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Is the Stock Market Overvalued?*

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ABSTRACT

The value of U.S. corporate equities in the first half of 2000 was close to 1.8 times U.S. gross national income. Some stock market analysts have argued that the market is overvalued at this level. We use standard economic theory and find that the market is correctly valued. In theory, the market value of equity plus debt liabilities should equal the value of productive assets plus debt assets. Since the net value of debt is currently low, the market value of equity should be approximately equal to the market value of productive assets. We find that the market value of productive assets is roughly 1.8 GNPs and is therefore in line with the market value of equity.

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1. Introduction

In the United States, the value of corporate equities relative to income has nearly doubled since 1994. In the first half of 2000, the value of corporate equities was close to 1.8 times the U.S. gross national income, or equivalently gross national product (GNP).[1] This ratio is high by historical standards. Its previous post-World War II peak was 1.0, which occurred in 1968. Over the 1946-1999 period, the value of all U.S. corporate equity averaged only 0.67 GNPs. [See Figure 1.] Thus, at 1.8 times GNP, the ratio is two and a half times the ratio's average in the postwar period.

Is the stock market value too high? Glassman and Hassett (1999) have argued that it is not. In fact, they say that it is undervalued by a factor of three. But others are concerned that, at 1.8 times GNP, the market is overvalued. In Congressional testimony, Federal Reserve Chairman Alan Greenspan has suggested that the high value of the market may reflect "irrational exuberance" among investors. Shiller (2000) has reiterated this concern and views a 50 percent drop in the value as plausible. General concern about an overvalued market is fueled by the experience of Japan in the 1990s. The value of corporate equity fell in 1990 by 60 percent, and subsequently the economy stagnated.

We use standard economic theory to value U.S. corporate equities and find that a value of 1.8 times GNP is justified. An implication of the theory is that the value of corporate equity should be equal to the value of productive assets in the corporate sector.[2] Our basic method, then, is to estimate the value of corporations' productive assets and compare that value to the value of corporate equities. This is not as easy as it may seem.

Productive assets include *tangible* assets–like factories, office buildings, and machines–and *intangible* assets–like patents, brand names, and firm-specific human capital. And a good measure of the value of these assets must include not only those used by U.S. corporations

in the United States itself, but also those used outside the United States, by their foreign subsidiaries.

Estimates of the value of some of these assets are reported by the U.S. government. The Commerce Department's Bureau of Economic Analysis (BEA) provides estimates of the value of tangible corporate assets located in the United States. In the 1990s, the estimate is slightly above 1.0 GNP. However the BEA does not estimate the value of assets of U.S. corporate foreign subsidiaries or the value of intangible assets in the corporate sector.

To estimate the value of assets of U.S. corporate foreign subsidiaries, we use profits of these subsidiaries divided by an estimate of the return on tangible capital in the United States. Our estimate is close to 0.4 GNPs. To estimate the value of corporate intangible assets, we use data on corporate profits and tangible assets, and an estimate of the return on capital used in the corporate sector. We find that corporate profits are larger than can be justified with tangible assets alone. If we redo the national accounts with intangible assets included, we can derive formulas that allow us to residually determine the value of these assets. The key assumption is that the after-tax returns on tangible and intangible capital are equal. We find that the value of intangible capital is roughly 0.4 GNPs.

A value of corporate intangible assets of 0.4 GNP is large, being nearly one-quarter of corporate equity. We think this estimate is reasonable in light of direct evidence. The value of high-technology companies can only be justified by their intangible capital, particularly human capital.[3] A significant fraction of the value of drug companies must be assigned to the value of patents that they own. And, as Bond and Cummins (2000), point out, brand names such as Coca-Cola account for much of the value of many companies.

Adding together the values of corporate tangible assets located in the United States, assets of foreign subsidiaries, and intangible capital gives us 1.8 GNPs as the value of produc-

tive assets in the corporate sector. As theory predicts, both the value of corporate equities and the value of productive assets were 1.8 GNPs in the first half of 2000.

Although our focus is the value of corporate equities, our theory has predictions for average real returns on debt and equity. Theory predicts that average returns on both debt and equity will be near 4 percent in the future. This assumes that there will be no important policy changes that significantly affect the pricing of financial assets. We see already that interest rates on U.S. Treasury inflation-protected securities with various maturities are consistent with this 4-percent prediction.

2. Theory

Our method of estimating the value of corporate assets involves constructing a standard growth model and quantifying it.[4] The growth model we use is established aggregate economic theory and is fast becoming the textbook model in intermediate and advanced undergraduate macroeconomic courses. In this section, we derive formulas for the value of corporate equity and asset returns. In the next, we use national income and product data to derive estimates for the United States.[5]

Our model economy includes two sectors, a corporate sector and a noncorporate sector. Since our focus is on the value of domestic corporations, output from the corporate sector is the gross domestic product of corporations located in the United States. Output of the noncorporate sector of our model is the remaining product of U.S. GNP. Our noncorporate sector thus includes the household business sector, the government sector, the noncorporate business sector, and the rest-of-world sector.

A. Willingness to Substitute

Our model economy is inhabited by infinitely lived households with preferences ordered by the expected value of

$$\sum_{t=0}^{\infty} \beta^t \left[(c_t \, \ell_t^{\psi})^{1-\sigma} / (1-\sigma) \right] N_t \tag{1}$$

where t indexes time, c_t is per-capita consumption, ℓ_t is the fraction of productive time allocated to nonmarket activities such as leisure, and N_t is the number of household members. The fraction of time allocated by households to market activities is denoted by $n = 1 - \ell$. The size of a household is assumed to grow at the rate of population growth, η. The curvature parameter on consumption, $\sigma \geq 0$, measures how risk averse a household is. The larger this parameter's value, the more risk averse is the household. The parameter $0 < \beta < 1$ measures impatience to consume, with a smaller value implying more impatience. The parameter ψ measures the relative importance of leisure and consumption to the household. The larger ψ is, the more important is leisure.

B. Ability to Transform

The model economy has two intermediate good sectors – a corporate sector, denoted by 1, and a noncorporate sector, denoted by 2. These provide the inputs to produce the economy's final good.

The noncorporate production technology is simple:

$$y_{2,t} \leq (k_{2,t})^{\theta} (z_t n_{2,t})^{1-\theta}. \tag{2}$$

Here y_2 is sector output, k_2 is capital services, n_2 is labor services, z is a stochastic technology parameter, and θ is the capital share parameter, $0 < \theta < 1$.

For our purposes, the corporate sector is the important sector and is more complicated. It has both tangible and intangible assets. U.S. corporations make large investments

in such things as on-the-job training, R&D, organization building, advertising, and firm-specific learning by doing. These investments are large, and the stock of intangible assets has important consequences for the pricing of corporate assets. So we assume that production in the corporate sector requires both tangible assets, which are measured, k_{1m}, and intangible assets which are unmeasured, k_{1u}. In addition to capital, labor services n_1 are required. The aggregate production function for the corporate sector is

$$y_{1,t} \leq (k_{1m,t})^{\phi_{mt}} (k_{1u,t})^{\phi_{ut}} (z_t n_{1,t})^{1-\phi_{mt}-\phi_{ut}} \tag{3}$$

where ϕ_{mt} and ϕ_{ut} are the random capital shares for measured and unmeasured capital, respectively. In order to capture variations in profit shares over the business cycle, we make the nonstandard assumption that capital shares vary. Variations in profit shares affect the equity risk premium, which we want to estimate.

The three per capita capital stocks in this economy depreciate geometrically and evolve according to

$$k_{i,t+1} = [(1-\delta_i)k_{i,t} + x_{i,t}]/(1+\eta) \tag{4}$$

where $i = 1m$, $1u$, or 2; δ_i is the rate of depreciation for capital of type i; and $x_{i,t}$ is gross investment of type i in period t. The right side of the capital accumulation equation (4) is divided by the growth in population $(1+\eta)$ because k_i and x_i are in per capita units.

The model also has a final good sector, which combines the intermediate inputs from the corporate and noncorporate sectors to produce a composite output good that can be used for consumption and investment. This production function is

$$c_t + g_t + x_{1m,t} + x_{1u,t} + x_{2,t} \leq y_t \equiv A\left(\mu y_{1,t}^{\rho} + (1-\mu) y_{2,t}^{\rho}\right)^{1/\rho} \tag{5}$$

where g is government consumption, $0 < \mu < 1$ is a parameter that determines the relative

sizes of the corporate and noncorporate sectors, $\rho \leq 1$ is a parameter that governs the substitutability of corporate and noncorporate goods, and $A > 0$ is a scale parameter.

Government production is assumed to be included in noncorporate production. However, the government plays a special role in the economy: it taxes various activities to finance government purchases and transfers. In particular, the government taxes consumption, labor income, property, and profits. Taxes are proportional in our model economy.

C. Equilibrium

There are two ways to decentralize our model economy and they lead to the same equilibrium outcome. One way is to assume that firms hire workers, make investment decisions, pay taxes directly to the government, and pay dividends to the households. Because of the investment decision, the firms' problem, in this decentralization, is dynamic. The other way to decentralize is to assume that firms rent capital and labor from households. Households make the investment decisions and pay taxes to the government. In this decentralization, the firms' problem is simple and static. The relevant equilibrium outcomes are the same in the two decentralizations because the households effectively own the capital in both cases. Here, we describe an equilibrium for the second type of economy. We find this economy easier to work with because we can consolidate all of the interesting transactions for a particular period into the household's budget constraint.

The household budget constraint in period t is

$$\begin{aligned}(1+\tau_{c,t})c_t + x_{1m,t} + x_{1u,t} + x_{2,t} \\ = r_{1m,t}k_{1m,t} + r_{1u,t}k_{1u,t} + r_{2,t}k_{2,t} + w_t n_t \\ -\tau_{1k,t}k_{1m,t} - \tau_{2k,t}k_{2,t} - \tau_{n,t}w_t n_t\end{aligned}$$

$$-\tau_{1,t}\left[(r_{1m,t}-\delta_{1m,t})k_{1m,t}+r_{1u,t}k_{1u,t}-x_{1u,t}-\tau_{1k,t}k_{1m,t}\right]$$

$$-\tau_{2,t}\left[(r_{2,t}-\delta_2)k_{2,t}-\tau_{2k,t}k_{2,t}\right]+\pi_t. \tag{6}$$

Households rent tangible and intangible capital to corporations at rental rates r_{1m} and r_{1u}, respectively. Households also rent capital to noncorporate firms at a rental rate of r_2. Wage income is wn, where $n = n_1 + n_2$ is total labor services. Taxes are paid on consumption expenditures, wage income, property, and profits. The tax rate on consumption is τ_c; that on wage income is τ_n; tax rates on property in the corporate and noncorporate sectors are τ_{1k} and τ_{2k}; and the rate on corporate profits is τ_1. Note that corporations can subtract depreciation and property taxes when they compute their corporate profits tax. Note also that unmeasured investment, for things like R&D, is untaxed. It, too, is subtracted from income when taxable income is computed. Noncorporate profits are taxed at a rate τ_2. Again, depreciation and property taxes are subtracted when taxable income is computed. Finally, transfers from the government to households are denoted by π.

Now consider equilibrium in this economy. Households maximize their expected utility (1) subject to the sequence of budget constraints (6) and the capital accumulation equations (4). Households take as given initial capital stocks as well as current and future prices and tax rates. Firms in all sectors behave competitively and solve simple, static optimization problems. The intermediate good firms choose capital and labor to maximize profits subject to the constraint on their production, namely, functions (3) or (2). Thus, wages and rental rates in the corporate and noncorporate sectors are equal to their marginal value products. The final good firms choose the intermediate inputs to maximize $y - p_1 y_1 - p_2 y_2$, where p_i is the price of the intermediate goods of sector i. Maximization is done subject to (5). If households and firms choose allocations optimally, then equilibrium prices are set so that

markets for goods, labor, and capital services all clear.

In this economy, the value of corporate equities is equal to the value of the end-of-period stock of capital used in the corporate sector. If we use the price of output as the unit of account, then the value is given by

$$V_t = \left[k_{1m,t+1} + (1-\tau_{1,t})k_{1u,t+1}\right] N_{t+1}. \tag{7}$$

This follows from the facts that the cost, on margin, of a unit of measured capital is 1 and the cost, on margin, of a unit of unmeasured capital is 1 minus the corporate income tax rate. Expenditures on unmeasured investment are expensed and reduce taxable corporate income. [See the budget constraint (6).]

The return on corporate equities is given by

$$r^e_{t,t+1} = [V_{t+1} + d_{t+1}N_{t+1}/V_t] - 1 \tag{8}$$

where $\{d_t\}$ is the stream of payments to the shareholders of the corporation (that is, the households). Payments to shareholders are given by

$$\begin{aligned} d_t &= p_{1,t}y_{1,t} - w_t n_{1,t} - \tau_{1k,t}k_{1m,t} \\ &\quad -\tau_{1,t}\left[(r_{1m,t} - \delta_{1m} - \tau_{1k,t})k_{1m,t} + r_{1u,t}k_{1u,t} - x_{1u,t}\right] - x_{1m,t} - x_{1u,t}. \end{aligned} \tag{9}$$

This represents what the corporation has left over after workers have been paid, taxes on property and profits have been paid, and new investments have been made.

The return on a one-period bond, which we refer to as the *risk-free rate*, is given by

$$r_{f,t} = \left\{\beta E_t\left[(c_{t+1})^{-\sigma}(\ell_{t+1})^{\psi(1-\sigma)}/(c_t)^{-\sigma}(\ell_t)^{\psi(1-\sigma)}\right]\right\}^{-1} - 1, \tag{10}$$

where $c^{-\sigma}\ell^{\psi(1-\sigma)}$ is the marginal utility of consumption. The value, or price, of the bond is simply the inverse of $1 + r_{f,t}$.

3. Findings

We can use the formulas for the asset values and returns just described to assess whether our model is consistent with U.S. observations. It is. To demonstrate that, we first abstract from uncertainty and price corporate equity and risk-free debt using a deterministic version of the model. Without uncertainty, calculations of the relevant quantities are trivial. We then establish that, for all practical purposes, the results are the same in the deterministic and stochastic versions of the model when we introduce uncertainty consistent with the behavior of the U.S. economy.[6]

A. Deterministic version

Again, we work first with the steady state of a deterministic version of the model. We derive an estimate for the return to capital using data from the U.S. noncorporate sector. We then derive an estimate for the size of the intangible capital stock. We choose the level of intangible capital so that the returns to capital in the corporate and noncorporate sectors are equated. With the estimate for intangible capital and data on measured corporate capital and taxes paid in the corporate sector, we can estimate the value of the stock market.

With no uncertainty, the after-tax return to corporate equities and the after-tax return to a bond that pays 1 for sure in the following period are both equal to the after-tax interest rate, which we denote by i and define to be

$$i = [(1+\gamma)^{\sigma}/\beta] - 1 \tag{11}$$

where γ is the growth of the technology parameter z_t. This follows directly from the first-order conditions of the household. In fact, if there is no uncertainty, then the after-tax return to each type of capital is also given by i, and the following is true:

$$i = (1-\tau_1)(r_{1m} - \delta_{1m} - \tau_{1k}) = r_{1u} - \delta_{1u} = (1-\tau_2)(r_2 - \delta_2 - \tau_{2k}). \tag{12}$$

Assuming that the U.S. economy is roughly in a steady state, we can estimate i using data from the BEA's national income and product accounts (NIPA). In Table 1, we report average values for income, product, and capital stocks of the United States during 1990-99. The first column of the table lists the accounting concepts used for NIPA data. The second column lists the average values over the period 1990-99 relative to GNP. We make adjustments to these values as theory requires, in order that the accounts are consistent with our model. The third column describes and quantifies the adjustments, and the fourth column lists the final, adjusted averages. (In Appendix B, we provide details about the calculations made for Table 1.) In Table 2, the adjusted averages are matched up with model counterparts.

Our estimate of the return to capital comes from noncorporate data because we observe the relevant quantities needed to infer $(1-\tau_2)(r_2-\delta_2-\tau_{2k})$. However, before we can construct an estimate of the return to capital in the noncorporate sector, we need to consider several of the adjustments made to the NIPA data. Two sets of adjustments are relevant: those to noncorporate profits and those to capital.

Consider first noncorporate profits. We make two adjustments to this item. One is to reduce the net interest payments of the sector by an estimate of the sector's purchases of intermediate financial services. We estimate that of the 0.042 GNPs of this sector's net interest payments, 0.022 should be treated as intermediate service purchases. So we reduce GNP 2.2 percent, with the reduction on the product side being in consumption of financial services and that on the income side, in imputed net interest income of households. Most of this adjustment is simply the difference in interest paid by people with home mortgages and the interest received by households who lend to the financial institutions that issue the mortgages.

The imputed net interest income that remains is 0.02 of GNP, which we see as a rea-

sonable number. Some of this is forgone interest of people who hold currency and checking accounts that pay less than the short-term interest rate. Some of it is the reduction in insurance premiums that is possible because the insurance company earns interest on premiums for a period prior to making claims. In these cases, the household is receiving services for forgone interest, and there should be an imputation to income and product.

The other adjustment that we make to noncorporate profits is the addition of imputed capital services to government capital and to consumer durables. The U.S. system of accounts uses a 0 percent interest rate when imputing services to government capital. We instead use the average return on capital in the noncorporate sector. So that income equals product, we add imputed services both to profits in the noncorporate sector and to government consumption. In the U.S. system, consumer durables are treated as consumption. We treat them instead as investment and impute services to these durables. These imputed capital services are added to profits in the noncorporate sector and to private consumption.

We must make one addition to the capital stock of the noncorporate sector. Capital stocks reported by the BEA include only capital located in the United States. But our measure of noncorporate profits includes profits of U.S. foreign subsidiaries equal to 0.012 of GNP. To estimate the capital stock used to generate these profits, we divide 0.012 by our estimate of the return on capital i.

We are now ready to compute the after-tax return on capital in the noncorporate sector (which is equal to $(1-\tau_2)(r_2-\delta_2-\tau_{2,k})$ and to i):

$$i = \frac{\text{accounting returns } + \text{ imputed returns}}{\text{noncorporate capital } + \text{ capital of foreign subsidiaries}} \tag{13}$$

$$= \frac{.064 + (.592 + .287)i}{2.153 + .012/i} \tag{14}$$

where 0.064 GNPs is noncorporate profits plus net interest less intermediate financial services, 0.592 GNPs is the net stock of government capital, 0.287 GNPs is the net stock of consumer durables, 2.153 GNPs is the sum of stocks of government capital, consumer durables, and noncorporate business, and 0.012 GNPs is net profits from foreign subsidiaries. We have assumed that τ_2 is 0 because the main categories of noncorporate income – namely services of owner-occupied housing, government capital, and consumer durables – are untaxed. The value of i that satisfies (14) is 4.08%. Therefore, our estimate of the imputed services to capital is 0.036, and our estimate of the capital associated with the net profits of 1.2 percent is 0.294.

So, theory predicts that, on average, the return to capital in the noncorporate sector should be 4.08 percent. This is close to the average values of the risk-free rate on inflation-protected bonds issued by the U.S. Treasury. In the first quarter of 2000, the average return on 5-year inflation-protected bonds was 3.99 percent, and the average return on 30-year inflation-protected bonds was 4.19%.

We turn next to the value of domestic corporations. To compute our estimate, we need the value of measured tangible capital, the corporate income tax rate, and an estimate of the value of unmeasured intangible capital. [See equation (7).]

In Table 1, measured tangible capital as reported in the BEA's *Fixed Reproducible Tangible Wealth* is 0.821 GNPs. However, this measure does not include inventories or land. Inventories are however available in NIPA so we add them. Land is not included in NIPA estimates but it is in the *Flow of Funds Accounts* for nonfinancial corporate business. The difference between real estate values reported by the Fed and nonresidential structures reported by the BEA is 0.06 GNPs. Thus, our estimate of measured capital, with land and inventories included, is 1.042 times GNP.

In Table 1, the corporate profits tax liability is 0.026 GNPs, and corporate profits are 0.073 GNPs. The tax rate is taken to be the average tax and is, therefore, equal to 0.356.

The next step is obtaining an estimate for unmeasured capital in the corporate sector. In the deterministic version of our model, the after-tax returns for the three types of capital must be equal, and this requirement ties down the size of unmeasured corporate capital. Above we computed one of these after-tax returns, namely the return on noncorporate capital. We can use this as our estimate of $r_{1u} - \delta_{1u}$ and our estimate of $(1 - \tau_1)(r_{1m} - \delta_1 - \tau_{1k})$. We can also use the fact that profits in the model economy's corporate sector are equal to NIPA corporate profits plus unmeasured investment, so

$$(r_{1m} - \delta_1 - \tau_{1k})k_{1m} + r_{1u}k_{1u} = \text{NIPA profits} + x_{1u} \tag{15}$$

Replacing $r_{1m} - \delta_1 - \tau_{1k}$ by $i/(1 - \tau_1)$ in (15) and rearranging we have

$$\begin{aligned} i &= (1 - \tau_1)\left[\text{NIPA profits } + x_{1u} - r_{1u}k_{1u}\right]/k_{1m} \\ &= (1 - \tau_1)\left[\text{NIPA profits } + ((1 + \eta)(1 + \gamma) - i)k_{1u}\right]/k_{1m} \end{aligned} \tag{16}$$

where we have used the fact that x_{1u} is proportional to k_{1u} on the steady state growth path. The only unknown in equation (16) is intangible capital. Using U.S. averages from Tables 1 and 2, we have

$$0.0408 = (1 - 0.026/0.073)(0.073 + 0.03k_{1u} - 0.0408k_{1u})/1.042 \tag{17}$$

where 0.026 GNPs is the tax paid on domestic corporate profits, 0.073 is NIPA profits, 0.03 is the growth rate of GNP, and 0.03 k_{1u} is the value of unmeasured net intangible investment in the steady state. The solution to this equation is $k_{1u} = 0.645$. Therefore, unmeasured intangible investment is equal to 0.019 GNPs.

With our estimate for unmeasured capital, we can now compute the model's market value of domestic corporations using the formula (7). Assuming that the time period is not too long, the total value, that is, N times the per capita value, is

$$V = [k_{1m} + (1 - \tau_1)k_{1u}]\, N = 1.457N \tag{18}$$

where $\tau_1 = 0.356$ (which is value of corporate income taxes divided by the value of taxable corporate income).

To compare this estimate to the data's market value of U.S. corporations, we need to add in the value of U.S. foreign subsidiaries. Profits from U.S. foreign subsidiaries averaged 1.56 percent of GNP over the period 1990-99.[7] Using an interest rate of 4.08 percent, we estimate that capital of U.S. foreign subsidiaries has a value of 0.382 GNPs. Let V_{US} be the market value of U.S. corporations. Then,

$$V_{US} = V + .382N \;=\; 1.84\,N \;=\; 1.84\,\text{GNPs}. \tag{19}$$

We write this in terms of GNPs because per capita GNP is normalized to 1, and total GNP is, therefore, N.

According to the Fed's *Flow of Funds* data, the market value of domestic corporations at the end of the first quarter of 2000 was 1.83 times GNP of that quarter. In the second quarter of 2000, the market value was 1.71 times GNP. The average thus far in 2000 is 1.77.

This number is equal to our estimate of the value of corporate capital if corporate debt is taken into account. In 2000, corporate debt was roughly 7 percent of GNP, which implies that the total value of US corporations – debt plus equity – is 1.84 times GNP. Our total value is 1.84 as well.

Thus far, we have assumed that the premium for nondiversifiable risk is small.

B. Stochastic version

Now we work out the implications of a stochastic version of the model. With uncertainty, we expect that risky assets, like corporate equities, would be paid a risk premium. So here, we quantify this premium. We find that, in fact, the premium is very small. Thus, the results of the stochastic version of the model are essentially those of the deterministic version.

Calibration

To determine the implications of the stochastic version of the model, we must first calibrate the model. We do this in three steps. First, we compute a steady state for the model that is consistent with the adjusted accounting measures in Table 1. Second, we choose parameters for the model – including means of stochastic parameters – that are consistent with these steady state values. Third, we choose stochastic processes for shocks that lead to fluctuations in the key variables that are comparable to their U.S. counterparts. The key variables for asset pricing are output, consumption, labor, and after-tax corporate earnings.

Steady State

To compute a steady state for the model we need to make some further adjustments to the NIPA data so that they are consistent with the model concepts. The adjustments that we have discussed so far are the addition of unmeasured investment; the subtraction of intermediate financial services; the imputation of consumer durable and government capital services; and adjustments to the capital stocks. The final adjustments needed are adjustments for sales and excise taxes, for depreciation of consumer durables, and adjustments for foreign subsidiary capital.

NIPA includes sales taxes in the measure of private consumption. In our model, we

treat consumption as pretax. Therefore, we must subtract sales taxes from NIPA private consumption. Consumer durables are treated as private consumption by NIPA and as investment in our model. Therefore, we add the depreciation of consumer durables to noncorporate depreciation and to consumption. Finally, because profits of foreign subsidiaries are included in national income (and therefore in noncorporate profits), we add an estimate of investment and depreciation for foreign subsidiaries. To do this, we use the same rate of depreciation as other noncorporate capital in the United States.

The adjusted values for income, product, and capital stocks are treated as a steady state for the model. These values are reported in Table 2 along with the relevant expressions for the model.

Also in this table are values and expressions for hours worked, growth rates, and tax rates. In the United States, hours worked per person are roughly one-quarter of discretionary time. The growth rates are averages over 1990-99 of total factor productivity and population. With the exception of the labor tax rate, we use NIPA values reported in Table 1 to calculate tax rates. The corporate and noncorporate income tax rates – which we used in earlier calculations – are set equal to 0.356 and 0, respectively. Property taxes and consumption taxes are the two parts of indirect business taxes. Consumption taxes are 0.047 GNPs, and property taxes are 0.032 GNPs. Our tax rate of 0.086 for consumption is found by dividing the total tax of 0.047 by private consumption, which is equal to 0.544. (See Table 2.) Our tax rates on property are found by dividing total property taxes by the capital stocks in the respective sectors. For corporate property, the rate is 0.02/1.042 or 0.019. For noncorporate property the rate is 0.012/2.447 or 0.005.

The labor tax rate is more difficult to estimate since the U.S. income tax is progressive, while taxes in our model economy are proportional. Households in the federal tax bracket of

28 percent or higher pay nearly all of the income tax. However, because of fringe benefits and before-tax contributions to retirement plans, the marginal tax rates of these households are effectively lower than 28 percent. Therefore, we choose the tax rate on labor income to be 25 percent. But our analysis is not sensitive to the exact rate used. The difference between tax revenues and government expenditures is a lump-sum transfer.

Parameters

In Table 3, we derive depreciation rates, capital shares, and parameters for the final good technology and the utility function. Most of these parameters can be pinned down by steady-state values.

There are two exceptions: the elasticity of substitution of corporate and noncorporate goods $1/(1-\rho)$ and the curvature parameter on consumption σ, which measures the degree of risk aversion. For these parameters, we experiment with different values – in such a way as to get reasonable predictions for the variability of consumption relative to GNP and the variability of corporate share to product. Our baseline values are $\sigma = 1.5$ and $\rho = -2$.

Stochastic Shock Processes

The final choices necessary for the stochastic version of the model are the stochastic processes. We assume that the technology parameter z_t is stochastic, with the process given by

$$\log z_{t+1} = \log z_t + \log(1+\gamma) + \varepsilon_{zt+1} \tag{20}$$

where ε_{zt} is an independent and identically distributed normal random variable with a mean of zero. Notice that z_t grows at rate γ, as do other nonstationary variables in this economy. We choose the variance of ε_z so that the standard deviation of U.S. GNP and our model's

output are roughly the same once we log the series and run them through the Hodrick-Prescott filter. The standard deviation of U.S. GNP is 1.74 percent for the postwar period.

In our baseline economy, we assume that the only shocks hitting the economy are technology shocks for two reasons. First, technology shocks in the postwar period are important sources of aggregate fluctuations. Second, correctly identifying the shocks matters little for the size of the equity premium provided the model has been calibrated to the steady-state observations and provided the model's variances and covariances of consumption and corporate profits match their empirical counterparts.

Table 4 summarizes the parameters for the baseline economy. One parameter included in this table that has not yet been discussed is that for the adjustment cost b. Because the cyclical variation of consumption is crucial for asset pricing, we include adjustment costs on all types of capital of the form $\varphi(x/k) = b/2(x/k - \hat{\delta})^2 k$, where $\hat{\delta} = \delta + \gamma + \eta$.[8] We do this to ensure that the relative volatility of consumption and output in the model is approximately equal to the observed relative volatility.

Simulation Results

Given parameter values, we compute an equilibrium for the economy, simulate time series, and compute asset values and returns. Following Jermann (1998), we compute a linear approximation to the decision rules for capital. All other variables, including equity returns, can be determined in a nonlinear way once we have values for the capital stocks and the stochastic shocks.

Shocks Only to Technology

With no other shocks, we find that the ratio of the value of corporate equities to GNP is 1.85, about what we found in the determinstic version of our model; the return on equity

is 4.1; and the return on debt is 4.07. (See Table 4.) The equity risk premium in this case is small, being only 0.03 percent, which is close to the deterministic case with no equity premium.

In this economy with only technology shocks, hours of work are too smooth relative to U.S. data, and corporate earnings are too volatile. We need to get the right variations in hours as well as consumption since both are arguments of marginal utility; movements in marginal utility are what is relevant for asset pricing. We also need to get the right variation and co-variation in corporate earnings since this is relevant for stock returns and the equity premium paid to stocks. Thus, we consider several variations on our baseline economy that should move the model toward greater volatility in hours and less volatility in corporate earnings. The parameters used in these variations are summarized in Table 4.

Shocks Also to Labor Taxes

To get more volatility in hours and leisure, we assume that labor tax rates are stochastic. Assume, for example, that τ_{nt} is an autoregressive process with

$$\tau_{nt+1} = (1 - \rho_n)\bar{\tau}_n + \rho_n \tau_{nt} + \varepsilon_{nt+1} \tag{21}$$

where $\bar{\tau}_n$ is the mean of the process and ε_{nt} is a i.i.d. normal shock with a mean of zero. We set $\bar{\tau}_n$ equal to 0.25. In order to get a high value for the autocorrelation of hours, as is observed in U.S. data, we set ρ_n equal to 0.95 to The variances of ε_{zt} and ε_{nt} are chosen to make the standard deviations of GNP and hours in the model match those in the U.S. data (which are 1.74 percent and 1.52 percent, respectively, for the postwar period). The adjustment cost parameter is set so that the relative volatility of consumption and output is roughly 0.5, as in the data.

In Table 4, we report the results of this experiment. Notice that little has changed from

the economy with only technology shocks. The average ratio of the stock value to GNP is the same, and the equity and debt returns are not much different from the baseline economy's. Note also that the variation in tax rates actually leads to a fall in the premium from 0.03 to 0.01. This happens because the greater variation in hours reduces the correlation between consumption and earnings. But with shocks to technology and labor tax rates, the variation in corporate earnings and the correlation between earnings and consumption are still high relative to that for the U.S. data.

Shocks Also to Corporate Capital Share

So now we try a shock to a variable that has a significant effect on corporate earnings and consumption: the share of corporate profits in income. We assume here, as with the labor tax rate, that this variable follows an autoregressive process, with

$$\phi_{mt+1} = (1 - \rho_\phi)\bar{\phi}_m + \rho_\phi \phi_{mt} + \varepsilon_{\phi t+1} \tag{22}$$

where $\bar{\phi}_m$ is the mean of the process and $\varepsilon_{\phi t}$ is i.i.d. normal with a mean of zero. If we choose ρ_ϕ and the variance of $\varepsilon_{\phi t}$ to replicate the variability in U.S. corporate shares, then the results show little difference from the benchmark economy. In fact, with shocks to both the labor tax rate and the corporate profits share, we find that we are effectively back to the deterministic version of the model with the equity premium equal to zero.

We tried some other experiments to see if we could generate a large risk premium. Introducing random corporate profit tax rates led to counterfactually high variation in corporate earnings. With larger values of σ, we found the volatility of consumption too high and the volatility of hours too low. Different values of ρ, the parameter which affects the substitutability of corporate and noncorporate goods, changed the results little.

Effects of More Rapid Growth

If we increased the growth rate in technology, we got a higher risk-free rate but a similar risk premium. The media have suggested that higher future growth justifies higher equity values. We found that this is not so. There are two consequences of higher growth for the value of the stock market. One is that with more rapid growth, future corporate payouts are larger. If market discount factors remained fixed, then these higher payouts imply higher stock market values. But higher growth also leads to greater discounting of future payouts, which reduces the current value of these future payouts. We find that these two consequences of more rapid growth for the value of corporate equities roughly offset each other. The expectation of more rapid economic growth does not justify higher equity values relative to GNP.

A change that would justify higher corporate value relative to income is an increase in the corporate after-tax earnings share of income. This we see as very unlikely because of the historic stability of this variable, once it is corrected for business cycle variation.

From the exercises of this section, we can summarize our principle findings. First, the equity premium is small. The equity premium is less than 0.1 percent and our best estimate is that it is close to 0. Second, the value of the stock market relative to GNP should be near 1.8 GNPs and risk-free real return should be near 4 percent. These conclusions depend crucially on our assumption that there is unmeasured intangible capital in the corporate sector. But they are robust to our choices of key parameters and shocks.

4. Conclusions

Some stock market analysts have argued that corporate equities are currently overvalued. But such an argument requires a point of reference: overvalued relative to what? In this paper, we use the basic growth model that is standard for macroeconomists as our reference

point. We make full use of the U.S. national income and product accounts in matching up all variables in the model with NIPA data. We show that theory has a clear prediction for the value of corporate equities. It should be equal to the value of productive assets less net debt. We find that it is. We also find that the risk-free return should be near 4 percent, as it currently is. Barring any institutional changes, we predict a small equity premium in the future.

Appendix A. Some Financial Facts

In this appendix, we report some facts about U.S. household asset holdings that guided the selection of the model that we use to determine whether the U.S. stock market is currently overvalued.

We assumed that individuals in our model are not on corners with respect to their asset choices. There is some evidence that most are not. Households hold a lot of both debt and equity. Table A1 reports the balance sheet of U.S. households in 1999 and on average for the 1946-99 period, all relative to gross national product. Their holding of debt is 1.46 GNP. Some of this debt is held for liquidity purposes, but the total holding is significantly above what financial planners typically recommend for emergencies and unforeseen contingencies.

Of the non-liquid assets, approximately 50 percent are currently in retirement accounts. In Table A2, we report holdings in retirements accounts in 1999 – by type of account and by type of asset.[9] These pension fund assets are roughly split between debt and equity. The holdings can cheaply be shifted by pension managers or, in many cases, by individuals themselves.

Survey data find that many people do in fact shift between debt and equity. [See Vissing-Jorgenson 2000.] Figure 2 captures this switching in a graphic manner. The figure is a scatter plot of the fraction of financial assets in equity in two different years for a sample of people. A circle depicts the positions of a person in the sample in 1984 and 1994. The circle for a person with the same equity share in the two years falls on the 45-degree line. The large number of circles that are far from the 45-degree line establishes that many people made large changes in the share of their portfolio in equity.

We assumed that tax rates on dividends and interest were effectively zero. Corporations do pay taxes on capital income. But taxes on dividends and realized capital gains from

the sale of corporate equity are not taxes on corporate capital income. Someone can avoid taxes on dividends and capital gains by managing his portfolio is such a way that gains are unrealized capital gains. Dividends paid to pension funds, which now own half of corporate equity, are not subject to the personal income tax. Similarly pension funds' realized capital gains from the sale of corporate equity are not taxed. There are also tax-managed mututal funds, introduced in the mid-1999s, which are used to minimize taxes and financial fees while allowing people to hold well-diversified portfolios.[10]

Appendix B. NIPA Data and the Model

In this appendix, we describe in detail the adjustments that we made to the U.S. BEA's NIPA data (as reported in the *Survey of Current Business* (SCB)) before we compared these data to our model's estimates.

The Data

On the left side of Table 1, we report average values for income, product, and capital of the United States during 1990-99. The first column of the table lists the accounting concepts of the NIPA. In the second column, we report average values relative to GNP. Thus, GNP is normalized to 1. Notice also that the sum of value added for the corporate and noncorporate sectors is equal to GNP.

Corporate income is domestic income of corporations with operations in the United States (see Table 1.15, SCB). *Noncorporate income* is the difference between gross national income (Table 1.14, SCB) and corporate income. Thus, noncorporate income includes income of households, the government, noncorporate business, and foreign subsidiaries. For compensation in the noncorporate sector, we include total employee compensation and 80 percent of proprietors' income. Profits of the noncorporate sector include profits of foreign subsidiaries, rental income, and 20 percent of proprietors' income.

Total product is the sum of private consumption, public consumption, and investment in the three types of capital, namely measured corporate, unmeasured corporate, and noncorporate (Table 1.1, SCB). We include net exports in noncorporate investment since production in the rest of the world is included in our model's notion of noncorporate production.

Adjustments

On the right side of Table 1, we provide descriptions and values of the adjustments that we made to NIPA data in order to make them consistent with the theory. We now

describe each adjustment in detail.

NIPA includes sales taxes in its measure of private consumption. On the income side, these taxes are included in indirect business taxes. In our model, we treat consumption as pre-tax, and therefore, subtract sales taxes from both sides of the accounts. We estimate that of the 0.079 GNPs of total indirect business taxes, 0.047 GNPs is sales or excise taxes, which we model as taxes on private consumption. The remainder is attributed to property taxes in the corporate and noncorporate sectors.

NIPA does not include a measure of intangible investment because this type of investment is expensed. We estimate it to be 0.019 GNPs. We include an estimate of intangible investment in our notion of GNP because it raises both after-tax corporate profits and investment.

We make an adjustment to net interest – in both the corporate and noncorporate sectors. We subtract the part of financial services purchased by businesses that we estimate consists of intermediate financial goods. NIPA treats net interest of financial intermediaries as purchases of services by the lender, typically, the household. The United Nations system of accounts treats it, instead, as purchases of services by the borrower. Thus, in the U.N. system, no entry for imputed interest is made, so imputed interest and consumption services are lower. Here, we compute lenders' (borrowers') purchases of financial services as the product of the short-term interest rate less interest received and the amount loaned (borrowed).

We assume that all of the NIPA net interest in the corporate sector, totaling 0.015 GNPs, is intermediate services and we subtract it. We assume that only part of the net interest in the noncorporate sector, equal to 0.022 GNPs, is intermediate. The remainder of noncorporate net interest is included in profits. Most of the 0.022 GNPs adjustment is for services implicitly purchased by homeowners with mortgages. On the product side, we lower

consumption services by 0.037 GNPs.

Consumer durables are treated as private consumption in NIPA and as investment in our model. Therefore, we include depreciation of consumer durables. On the income side, this depreciation appears in noncorporate capital consumption. On the product side, it is added to consumption services. This is the procedure used for housing services which are included in NIPA.

Because profits of foreign subsidiaries are included in NIPA (and therefore in noncorporate profits), we add an estimate of the capital of these subsidiaries to noncorporate capital. To make the depreciation and investment of the noncorporate sector comparable to the capital stock, we include depreciation and net investment for the foreign subsidiaries. In making these estimates, we are assuming that depreciation rates and growth rates are the same at home and abroad.

To noncorporate profits, we add imputed capital services to government capital and to consumer durables. NIPA uses a zero percent interest rate when imputing services to government capital. We instead use the average return on capital in the noncorporate sector. So that income equals product, we add imputed services to profits in the noncorporate sector and to government consumption. In NIPA, consumer durables are treated as consumption. We instead treat them as investment and impute services to these durables. These imputed capital services are added to profits in the noncorporate sector and to private consumption.

We make several adjustments to the capital stocks reported in the SCB *Fixed Reproducible Tangible Wealth*. Measured capital is 0.821 GNPs. This measure does not include the value of inventories or land. A measure of inventories is, however, available in NIPA so we add them. A measure of land is not included in NIPA but it is in the Federal Reserve's *Flow of Funds Accounts* for nonfinancial corporate business. The difference between real estate

values reported by the Fed and nonresidential structures reported in NIPA is 0.06 GNPs. Thus, our estimate of measured capital, with land and inventories included, is 1.043 GNPs.

We make one adjustment to the capital stock of the noncorporate sector. Capital stocks reported by the BEA include only capital located in the United States. However, some national income is produced by capital abroad, namely corporate profits of foreign subsidiaries. We estimate that the capital used abroad is equal to the profits divided by the return on capital.

The Steady State of the Model

We treat the adjusted values for income, product, and capital as steady-state values for the model. In Table 2, notice that the values for income, product, and capital stocks are the adjusted values of Table 1. In the last column of Table 2, we show the relevant expression for the model.

We also include in Table 2 values for hours worked, growth rates, and tax rates. In the United States, hours per person are roughly one-quarter of discretionary time. The growth rates are averages for 1990-99 for total factor productivity and population. With the exception of the labor tax rate, we use NIPA values reported in Table 1 to calculate tax rates. We have data for corporate profits before- and after-tax. We assume that the noncorporate tax rate is zero because the main categories of noncorporate income are untaxed. Property taxes and consumption taxes are indirect business taxes. Consumption taxes are 0.047 GNPs, and property taxes are the remaining 0.032 GNPs.

The labor tax rate is more difficult to estimate since the U.S. income tax is progressive, while taxes in our model economy are proportional. Households in the federal tax bracket of 28 percent or higher pay nearly all of the income tax. However, because of fringe benefits and before-tax contributions to retirement plans, these households' marginal tax rates are

effectively lower than 28 percent. Therefore, we choose the tax rate on labor income to be 25 percent. But our analysis is not sensitive to the exact rate used.

In Table 3, we back out depreciation rates, capital shares, and parameters for the model's final good technology and the utility function. Most of these parameters can be tied down by steady state values. There are two exceptions: the elasticity of substitution of corporate and noncorporate goods $1/(1-\rho)$ and the curvature parameter on consumption σ, which measures the degree of risk aversion. For these we experiment with different values and compare second moments of the model and data.

Notes

[1]The two main data sources used in this article are the U.S. Department of Commerce's *National Income and Product Accounts* and the Board of Governors of the Federal Reserve System *Flow of Funds Accounts of the United States.*

[2]Actually, the market value of equity plus the market value of debt liabilities should equal the market value of debt assets plus the value of productive assets. Since net indebtedness of corporations is currently small, we simplify the discussion and ignore corporate debt holdings and liabilities when modeling the U.S. economy.

[3]In fact, Hall (2000) argues that 'e-capital,' which is human capital created by combining skilled labor and computers, is an important factor for the recent rise in equity prices.

[4]In Appendix A, we provide evidence on U.S. household asset holdings to justify some of the assumptions of our model.

[5]Much work in the asset pricing literature abstracts from production and stops short of matching variables in th theory with national income and product data. Notable exceptions include Cochrane (1991) and Mehra (1998).

[6]Readers familiar with the literature on the equity premium puzzle launched by Mehra and Prescott (1985) should not be surprised by this finding. See Kocherlakota (1996) for a nice survey on the literature. See also Jagannathan, McGrattan, and Scherbina (2000) for estimates of the current equity premium.

[7]Above, we used net profits, which subtracts factor payments sent abroad. This is the relevant figure for computing GNP. To calculate the value of U.S. domestic corporations, we want to use gross profits from U.S. foreign subsidiaries.

[8]With adjustment costs, we need to modify our formula for the equity value as follows: $V = [k_{1m}/(1 - \varphi'(x_{1m}/k_{1m})) + (1 - \tau_1)k_{1u}/(1 - \varphi'(x_{1u}/k_{1u}))]N$.

[9]We consolidate pension fund reserves and life insurance reserves.

[10]See Miller (1977) for an insightful discussion of taxes, and how they can be avoided.

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