\omega}{\omega-1}} \right) \end{aligned} \quad (89)$$

where the second line substitutes for the equilibrium levels of labor supply. Once again, the disutility of labor supply is identical for all households: so $\mathcal{N}_t^s = \mathcal{N}_t, \forall s$.

This means that the Euler equation can be written as:

$$C_{t+1}^s - \mathcal{N}_{t+1} = \beta^{\frac{1}{\gamma}} (1+r_t)^{\frac{1}{\gamma}} \left(\frac{P_t^c}{P_{t+1}^c} \right)^{\frac{1}{\gamma}} [C_t^s - \mathcal{N}_t]$$

⁵Specifically that human wealth does not grow faster than the interest rate: $\lim_{j \rightarrow \infty} (1+r_{t+j})^{-1} \Omega_{t+j+1}^s = 0$.

which implies that

$$P_{t+1}^c C_{t+1}^s - P_{t+1}^c \mathcal{N}_{t+1} = \beta^{\frac{1}{\gamma}} (1 + r_t)^{\frac{1}{\gamma}} \left(\frac{P_t^c}{P_{t+1}^c} \right)^{\frac{1}{\gamma}-1} [P_t^c C_t^s - P_t^c \mathcal{N}_t]$$

Iterating the Euler equation forward implies that

$$P_{t+j}^c C_{t+j}^s = P_{t+j}^c \mathcal{N}_{t+j} + \beta^{\frac{j}{\gamma}} \mathcal{D}_{t+j}^{\frac{-1}{\gamma}} \left(\frac{P_t^c}{P_{t+j}^c} \right)^{\frac{1}{\gamma}-1} [P_t^c C_t^s - P_t^c \mathcal{N}_t]$$

Using this expression in the household's inter-temporal budget constraint gives:

$$\begin{aligned} \Omega_t - P_t B_t^s - B_t^{*,s} &= \sum_{j=0}^{\infty} \mathcal{D}_{t+j} \left[P_{t+j}^c \mathcal{N}_{t+j} + \beta^{\frac{j}{\gamma}} \mathcal{D}_{t+j}^{\frac{-1}{\gamma}} \left(\frac{P_t^c}{P_{t+j}^c} \right)^{\frac{1}{\gamma}-1} (P_t^c C_t^s - P_t^c \mathcal{N}_t) \right] \\ &= (P_t^c C_t^s - P_t^c \mathcal{N}_t) \sum_{j=0}^{\infty} \beta^{\frac{j}{\gamma}} \mathcal{D}_{t+j}^{1-\frac{1}{\gamma}} \left(\frac{P_t^c}{P_{t+j}^c} \right)^{\frac{1}{\gamma}-1} + \sum_{j=0}^{\infty} \mathcal{D}_{t+j} P_{t+j}^c \mathcal{N}_{t+j} \end{aligned}$$

This implies that the household's consumption function can be written as:

$$P_t^c C_t^s = P_t^c \mathcal{N}_t + \Psi_t^{-1} \tilde{\Omega}_t - \Psi_t^{-1} (P_t B_t^s + B_t^{*,s})$$

where Ψ_t is the inverse of the marginal propensity to consume and $\tilde{\Omega}_t$ is adjusted non-financial wealth, given respectively by:

$$\Psi_t = 1 + \beta^{\frac{1}{\gamma}} \left(\frac{P_{t+1}^c}{P_t^c (1 + r_t^*)} \right)^{\frac{\gamma-1}{\gamma}} \Psi_{t+1} \quad (90)$$

$$\tilde{\Omega}_t = W_{T_t} n_{T_t} + W_{N_t} n_{N_t} + \Pi_t - P_t^c \mathcal{N}_t + \frac{1}{1 + r_t^*} \tilde{\Omega}_{t+1} \quad (91)$$

Aggregation across households is straightforward. The consumption function is an affine function of financial wealth. The non-financial wealth components are common across all households. So the aggregate consumption function is also an affine function of (aggregate) financial wealth.

Aggregate consumption is equal to:

$$P_t^c C_t^{\text{agg}} \equiv Z_{t-1} P_t^c C_t^o + (Z_t - Z_{t-1}) P_t^c C_t^n$$

where C_t^o and C_t^n are *per capita* consumption levels of 'old' households (i.e., those alive in period $t - 1$) and newborn households respectively.

Since newborn households enter the model with no financial wealth, we have:

$$C_t^n = P_t^c \mathcal{N}_t + \Psi_t^{-1} \tilde{\Omega}_t$$

The consumption functions of all old agents are affine in financial wealth/debt, so

$$C_t^o = P_t^c \mathcal{N}_t + \Psi_t^{-1} \tilde{\Omega}_t - \Psi_t^{-1} (P_t B_t + B_t^*)$$

where B_t and B_t^* are *per capita* debt stocks (since the date t 'old' households represent the entire population in period $t - 1$).

This implies that:

$$P_t^c C_t^{\text{agg}} = Z_t \left(P_t^c \mathcal{N}_t + \Psi_t^{-1} \tilde{\Omega}_t \right) - Z_{t-1} \Psi_t^{-1} (P_t B_t + B_t^*)$$

and dividing both sides by Z_t gives the *per capita* consumption function:

$$P_t^c C_t = P_t^c \mathcal{N}_t + \Psi_t^{-1} \tilde{\Omega}_t - \Psi_t^{-1} \frac{P_t B_t + B_t^*}{1 + \vartheta} \quad (92)$$

where C, B and B^* are *per capita* consumption and debt stocks.

Note that the case of log utility, $\gamma = 1$, implies that the expression for Ψ_t simplifies substantially to:

$$\Psi_t = (1 - \beta)^{-1}, \forall t$$

which implies that the consumption function under log utility is

$$P_t^c C_t = P_t^c \mathcal{N}_t + (1 - \beta) \tilde{\Omega}_t - \frac{1 - \beta}{1 + \vartheta} (P_t B_t + B_t^*)$$

The aggregate household budget constraint is given by:

$$P_t^c C_t + \frac{P_t B_t + B_t^*}{1 + \vartheta} = W_{Tt} n_{Tt} + W_{Nt} n_{Nt} + \Pi_t + P_t \frac{B_{t+1}}{1 + r_t} + \frac{B_{t+1}^*}{1 + r_t^*} \quad (93)$$

reflecting the same logic as above.⁶

We can eliminate non-financial wealth from the consumption function to derive an aggregate Euler equation. Rearranging the consumption function gives:

$$\tilde{\Omega}_t = \Psi_t (P_t^c C_t - P_t^c \mathcal{N}_t) + \frac{P_t B_t + B_t^*}{1 + \vartheta}$$

⁶The household budget constraint holds for all households, but newborns have no initial financial wealth/debt: $B_t^t = B_t^{*t} = 0$. So the per capita value of previously accumulated debt is equal to the per capita value of debt held last period, divided by the change in population.

which we can substitute into the difference equation for $\tilde{\Omega}$ to give:

$$\begin{aligned} \Psi_t (P_t^c C_t - P_t^c \mathcal{N}_t) + \frac{P_t B_t + B_t^*}{1 + \vartheta} &= W_{Tt} n_{Tt} + W_{Nt} n_{Nt} + \Pi_t - P_t^c \mathcal{N}_t \\ &+ \frac{1}{1 + r_t^*} \left(\Psi_{t+1} (P_{t+1}^c C_{t+1} - P_{t+1}^c \mathcal{N}_{t+1}) + \frac{P_{t+1} B_{t+1} + B_{t+1}^*}{1 + \vartheta} \right) \end{aligned}$$

Using the aggregate budget constraint to substitute for $\frac{P_t B_t + B_t^*}{1 + \vartheta}$ gives:

$$\begin{aligned} \Psi_t (P_t^c C_t - P_t^c \mathcal{N}_t) + P_t \frac{B_{t+1}}{1 + r_t} + \frac{B_{t+1}^*}{1 + r_t^*} &= P_t^c C_t - P_t^c \mathcal{N}_t \\ + \frac{1}{1 + r_t^*} \left(\Psi_{t+1} (P_{t+1}^c C_{t+1} - P_{t+1}^c \mathcal{N}_{t+1}) + \frac{P_{t+1} B_{t+1} + B_{t+1}^*}{1 + \vartheta} \right) \end{aligned}$$

which can be rearranged to give:⁷

$$(\Psi_t - 1) (P_t^c C_t - P_t^c \mathcal{N}_t) = \frac{\Psi_{t+1}}{1 + r_t} (P_{t+1}^c C_{t+1} - P_{t+1}^c \mathcal{N}_{t+1}) - \frac{\vartheta}{1 + \vartheta} \frac{P_{t+1} B_{t+1} + B_{t+1}^*}{1 + r_t^*}$$

Finally, noting from (90) that $\Psi_t - 1 = \beta^{\frac{1}{\gamma}} \left(\frac{P_{t+1}^c}{P_t^c (1 + r_t)} \right)^{\frac{\gamma-1}{\gamma}} \Psi_{t+1}$, gives

$$\begin{aligned} P_t^c C_t - P_t^c \mathcal{N}_t &= \left(\frac{P_{t+1}^c}{P_t^c} \right)^{\frac{1-\gamma}{\gamma}} (\beta (1 + r_t))^{-\frac{1}{\gamma}} (P_{t+1}^c C_{t+1} - P_{t+1}^c \mathcal{N}_{t+1}) \\ &- \frac{\vartheta}{1 + \vartheta} (\Psi_t - 1)^{-1} \frac{P_{t+1} B_{t+1} + B_{t+1}^*}{1 + r_t^*} \end{aligned}$$

This demonstrates that the aggregate Euler equation depends on total asset holdings. In the case of no population growth, $\vartheta = 0$, the aggregate and individual household Euler equations coincide.

G.3. Firms

Firms maximize dividends over an infinite horizon and distribute them lump sum to households.

The non-tradable firm maximizes the present discounted value of dividend payments (expressed in units of *tradable* output):

$$\max \sum_{i=0}^{\infty} \lambda_{t,t+i} \left[\begin{array}{l} P_{t+i} a_{Nt+i} K_{Nt+i}^{\alpha_N} (X_{Nt+i} n_{Nt+i})^{1-\alpha_N} - P_{t+i} \frac{\phi_N}{2} \left(\frac{K_{Nt+i+1}}{K_{Nt+i}} - \bar{g}_N \right)^2 K_{Nt+i} \\ - P_{t+i} I_{Nt+i} - w_{Nt+i} n_{Nt+i} \end{array} \right]$$

subject to: $K_{Nt+i+1} = (1 - \delta_N) K_{Nt+i} + I_{Nt+i}$

⁷The no-arbitrage condition for asset returns implies that $\frac{P_t}{1+r_t} = \frac{P_{t+1}}{1+r_t^*}$, which allows us to collect terms in both types of bonds.

where $\lambda_{t,t+1}$ is a (compound) discount factor (discussed below) and the term in brackets is the per-period dividend.

Substituting for I_{Nt+1} implies that the firm maximizes:

$$\max \sum_{i=0}^{\infty} \lambda_{t,t+i} \left[\begin{array}{l} P_{t+i} a_{Nt+i} K_{Nt+i}^{\alpha_N} (X_{Nt+i} n_{Nt+i})^{1-\alpha_N} - P_{t+i} \frac{\phi_N}{2} \left(\frac{K_{Nt+i+1}}{K_{Nt+i}} - \bar{g}_N \right)^2 K_{Nt+i} \\ - P_{t+i} K_{Nt+i+1} + (1 - \delta_N) P_{t+i} K_{Nt+i} - w_{Nt+i} n_{Nt+i} \end{array} \right]$$

The first order conditions are:

$$0 = W_{Nt} - P_t (1 - \alpha_N) a_{Nt} K_{Nt}^{\alpha_N} (X_{Nt} n_{Nt})^{-\alpha_N} X_{Nt} \quad (94)$$

$$0 = -\lambda_{t,t} P_t \left(1 + \phi_N \left(\frac{K_{Nt+1}}{K_{Nt}} - \bar{g}_N \right) \right) + \lambda_{t,t+1} P_{t+1} \left(\begin{array}{l} \alpha_N a_{Nt+1} K_{Nt+1}^{\alpha_N-1} (X_{Nt+1} n_{Nt+1})^{1-\alpha_N} + (1 - \delta_N) \\ - \frac{\phi_N}{2} \left(\frac{K_{Nt+2}}{K_{Nt+1}} - \bar{g}_N \right)^2 + \phi_N \left(\frac{K_{Nt+2}}{K_{Nt+1}} - \bar{g}_N \right) \frac{K_{Nt+2}}{K_{Nt+1}} \end{array} \right) \quad (95)$$

The first order condition for labor is identical to the baseline model. The second equation looks slightly different because of the different ownership structure (ie firms are now assumed to own the capital stock).

The first order condition for capital can be written as:

$$\alpha_N a_{Nt+1} K_{Nt+1}^{\alpha_N-1} (X_{Nt+1} n_{Nt+1})^{1-\alpha_N} + 1 - \delta_N = \frac{\lambda_{t,t} P_t}{\lambda_{t,t+1} P_{t+1}} \left(1 + \phi_N \left(\frac{K_{Nt+1}}{K_{Nt}} - \bar{g}_N \right) \right) + \frac{\phi_N}{2} \left(\frac{K_{Nt+2}}{K_{Nt+1}} - \bar{g}_N \right)^2 - \phi_N \left(\frac{K_{Nt+2}}{K_{Nt+1}} - \bar{g}_N \right) \frac{K_{Nt+2}}{K_{Nt+1}}$$

The left hand side is the return on capital, net of depreciation, measured in units of non-tradable output. The right hand side captures the inter-temporal cost of substituting non-tradable output across time and adjustment costs. The right hand side depends only on the ratio of discount factors between time periods. This is the same for all households and is given by the inverse of the rate of return on intermediary bonds. This means that the first order condition for non-tradable capital is:

$$\alpha_N a_{Nt+1} K_{Nt+1}^{\alpha_N-1} (X_{Nt+1} n_{Nt+1})^{1-\alpha_N} + 1 - \delta_N = \frac{P_t (1 + r_t^*)}{P_{t+1}} \left(1 + \phi_N \left(\frac{K_{Nt+1}}{K_{Nt}} - \bar{g}_N \right) \right) + \frac{\phi_N}{2} \left(\frac{K_{Nt+2}}{K_{Nt+1}} - \bar{g}_N \right)^2 - \phi_N \left(\frac{K_{Nt+2}}{K_{Nt+1}} - \bar{g}_N \right) \frac{K_{Nt+2}}{K_{Nt+1}}$$

The tradable firm solves an isomorphic problem. So the labor demand equation is

the same as the baseline model. The first order condition for capital is:

$$\begin{aligned} \alpha_T a_{T+1} K_{T+1}^{\alpha_T-1} (X_{T+1} n_{T+1})^{1-\alpha_T} + 1 - \delta_T &= (1 + r_t^*) \left(1 + \phi_T \left(\frac{K_{T+1}}{K_T} - \bar{g}_T \right) \right) \\ &+ \frac{\phi_T}{2} \left(\frac{K_{T+2}}{K_{T+1}} - \bar{g}_T \right)^2 - \phi_T \left(\frac{K_{T+2}}{K_{T+1}} - \bar{g}_T \right) \frac{K_{T+2}}{K_{T+1}} \end{aligned}$$

G.4. Market clearing

The market clearing conditions are the same as in previous derivations. Substituting into the *per capita* budget constraint gives:

$$\frac{B_t^*}{1 + \vartheta} = TB_t + \frac{B_{t+1}^*}{1 + r_t^*}$$

where we impose that non-tradable bonds are in zero net supply.

G.5. Stationary units

The redefinition of sector-specific growth rates to include deterministic population growth rates means that the transformations into stationary units in the baseline model continue to hold in almost all cases. The main exception is the consumption Euler equation and household budget constraints, which we have written in per capita terms, but without adjusting for non-stationary tradable productivity. Adjusting the Euler equation for productivity gives:

$$\begin{aligned} X_{T-1} (P_t^c c_t - P_t^c n_t) &= \left(\frac{P_{t+1}^c}{P_t^c} \right)^{\frac{1-\gamma}{\gamma}} (\beta (1 + r_t^*))^{-\frac{1}{\gamma}} X_{Tt} (P_{t+1}^c c_{t+1} - P_{t+1}^c n_{t+1}) \\ &- \frac{\vartheta X_{Tt}}{1 + \vartheta} (\Psi_t - 1)^{-1} \frac{P_{t+1} b_{t+1} + b_{t+1}^*}{1 + r_t^*} \end{aligned}$$

where lower case letters denote stationary units (as in the baseline model), which in this variant means adjusted for both productivity and population.

In particular,

$$n_t = X_{Tt}^{-1} \mathcal{N}_t = \omega^{-1} \left(\theta_T \left(\frac{w_{Tt}}{\theta_T p_t^c} \right)^{\frac{\omega}{\omega-1}} + \theta_N \left(\frac{w_{Nt}}{\theta_N p_t^c} \right)^{\frac{\omega}{\omega-1}} \right)$$

These considerations imply that the Euler equation in stationary units is given by:⁸

$$p_t^c (c_t - n_t) = \left(\frac{p_{t+1}^c}{p_t^c} \right)^{\frac{1-\gamma}{\gamma}} (\beta (1 + r_t^*))^{-\frac{1}{\gamma}} \frac{g_{Tt}}{1 + \vartheta} p_{t+1}^c (c_{t+1} - n_{t+1}) - \frac{\vartheta g_{Tt}}{1 + \vartheta} (\Psi_t - 1)^{-1} \frac{P_{t+1} b_{t+1} + b_{t+1}^*}{1 + r_t^*}$$

Similar arguments apply to the flow budget constraint:

$$\frac{X_{Tt-1} b_t^*}{1 + \vartheta} = X_{Tt-1} t b_t + \frac{X_{Tt} b_{t+1}^*}{1 + r_t^*}$$

so that

$$\frac{b_t^*}{1 + \vartheta} = t b_t + \frac{g_{Tt} b_{t+1}^*}{(1 + \vartheta) (1 + r_t^*)}$$

G.6. Steady state

In steady state (also imposing market clearing) the Euler equation implies:

$$p^c (c - n) \left[1 - (\beta (1 + r^*))^{-\frac{1}{\gamma}} \frac{\bar{g}_T}{1 + \vartheta} \right] = - \frac{\vartheta \bar{g}_T}{1 + \vartheta} \frac{b^*}{(\Psi - 1) (1 + r^*)}$$

Under the assumption that $c > n$ (so that marginal utility is positive in steady state), this expression reveals that the sign of the economy's foreign debt position depends on the relative patience of households. Specifically, it depends on the size of the discount factor β in relation to the (productivity) growth adjusted real interest rate. In the particular case in which $\beta = \beta_0 \equiv \left(\frac{1 + \vartheta}{\bar{g}_T} \right)^{-\gamma} \frac{1}{1 + r^*}$, the economy will hold no foreign debt or assets (and the trade balance will be zero) in steady state. If the economy is relatively less patient (so $\beta < \beta_0$) then the economy will be a net debtor, with $b^* > 0$, in steady state. Conversely, if households are more patient, then the economy will hold foreign bonds in steady state ($b^* < 0$ means that *debt* is negative and the economy holds positive assets).

These observations mean that we can calibrate β to deliver a desired steady-state foreign debt position (conditional on the values of the other model parameters). First note that the steady-state (inverse) marginal propensity to consume is given by:

$$\Psi = \frac{1}{1 - \beta^{\frac{1}{\gamma}} (1 + r^*)^{\frac{1}{\gamma} - 1}}$$

⁸Recall that P_t^c is cointegrated with tradable productivity, so $p_t^c = P_t^c$.

Plugging this into the steady-state Euler equation and rearranging gives:

$$\begin{aligned}
-\frac{\vartheta \bar{g}_T}{1 + \vartheta} \frac{b^*}{p^c (c - n) (1 + r^*)} &= \left(\frac{1}{1 - \beta^{\frac{1}{\gamma}} (1 + r^*)^{\frac{1}{\gamma} - 1}} - 1 \right) \left[1 - (\beta (1 + r^*))^{-\frac{1}{\gamma}} \frac{\bar{g}_T}{1 + \vartheta} \right] \\
&= \frac{\beta^{\frac{1}{\gamma}} (1 + r^*)^{\frac{1}{\gamma} - 1}}{1 - \beta^{\frac{1}{\gamma}} (1 + r^*)^{\frac{1}{\gamma} - 1}} \left[1 - (\beta (1 + r^*))^{-\frac{1}{\gamma}} \frac{\bar{g}_T}{1 + \vartheta} \right] \\
&= \frac{\mathcal{B}}{1 + r^* - \mathcal{B}} \left[1 - \mathcal{B}^{-1} \frac{\bar{g}_T}{1 + \vartheta} \right]
\end{aligned}$$

where the final line makes use of the following definition:

$$\mathcal{B} \equiv \beta^{\frac{1}{\gamma}} (1 + r^*)^{\frac{1}{\gamma}}$$

Rearranging the final equation allows us to solve for \mathcal{B} :

$$\mathcal{B} = \frac{\bar{g}_T \left(1 - \frac{\vartheta b^*}{p^c (c - n)} \right)}{(1 + \vartheta) \left(1 - \frac{\vartheta \bar{g}_T b^*}{(1 + \vartheta)(1 + r^*) p^c (c - n)} \right)}$$

as a function of other parameters, steady-state allocations and the desired steady-state foreign debt position, b^* .

Finally, we can solve for $\beta = \mathcal{B}^\gamma (1 + r^*)^{-1}$.

REFERENCES

- BETTS, C. M. AND T. J. KEHOE (2006): "U.S. real exchange rate fluctuations and relative price fluctuations," *Journal of Monetary Economics*, 53, 1297–1326.
- DE GREGORIO, J., A. GIOVANNINI, AND H. C. WOLF (1994): "International evidence on tradables and nontradables inflation," *European Economic Review*, 38, 1225–1244.
- GOODRIDGE, P., J. HASKEL, AND G. WALLIS (2018): "Accounting for the UK Productivity Puzzle: A Decomposition and Predictions," *Economica*, 85, 581–605.
- IRELAND, P. N. (2005): "The liquidity trap, the real balance effect, and the Friedman rule," *International Economic Review*, 46, 1271–1301.
- LOMBARDO, G. AND F. RAVENNA (2012): "The size of the tradable and non-tradable sectors: Evidence from input–output tables for 25 countries," *Economics Letters*, 116, 558–561.
- TENREYRO, S. (2018): "'The fall in productivity growth: causes and implication'," speech given at the Peston Lecture Theatre (Queen Mary, University of London), Bank of England.
- WEIL, P. (1989): "Overlapping families of infinitely-lived agents," *Journal of public economics*, 38, 183–198.
- (1991): "Is money net wealth?" *International Economic Review*, 37–53.