Measurement with Minimal Theory

Ellen McGrattan

Federal Reserve Bank of Minneapolis

July 2006

Working paper #643 available at minneapolisfed.org/research

Central Questions

- What are the driving forces of business cycles?
- Can we determine key forces using *minimal theory*?
- If so, then ultimately useful for
  - Isolating promising classes of models
  - Doing policy analysis

Minimal Theory: Hall's Conjecture

- Many DSGE models are representable as a VARMA

$$
\begin{aligned}
Y_t &= B_1 Y_{t-1} + B_2 Y_{t-2} + \ldots + e_t \\
&= (B_1 + M) Y_{t-1} + e_t - M e_{t-1} \qquad (1)
\end{aligned}
$$

with $E e_t e_t' = \Sigma$, $M = B_j B_{j-1}^{-1}$, $j \geq 2$.

- Estimate (1) via MLE

- Can use it to compute IRFs and variance decompositions

Good in Theory, Not in Practice

- Set up a laboratory
  - Map a DSGE model to a VARMA
  - Simulate N datasets from the VARMA
  - Estimate a VARMA for each dataset
  - Construct statistics of interest
- Show it's fine with $T = \infty$, not useful with $T = 200$
- Range of answers too wide to be a good guide

Imposing a Little More: C&K Conjecture

- Many DSGE models representable as a state space

$$X_{t+1} = AX_t + B\epsilon_t, \quad E\epsilon_t\epsilon_t' = I$$
$$Y_t = CX_t$$

  with zero elements of A, B, C known a priori

- Estimate without imposing cross-equation restrictions

- Can use it to compute IRFs and variance decompositions

Good in Theory, Not in Practice

- Set up a laboratory
  - Map a DSGE model to the SS model
  - Simulate N datasets from the SS model
  - Estimate SS model for each dataset
  - Construct statistics of interest
- Show it's fine with $T = \infty$, not useful with $T = 200$

Imposing Even More: McGrattan's Conjecture

- Work directly with the DSGE model (Restricted SS)
- May even need tight bounds on parameters, e.g.,
  - Loose: impose no bounds
  - Modest: impose economically plausible bounds
  - Tight: estimate only controversial parameters

Good in Theory, And in Practice

- Set up a laboratory
  - Simulate N datasets from the DSGE model
  - Estimate "deep structural" parameters for each
  - Construct statistics of interest
- Show dramatic improvement with DSGE for $T = 200$

Setting up the Lab

Prototype Economy

- The state vector is $[\log \hat{k}_t, s_t, 1]$ with

$$\log \hat{k}_{t+1} = \gamma_0 + \gamma_k \log \hat{k}_t + \gamma_s' s_t$$

$$s_{t+1} = P_0 + P s_t + Q \epsilon_{t+1}$$

  where $\log \hat{k}_t$ is detrended capital, s_t are shocks

- Observables in estimation are:

$$Y_t = C X_t, \quad X_t = [\log \hat{k}_t, s_t, s_{t-1}, 1]'$$

- CKM show equivalence of many models to prototype

Some Details

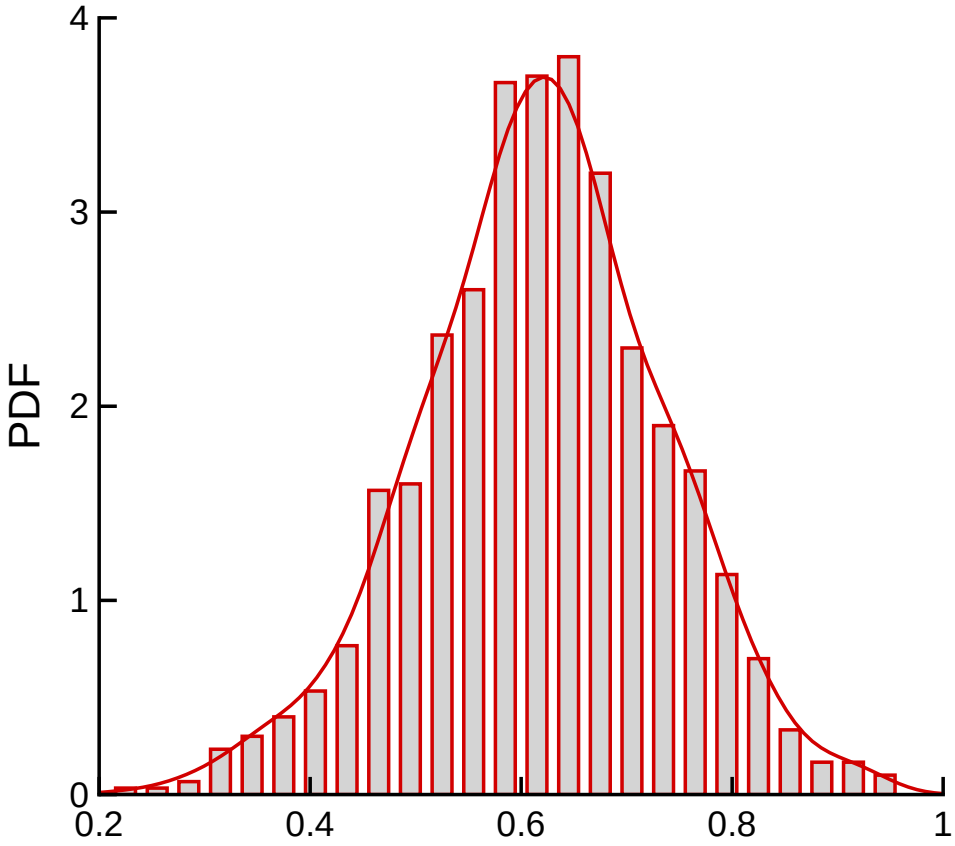
- Preferences: $(c(1-l)^{\psi})^{1-\sigma}/(1-\sigma)$
- Technology: Cobb-Douglas in capital/labor
- 3 shocks (all with innovation variance = 1%):
  - Unit-root technology
  - Labor wedge, AR(1) with $\rho = .95$
  - Investment wedge, AR(1) with $\rho = .95$
- 3 observables:
  - Growth in log labor productivity
  - Log of hours per person
  - Log of investment share

Statistics of Interest

- Impulse responses of observables
- Variance decompositions of observables
- K-P statistics for HP-filtered output, hours, investment
  - Standard deviations
  - Autocorrelations
  - Cross-correlations
  - Variance decompositions

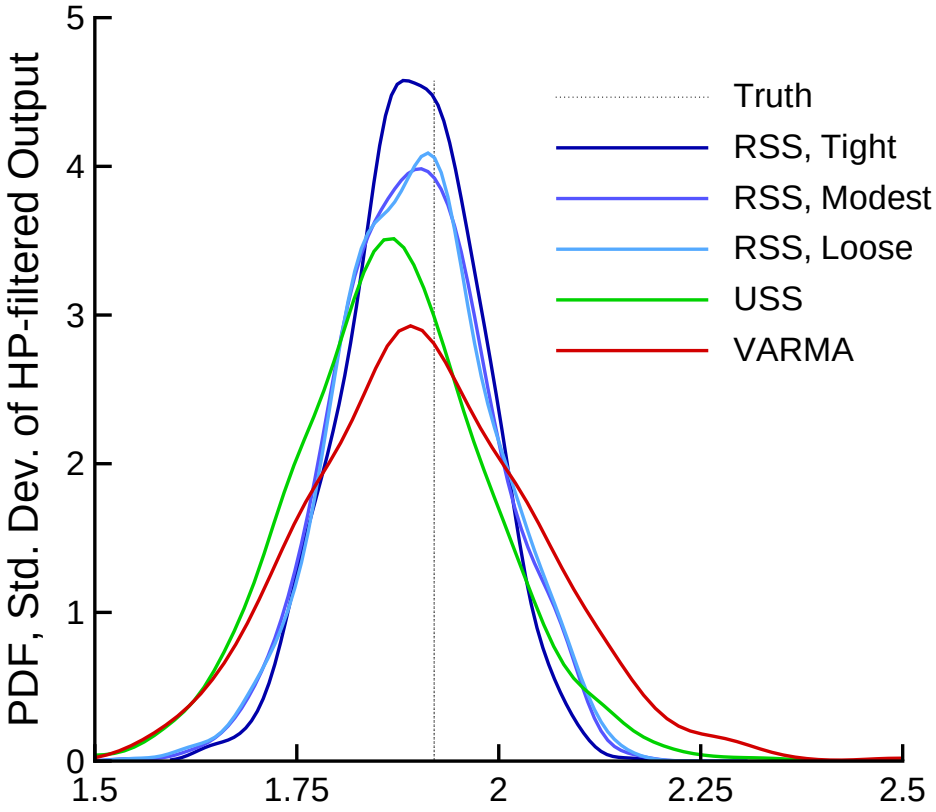
Results

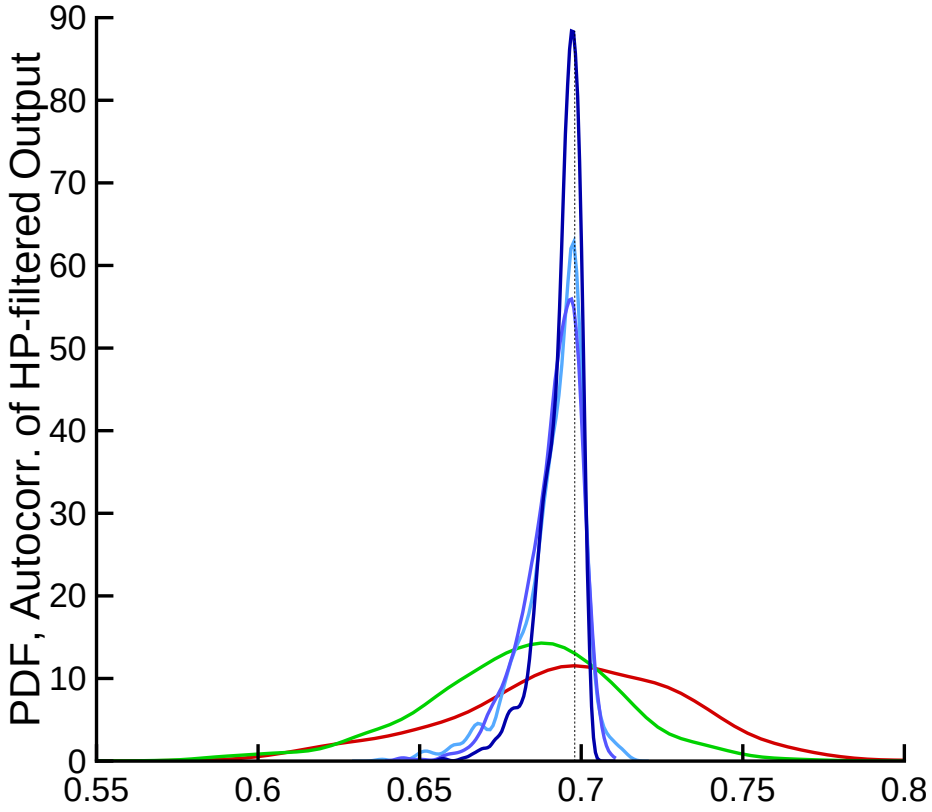
Displaying the Results



- Use kernel estimates to smooth
- Drop histogram when showing results

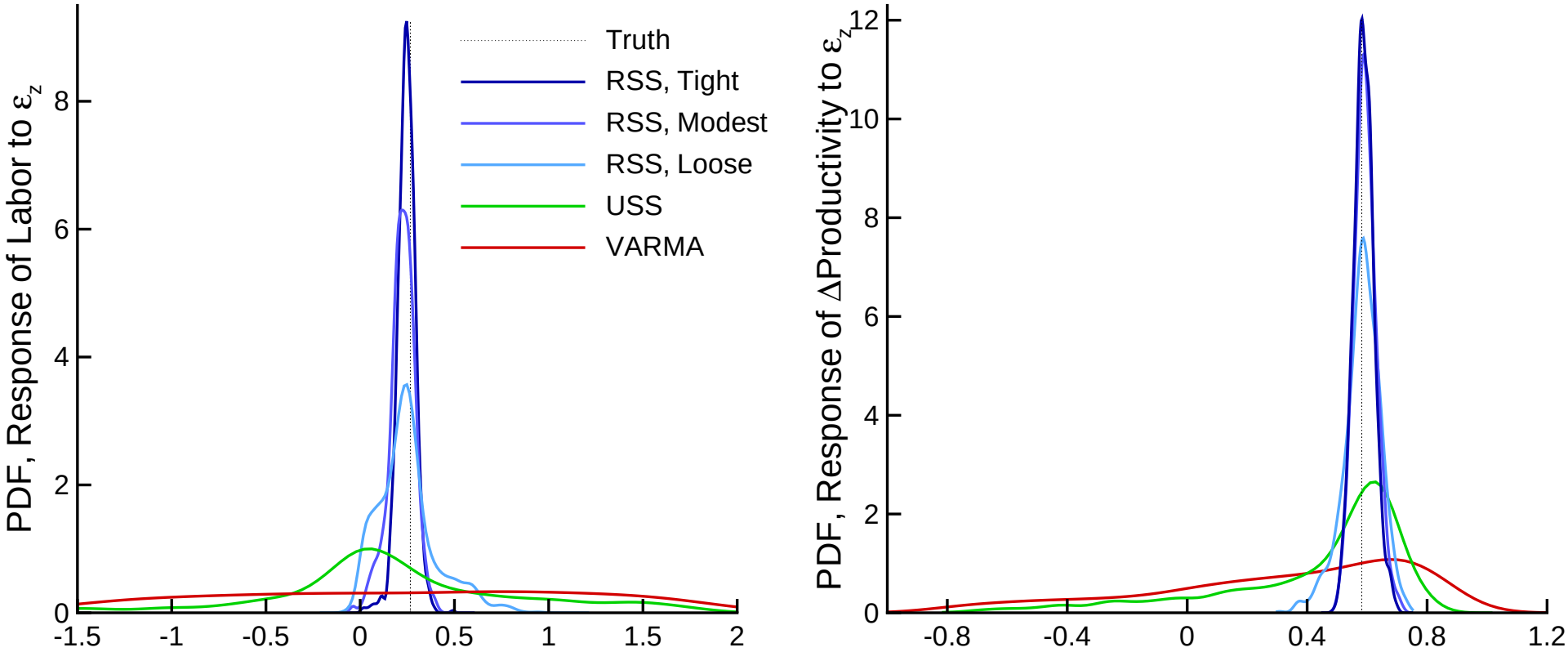
Statistics for HP-filtered Output





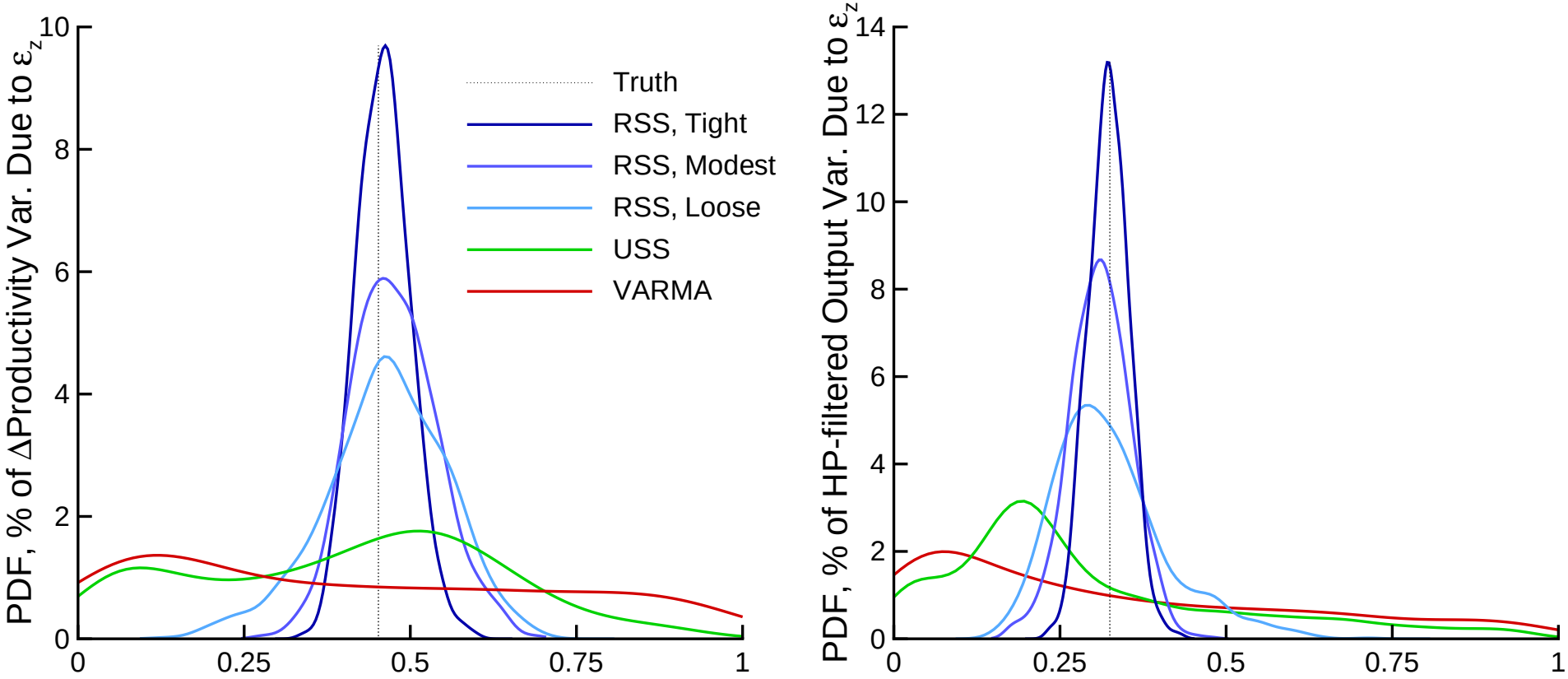
- All do well with SDs
- Some differences with ACs

Responses to Technology on Impact



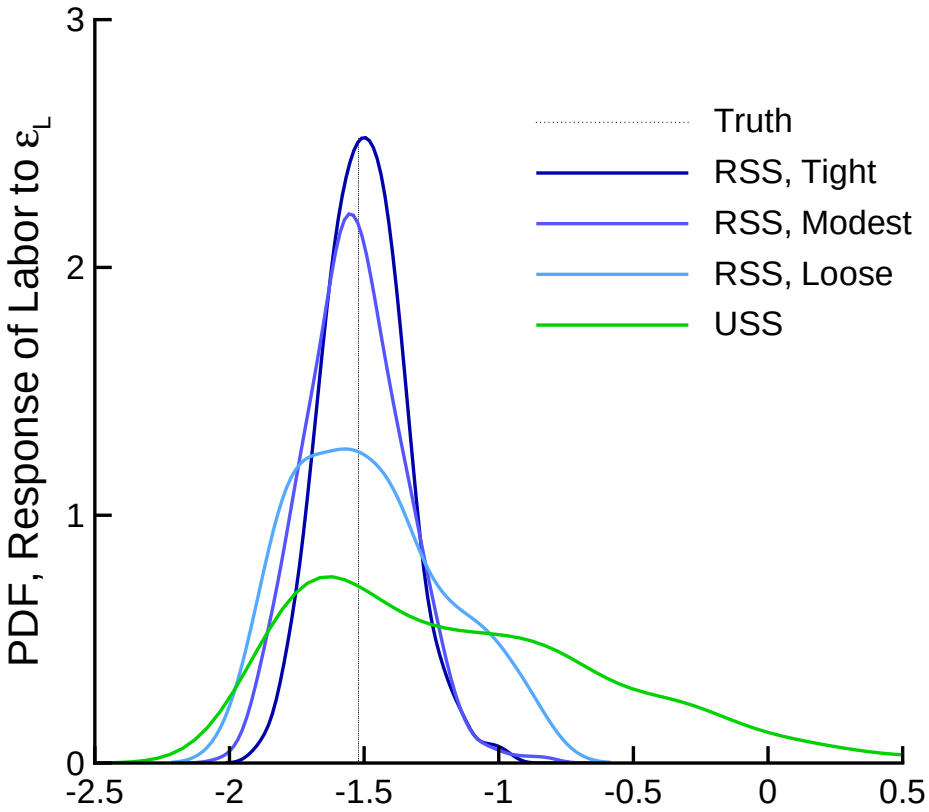
- VARMA and USS completely uninformative
- Whether or not technology is important shock

Variance Decompositions



- VARMA and USS completely uninformative
- Whether or not data unfiltered or filtered

Effects of Policy



- VARMA result not identifiable without more theory
- Need at least modest bounds in SS for policy analysis

Summary

- Imposing little theory, yields little
- Even with DSGEs, may need parameter constraints
- Ultimately essential for
  - Isolating promising classes of models
  - Doing policy analysis