



WHY ARE RETURNS TO PRIVATE BUSINESS WEALTH SO DISPERSED?

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## Motivation

Q: Why are private business *accounting* returns dispersed?

A: BGM investigate

- Partial insurance for business income risk
  - Collateral constraints limiting borrowing
- ⇒ Factor misallocation



## Main Exercise

- Parameterize model of entrepreneurial dynamics
- Ensure model predicts enough dispersion in ROEs
- Quantify roles of uninsurable risk and limited borrowing



## Main Exercise

- Parameterize model of entrepreneurial dynamics
- Ensure model predicts enough dispersion in ROEs
- Quantify roles of uninsurable risk and limited borrowing

Next, consider BGM model of business owner



# Problem of BGM Business Owner

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta}$$

$$\text{st } c_t + \underbrace{k_{t+1} - b_{t+1} - (k_t - b_t)}_{\text{change in equity, } a_{t+1} - a_t}$$

$$\leq \underbrace{z_t \epsilon_t (k_t^\alpha l_t^{1-\alpha})^\eta - W l_t - \delta k_t - r b_t}_{\text{net income, } \pi_t}$$

$$b_{t+1} \leq \xi k_{t+1}$$

$$\log z_{t+1} = \rho \log z_t + u_{t+1}$$



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plus timing assumption : choose  $k, l$  before shocks



**Let's Take a Closer Look...**



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# Problem of BGM Business Owner

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta} \quad \leftarrow \text{Only 1 owner}$$

$$\text{st } c_t + \underbrace{k_{t+1} - b_{t+1} - (k_t - b_t)}_{\text{change in equity, } a_{t+1} - a_t}$$

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$$b_{t+1} \leq \xi k_{t+1} \quad \text{Weathering Tukey g-h shocks}$$

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With only  
business  
income



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With only  
← business  
saving

$$\leq \underbrace{z_t \epsilon_t (k_t^\alpha l_t^{1-\alpha})^\eta - W l_t - \delta k_t - r b_t}_{\text{net income, } \pi_t}$$

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↑  
 ← Without  
 bankruptcy  
 or default



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Without  
rental  
markets



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Without  
independent  
contractors







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Without  
informal  
workers  
✓



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family  
workers  
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Without  
opportunities  
to publicly list



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Without  
possibility of  
paid-employment



$$b_{t+1} \leq \xi k_{t+1}$$

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Without  
government  
help  
↓

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Without  
spousal  
income  
↓

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Without  
Mom's  
basement!  
↓

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# Quantitative Findings





# Eliminating Collateral Constraints

- Increases
  - Productivity 0.02%
  - Output 0.15%
  - Wages 0.02%

⇒ Almost no change



# Eliminating Collateral Constraints

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Is this surprising?



# Eliminating Collateral Constraints

- Increases
  - Productivity 0.02%
  - Output 0.15%
  - Wages 0.02%

Is this surprising? **Not really**



# Insuring Risk

- Increases
  - Productivity 5.7%
  - Output 8.2%
  - Wages 15.5%
  - Value of businesses by 700%

⇒ Huge changes



# Insuring Risk

- Increases
  - Productivity 5.7%
  - Output 8.2%
  - Wages 15.5%
  - Value of businesses by 700%

Is this surprising?



## Surprising?

- Perhaps given
  - Magnitudes are so large
  - Impact of risk on saving/investment not obvious
- Perhaps not given
  - Few sources of insurance in the model
  - Owners have small-scale operations to avoid losses
  - Only firm-level data used for calibration



## A Look at Data from US



## In Related Work

- Assembled longitudinal database of business owners
- Estimated life-cycle income profiles for 35,000 groups
- Compared similar self- and paid-employed (SE&PE)
  - Growth and volatility patterns
  - Determinants of entrepreneurial choice

See: *On the Nature of Entrepreneurship*

(Bhandari, Kass, May, McGrattan, and Schulz)





## Of Particular Relevance for BGM

- Comparisons of *attached* SE and PE
  - Same employment status for 12+ years
  - Fewer than 2 switches in status during sample
  - No intermediate non-employment years
- Information from individual filings
  - All income sources (plus W2s, K1s, etc)
  - Plus related family incomes
- Information from business filings
  - Income statements
  - Balance sheets (if large enough)
  - Links to employees/other businesses/other owners



## Relevant Findings

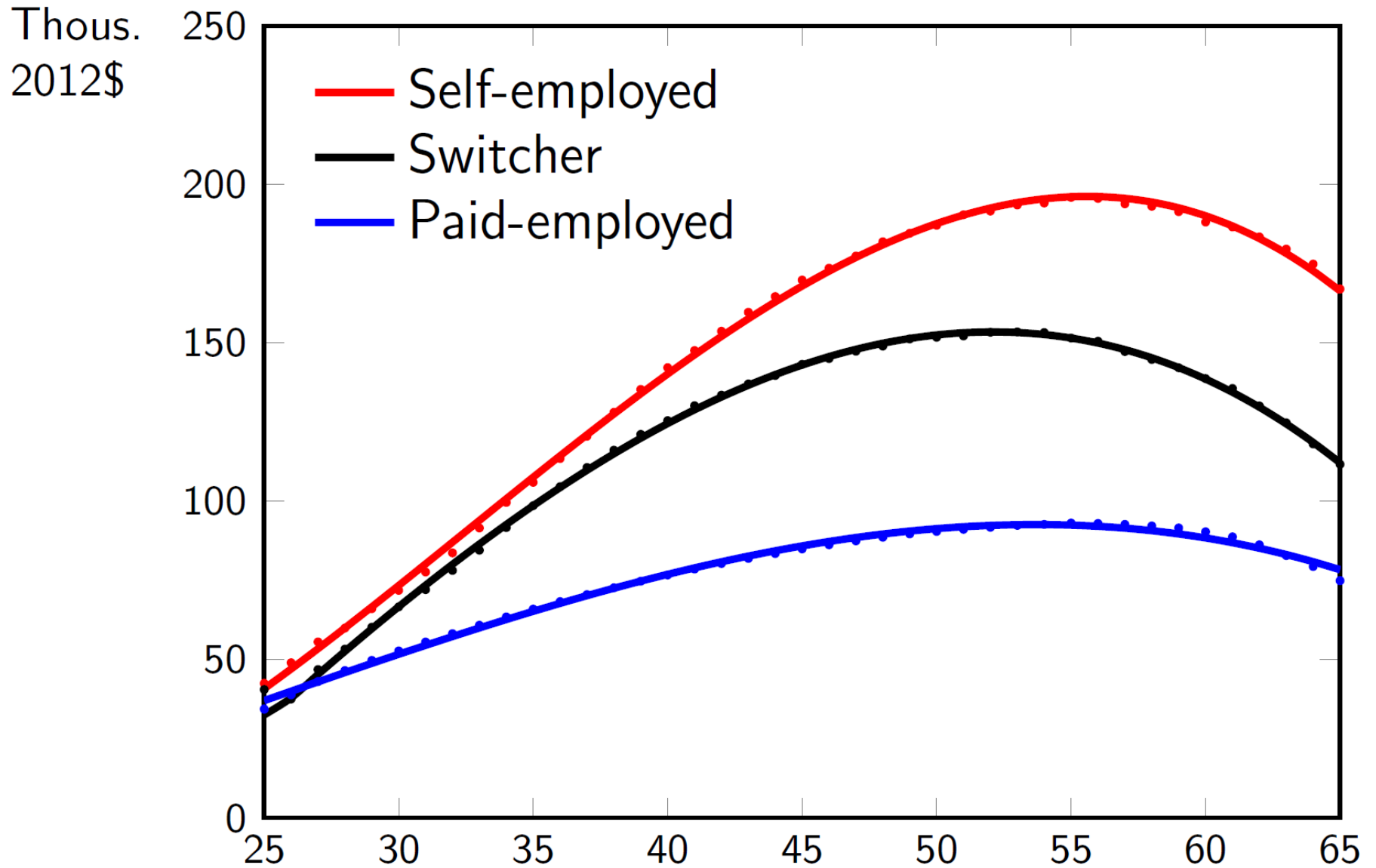
- For individual  $i$ , time  $t$ , cohort  $c$ , age  $a$ , group  $g$ , estimate:

$$y_{it} = \alpha_i + \beta_{g(i),t} + \sum_a \gamma_{c(i),g(i)}^a + \epsilon_{it}$$

- Find:
  - Attached SE:
    - Start out similarly to PE peers, *but*
    - Have higher and more persistent growth
    - Have losses and incomes 3 times more volatile
  - Entrants to SE (including non-attached):
    - Have lower asset income before entry
- How possible without some insurance?



# Estimated Income Profiles

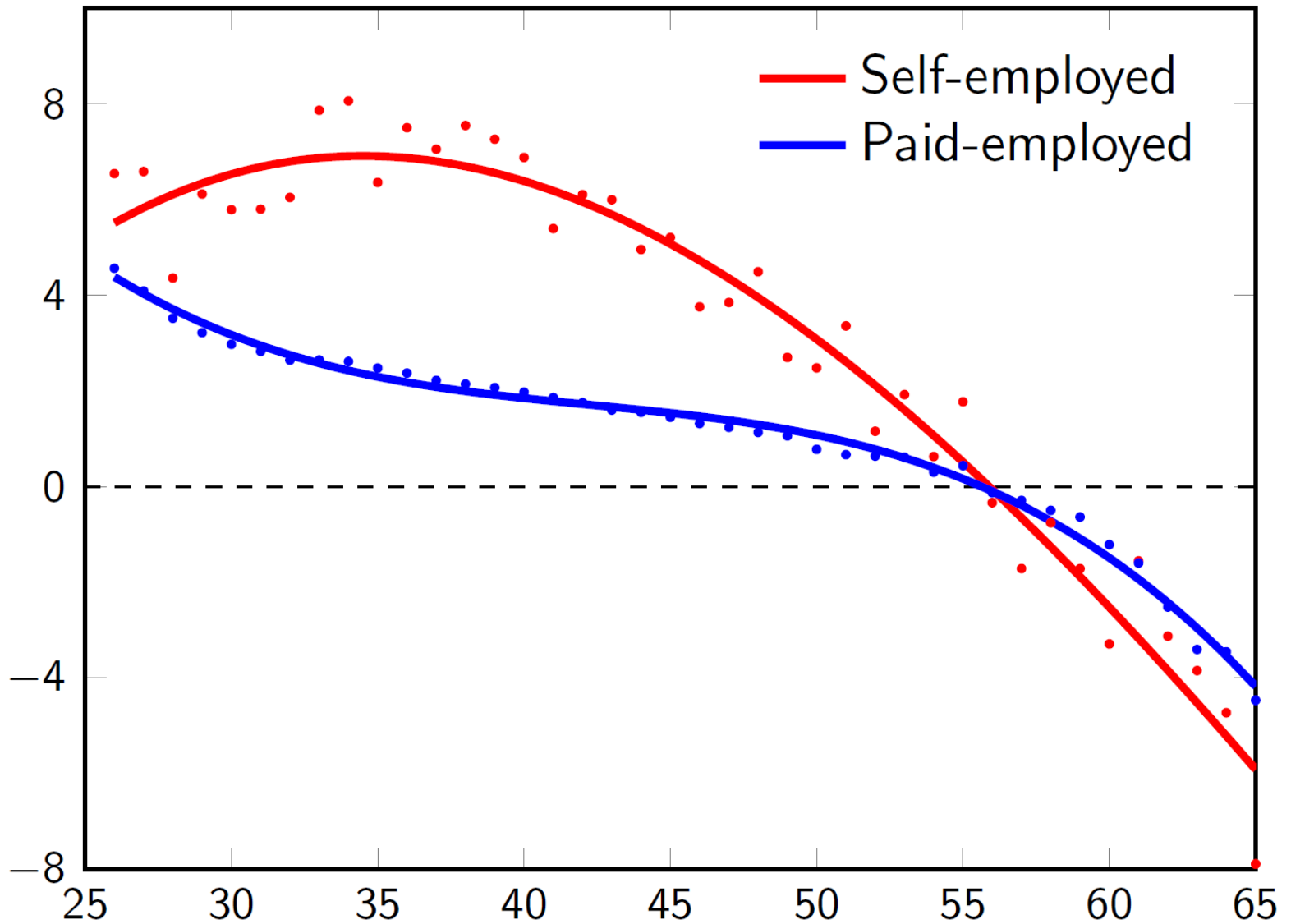


- SE start out similarly but grow much faster



# Estimated Growth

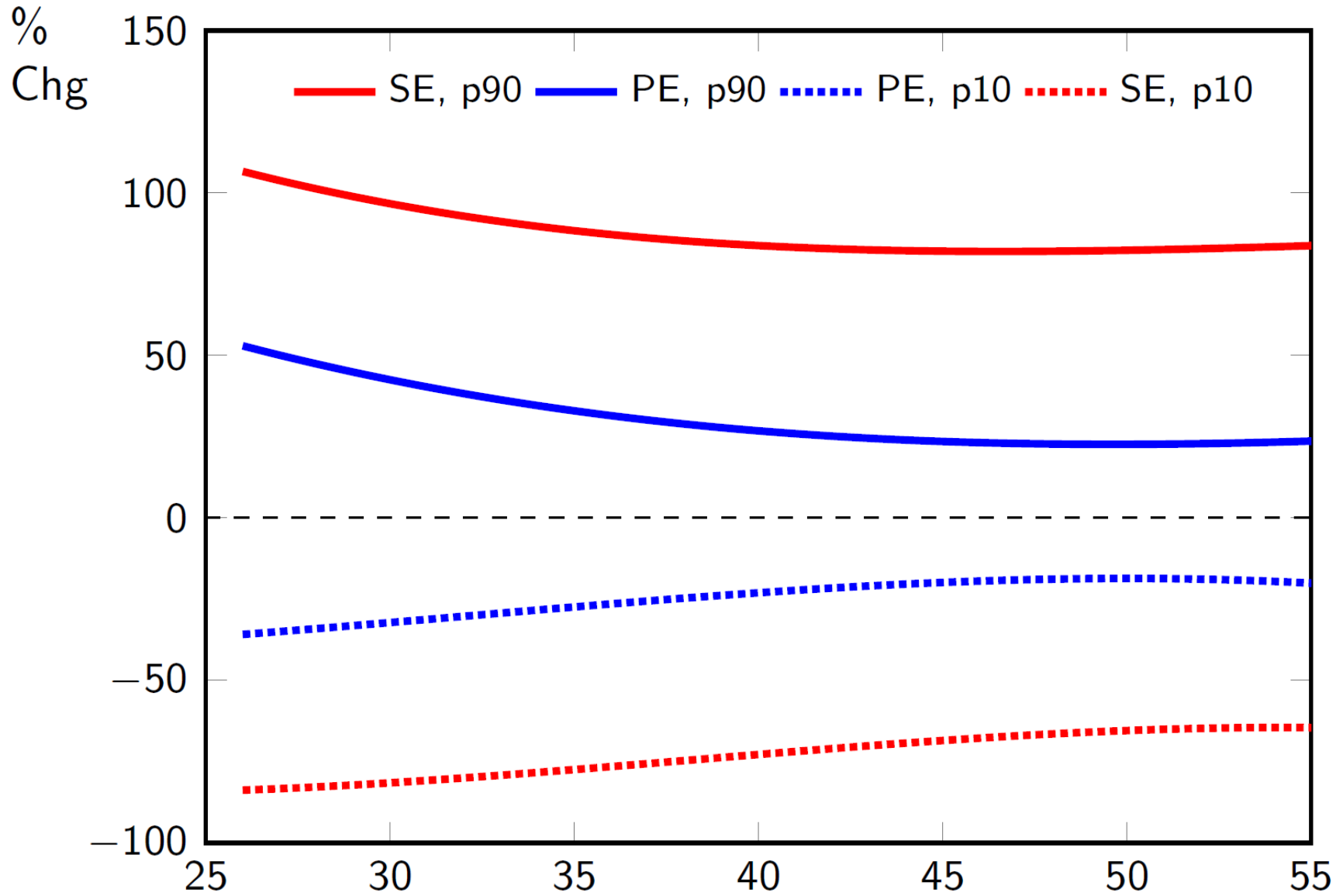
Thous.  
2012\$



- SE rewarded for early firm-specific investments



# Income Changes for Attached SE/PE

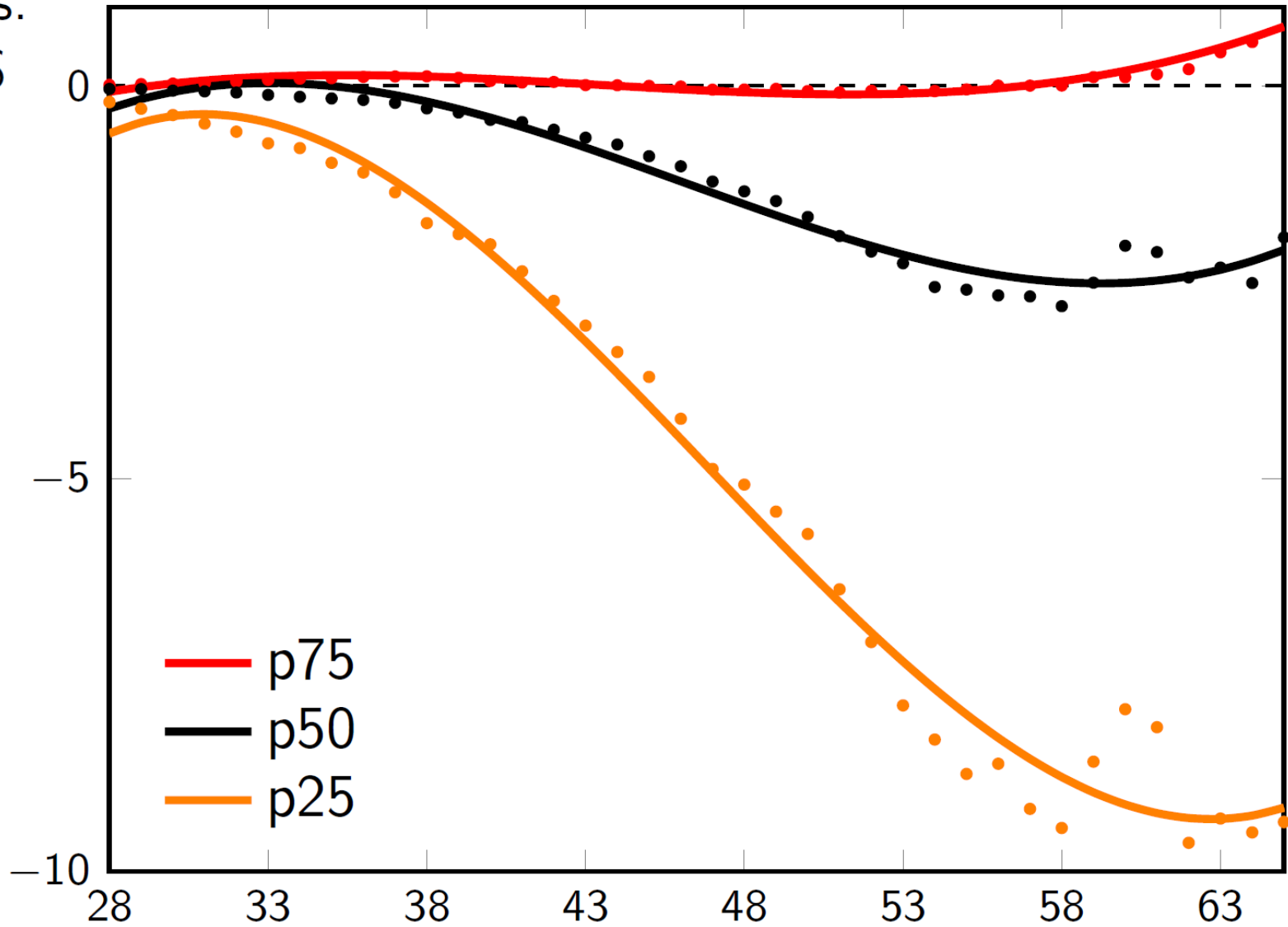


- Volatility in SE about 3 times that of PE



# Difference in Past Asset Incomes

Thous.  
2012\$



● Past asset incomes *lower* for entrants



## How Possible?

- Spousal wages provide insurance
- Businesses have multiple owners
- Owners have multiple businesses
- Rental/labor markets are fluid
- Debt financing not always needed
- Public listing always an option
- Paid-employment always an option



## How Possible?

- Spousal wages provide insurance
- Businesses have multiple owners
- Owners have multiple businesses
- Rental/labor markets are fluid
- Debt financing not always needed
- Public listing always an option
- Paid-employment always an option
- And...there is always Mom!





## Disclaimer

The authors thank Anne Parker and Barry Johnson for facilitating this project through the Joint Statistical Research Program of the Statistics of Income Division of the United States Internal Revenue Service. May and McGrattan are IRS employees without pay under an agreement made possible by the Intragovernmental Personnel Act of 1970 (5 U.S.C. 3371-3376). Any opinions and conclusions expressed herein are those of the authors and do not necessarily represent the views of the Internal Revenue Service or the U.S. Department of the Treasury, or the National Science Foundation. All results have been reviewed to ensure that no confidential information is disclosed. All data work for this project involving confidential taxpayer information was done at IRS facilities, on IRS computers, by IRS employees, and at no time was confidential taxpayer data ever outside of the IRS computing environment.