The CAPM Debate

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1. Introduction

The capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965), and Black (1972) is a theoretical description of how asset portfolios are chosen. An assumption of the model is that investors are concerned only with risk and return – they are willing to incur additional risk if they expect higher returns. In fact, the main prediction of the CAPM is a linear relationship between an asset's expected return and its "beta" which is a measure of risk. Beta is a measure of an asset's contribution to the total variation in the return on a diversified portfolio; stocks with high beta values are riskier than stocks with low beta values, and stocks with high beta values are expected to earn higher returns than stocks with low beta values. According to the CAPM, if the portfolio consists of an investment in every asset in proportion to its value, then beta suffices to explain the cross-sectional variability in asset returns. Whether or not this prediction is empirically plausible has been the focus of an intense debate. In this paper, we give an overview of the debate.[1]

A natural starting point for our discussion is the empirical work of Black, Jensen, and Scholes (BJS, 1972), who provide evidence in support of the CAPM. They consider two versions of the CAPM – the original model of Sharpe (1964) and Lintner (1965) which assumes that risk-free borrowing and lending rates are the same and the model of Black (1972) which relaxes this assumption. Black, Jensen, and Scholes (1972) find that the data are consistent with the prediction of a linear relationship between average returns and beta. However, their results indicate that the relationship is flatter than predicted by the Sharpe-Lintner CAPM; low-beta stocks do better than the model predicts and high-beta stocks do worse. BJS (1972) provide two possible explanations for their results. One explanation is that the data fit Black's (1972) model better than the traditional CAPM of Sharpe (1964) and Lintner (1965) because borrowing restrictions are nonnegligible. Another explanation is mismeasurement of the true market portfolio. Most applications of the CAPM neglect a significant fraction of wealth, and this error leads to biased estimates of the relationship between risk and return. But whatever is the reason for the flatter-than-expected relationship, the results of BJS

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[1] The literature on the CAPM is very large. Therefore, we do not attempt a comprehensive survey, and, as a result, we cannot discuss all of the important contributions to the literature.

(1972) indicate that the slope of the line relating expected return and beta is positive – which is consistent with the theory.

There were a number of papers after Black, Jensen, and Scholes (1972) that found other variables besides beta which help explain the cross-sectional variability in expected returns. For example, Banz (1981) shows that size (defined as the price of a stock times the number of shares outstanding) also helps to explain cross-sectional variability in returns. This is counter to the theory which says that only beta should matter. Fama and French (1992) try to make the case against beta even stronger. They argue that the cross-sectional relationship between betas and average returns is completely flat. In other words, they reject the notion that beta is a relevant predictor for expected returns. They find that two other variables, size and the ratio of a firm's book value of common equity to its market value, seem to describe the cross-section of average stock returns. Thus, Fama and French (1992) reject the main assumption of the CAPM and, hence, the theory itself.

The study of Fama and French (1992) has received a lot of attention because the CAPM has been widely used by both academics and businessmen during the last three decades.[2] For some proponents of the CAPM, the announcement that "beta is dead" is premature. Black (1993), for example, attributes the Fama-French results to a carefully chosen sample period and notes that, even for the sample period that they use, one cannot reject the hypothesis that the slope of the line relating return and beta is positive. This view is supported by Kothari, Shanken, and Sloan (1995) and Amihud, Christensen, and Mendelson (1992). However, despite the critiques of the Fama-French tests, some of their critics still conclude that variables such as size or book-to-market equity are good predictors of expected returns.

Some recent attempts to resurrect the CAPM have abandoned the assumption that the return on broad stock market indexes like the New York Stock Exchange Composite is a reasonable proxy for the return on the true market portfolio. For example, Jagannathan and Wang (1993,1995) include human capital in their measure of wealth. With human capital included, Jagannathan and Wang show that the CAPM is able to explain 28 percent of the cross-sectional variation in average returns in the 100 portfolios studied by Fama and French. Since only 1.4 percent of the cross-sectional variation can be explained by a traditional market portfolio that includes only stocks on the New York and American Stock Exchanges, the addition of human capital makes a significant difference. When Jagannathan and Wang (1993) also allow for time-varying betas, they show that the CAPM is able to explain 57 percent of the cross-sectional variation in average returns and that variables such as size have little ability to explain what is left unexplained.

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[2] "According to Alfred Rappaport, chairman of the financial advisory firm Alcar Group and an adjunct professor at Northwestern University's Kellogg Graduate School of Management, between one half and two thirds of large U.S. companies use CAPM in some form to find the hurdle rate for capital budgeting" (Tucker, et.al. (1994), page 220). "Beta and other forms of risk measurement decide how hundreds of billions of dollars are invested by pension funds and other institutional investors" (Dreman (1992), page 148).

On the basis of the studies that we survey in this article, we conclude with two observations. First, in our view, the CAPM debate is not yet over. Second, given the wide use of the model, the outcome of the debate is of great importance to both academics who teach the CAPM and to portfolio and investment managers who use it.

The paper is organized as follows. In Section 2, we document some historical facts about asset returns. In Section 3, we develop the CAPM and describe how to measure risk according to this theory. In Section 4, we describe some of the empirical findings and interpret them. In Section 5, we summarize our views on where the debate stands.

2. Historical Average Returns on Stocks, Bonds, and Bills

CAPM has been widely used to analyze asset prices and returns. An important implication of the model is its prediction for the cross-sectional variation in expected returns. Therefore, we start by summarizing some of the facts for historical data on asset returns. We focus on historical average returns since average returns over long time horizons are good estimates of expected returns. We consider three categories of assets: stocks, bonds, and bills.

Table 1 provides a summary of the average return history for portfolios of stocks, bonds, and treasury bills taken from Ibbotson Associates (1991). According to the description given in Ibbotson Associates (1991), the common stock returns are based on the Standard and Poor's (S&P) Composite Index. Currently the S&P Composite includes 500 stocks but it consisted of only 90 stocks prior to March 1957. For the period 1926-1981, the small stock index consists of stocks in the smallest quintile of firms in terms of their market value of equity (i.e., their share price times shares outstanding) listed in the New York Stock Exchange (NYSE); the portfolio composition is rebalanced once every five years. Starting in 1982, the small stock index fund corresponds to the Dimensional Advisors Small Company Fund. For the period 1926-1976, the total return on long-term government bonds are from the Center for Research in Security Prices (CRSP) at the University of Chicago Graduate School of Business. Each year one bond portfolio is constructed with a maturity of about 20 years. For the period 1977-1991, data from the Wall Street Journal is used to compute the total returns on bond funds. For treasury bill index returns, data from the CRSP U.S. Government Bond file are used through 1976. Data reported in The Wall Street Journal is used for the period thereafter.

For each sample period of Table 1, we report average monthly returns (multiplied by 12 to give an annual rate) in the first of four rows. If investors had rational expectations, then the average returns over a fairly long horizon would be a reasonable measure of expected returns. Notice that the historical returns on different types of securities vary substantially. The fact that they were content to hold such securities implies that they would demand vastly different rates of return for investing in different securities or projects. To the extent that these securities are claims to cash flows from a variety of real activities, these facts support the view that the cost of capital is not the same across different projects. During the 66 year period from 1926 to 1991, the S&P portfolio earned an average annualized return of 11.94% whereas the Treasury Bill portfolio earned only 3.64%. Since the average annualized inflation rate was 3.11%, the average real return on the treasury bill portfolio was hardly different from zero. The S&P portfolio, therefore, earned a hefty risk premium of 8.3%. The performance of the small stock portfolio was even more impressive; small stocks earned an average annualized return of 16.05%. To appreciate what these differences in average returns over such a long period of time implies, consider the value of a portfolio that had $1 invested in each one of these funds in 1926, which is listed in the third row for each sample period of Table 1. $1 invested in the S&P stocks would be worth $675.59 in 1991, whereas $1 invested in the treasury bill fund would only be worth $11.01, which is not much considering the fact that a basket of goods costing $1 in 1926 would cost $7.67 in 1991. For another perspective, consider what could have been purchased in 1991 if $10 had been invested in 1926. $10 invested in the small stock fund in 1926 would be worth an impressive $18,476 in 1991, which is enough to cover one year of tuition in most prestigious universities in the United States whereas $10 invested in the treasury bill fund would be worth only $110 in 1991 or dinner for two in a nice restaurant. The fourth row for each sample period of Table 1 reports the implied annual returns that would result in the values of row three. To calculate these returns we first calculate the value of the fund if $1 were invested in the first period. We then raise the value to 1 over the number of years for the subsample.

Notice from the second row of Table 1 that the portfolios with higher average returns over 1926 – 1991 also had more variable returns, i.e. the higher average returns were probably compensation for some perceived higher risk. For example, the standard deviation for small stocks, which yielded the highest return in this period, was 31.02% whereas the standard deviations for the S&P 500 index, treasury bonds, and treasury bills were 20.22%, 7.62%, and 0.94%, respectively. Similarly, in the 1926 – 1975 sample, small stocks had the highest return and the highest standard deviation.

From Table 1, we see that 1981 to 1991 was an unusual period. Long-term government bonds did extremely well. A dollar invested in the long-term government bond fund at the end of 1980 would have grown to $4.33 at the end of 1991, which implies a large compounded annual rate of return (14.24%). The risk premium on the S&P Composite fund was 7.75% which is not much different from the average annualized risk premium for the entire sample period. However, the average annualized return on treasury bills of 7.98% was substantially more than the average inflation rate of 4.25%. Clearly the time series of realized returns on these four portfolios exhibit substantial differences in both the sample mean as well as the sample volatility.

In Figure 1, we display the time series of the logarithm of the values for the four funds over the sample 1926 – 1991. For example, the values for December

1991 are logarithms of the numbers of row three in Table 1. We plotted the logarithms of the values because the levels are vastly different. The plot is intended to further illustrate the large differences in returns across the four funds that we are studying.

We take the stand that the large cross-sectional variation in the realized returns are not entirely accidental. If investors had reasonable expectations in 1926, they would have anticipated that something like this would be the outcome 66 years later, but still they were content to invest in a portfolio consisting of all of these different funds. To make a theoretical prediction of the choice of portfolio and the cross-sectional variability in returns, we now turn to a derivation of the CAPM.

3. The Capital Asset Pricing Model (CAPM)

To derive the capital asset pricing model, we start with the simple problem of choosing a portfolio for an arbitrarily chosen investor. To set up the problem, we need a few definitions. Let R_0 be the return (i.e., one plus the rate of return) on the risk-free asset (e.g., asset 0). By investing \$1, the investor will get \$$R_0$ for sure. In addition, assume that there are n risky assets. The risky assets have returns that are not known with certainty at the time the investments are made. Let α_i be the fraction of the investor's initial wealth that is allocated to asset i. R_i is the return on asset i. R_m is the return on the entire portfolio (e.g, $\sum_{i=0}^{n} \alpha_i R_i$). R_i is a random variable with expected value ER_i and variance $\text{var}(R_i)$. The covariance between the return of asset i and the return of asset j is represented by $\text{cov}(R_i, R_j)$.

Suppose that the investor's expected utility can be represented as a function of the expected return on his portfolio and its variance. In order to simplify notation without losing generality, we will assume that an investor can choose to allocate his wealth to three assets, i.e., $i = 0$, 1, or 2. In this case the problem is to choose fractions α_0, α_1, and α_2 that maximize

$$V(ER_m, \text{var}R_m) \tag{1}$$

subject to

$$\alpha_0 + \alpha_1 + \alpha_2 = 1, \tag{2}$$

$$ER_m = \alpha_0 R_0 + \alpha_1 ER_1 + \alpha_2 ER_2, \tag{3}$$

$$\text{var}(R_m) = \alpha_1^2 \text{var}(R_1) + \alpha_2^2 \text{var}(R_2) + 2\alpha_1\alpha_2 \text{cov}(R_1, R_2). \tag{4}$$

The objective function V is (i) increasing in expected return, $\partial V/\partial ER_m > 0$, ($ii$) decreasing in the variance of the return $\partial V/\partial \text{var}R_m < 0$, and ($iii$) concave. These properties imply that there is a tradeoff between expected returns and the variance of returns. The constraint in (2) ensures that the fractions sum to 1. Eqs. (3) and (4) follow from the definition of the rate of return on the wealth portfolio of the investor, R_m. Substituting $1 - \alpha_1 - \alpha_2$

for α_0 in Eq. (1) and taking the derivative of V with respect to α_1 and α_2 yields the following conditions that must hold at an optimum:

$$(ER_1 - R_0)V_1 + 2\big(\alpha_1 \mathrm{var}(R_1) + \alpha_2 \mathrm{cov}(R_1, R_2)\big)V_2 = 0, \qquad (5)$$

$$(ER_2 - R_0)V_1 + 2\big(\alpha_2 \mathrm{var}(R_2) + \alpha_1 \mathrm{cov}(R_1, R_2)\big)V_2 = 0, \qquad (6)$$

where V_j is the partial derivative of V with respect to its jth argument, $j = 1, 2$. Now consider multiplying Eq. (5) by α_1 and Eq. (6) by α_2 and adding them together:

$$\begin{aligned} &\{\alpha_1(ER_1 - R_0) + \alpha_2(ER_2 - R_0)\}V_1 \\ &\quad + 2\{\alpha_1\big(\alpha_1 \mathrm{var}(R_1) + \alpha_2 \mathrm{cov}(R_1, R_2)\big) \\ &\quad + \alpha_2\big(\alpha_2 \mathrm{var}(R_2) + \alpha_1 \mathrm{cov}(R_1, R_2)\big)\}V_2 = 0, \end{aligned} \qquad (7)$$

or equivalently,

$$(ER_m - R_0)V_1 + 2\mathrm{var}(R_m)V_2 = 0. \qquad (8)$$

We used the definitions of ER_m and $\mathrm{var}(R_m)$ to write Eq. (7) more succinctly as in (8). The equations in (5), (6), and (8), can all be written as explicit functions of the ratio V_2/V_1 and then the first two expressions (from (5) and (6)) can be equated to the third (from (8)). This yields the following two relationships:

$$ER_i - R_0 = \frac{\mathrm{cov}(R_i, R_m)}{\mathrm{var}(R_m)}(ER_m - R_0), \quad i = 1, 2. \qquad (9)$$

In fact, even for the more general case where n is not necessarily equal to 2, equation (9) holds. Setting $\beta_i = \mathrm{cov}(R_i, R_m)/\mathrm{var}(R_m)$, we have:

$$ER_i = R_0 + (ER_m - R_0)\beta_i, \qquad (10)$$

for all $i = 1, \ldots n$.

A portfolio is on the mean-variance frontier if there is no other choice of weights α_0, α_i, $i = 1, \ldots n$ that yields a higher expected return for the same variance. One can show that the optimally chosen portfolio for the problem in Eqs. (1) – (4) has this property. In fact, one can also show that equation (10) will continue to hold if the return R_m is replaced by the return on any mean-variance efficient portfolio other than the risk-free asset.

Note that the return R_m in (10) is the return for one individual's wealth portfolio. But Eq. (10) holds for every mean-variance efficient portfolio and V need not be the same for all investors. A property of mean-variance efficient portfolios is that portfolios of mean-variance efficient portfolios will also be mean-variance efficient. If we define the *market portfolio* to be a weighted sum of individual portfolios where the weights are determined by the fractions of total wealth held by individuals, then it too is mean-variance efficient. Therefore, an equation of the form given by Eq. (10) also holds for the market portfolio. In fact, equation (10) with R_m equal to the return on the market portfolio is the key relation for the CAPM. It implies that the ratios of reward measured as the expected return in excess of the riskless rate (i.e., $ER_i - R_0$) to risk (i.e., β_i) are the same across assets i. This is consistent with the notion that investors trade off returns and risk.

In specifying the problem of a typical investor (in (1) – (4)), we assumed that a riskless asset was available. If we drop this assumption and set $\alpha_0 = 0$ from the start, then we obtain a slightly different relationship between risk and return than is given in (10). In particular, Black (1993) shows that in the absence of a riskless asset expected returns on the risky assets satisfy the following relationship:

$$ER_i = ER_z + (ER_m - ER_z)\beta_i, \tag{11}$$

where R_z is the return on a zero-beta portfolio (i.e., cov(R_z, R_m)=0), R_m is the return on the market portfolio, and $\beta_i = \text{cov}(R_i, R_m)/\text{var}(R_m)$.

We now provide an interpretation of beta (β_i) in (10) or (11) as a measure of risk. Consider a portfolio p of securities that earns return R_p and has standard deviation $S_p = (\text{var}R_p)^{\frac{1}{2}}$. Let the standard deviation of an arbitrary security i be S_i and the covariance between security i's return and that of the portfolio be $C_{i,p}$. Now consider a new portfolio with x_i invested in asset i, $-x_i$ invested in the risk-free asset, and x_p invested in the original portfolio. That is, consider modifying the portfolio of an investor who currently holds x_p in portfolio p by borrowing $\$x_i$ and investing it in security i. The standard deviation of the new portfolio is then

$$S = \sqrt{x_i^2 S_i^2 + x_p^2 S_p^2 + 2x_i x_p C_{i,p}}. \tag{12}$$

Note that the derivative of S with respect to x_i is

$$\frac{dS}{dx_i} = (x_i S_i^2 + x_p C_{i,p})/S. \tag{13}$$

This derivative measures the change in the standard deviation (or risk) of the whole portfolio with a small change in the amount invested in security i. If we evaluate this derivative at $x_i = 0$ and $x_p = 1$, then we find

$$\left.\frac{dS}{dx_i}\right|_{x_i=0, x_p=1} = \frac{C_{i,p}}{S_p} = \frac{C_{i,p}}{S_p^2} \cdot S_p = \beta_i S_p. \tag{14}$$

Notice that dS/dx_i is equal to $\beta_i S_p$. That is, at the margin an additional dollar invested in security i (by borrowing the dollar) increases the standard deviation of the portfolio by $\beta_i S_p$ and not by S_i.[3] Since S_p does not depend

[3] Note that we could have started our derivations of equation (10) by using the fact that investors trade off expected returns (i.e., means) and risk (i.e., variances) when making portfolio decisions. Suppose that investors are indifferent between securities that yield the same return-risk ratios, i.e., $(dR/dx_i)/(dS/dx_i)|_{x_i=0,x_p=1}$ is constant for all i, where $R = x_i R_i + x_p R_p - x_i R_0$. Then, it follows that $(ER_i - R_0)/\beta_i$ is a constant and, hence, equation (10) holds.

on the particular asset, i, β_i measures the relevant risk up to a scale factor.

To see this more clearly, consider the following example. Suppose the investor is holding \$1000 in the fund that includes all stocks listed in the NYSE and the American Stock Exchange (AMEX) where the investment proportions are the same as the relative market capitalization of the stocks of the firms. Suppose that all dividends are reinvested in the fund. Consider borrowing \$1 and investing in one of the following securities: AT&T, Bristol Myers and Squibb, Coca Cola, ConEdison, Dayton Hudson, Digital Equipment, Exxon, Ford, IBM, McDonald's, or McGraw-Hill. In Table 2, we report the means and the standard deviations of the monthly percentage rates of returns for these securities, their betas, and the change in the portfolio's standard deviation with a \$1 change in the holdings of any of the securities. If it is the case that dS/dx_i is equal to $\beta_i S_p$, then we should observe that the numbers in the column marked $\partial S/\partial x_i$ are a scale multiple of the betas for the eleven securities. Figure 2 plots the incremental standard deviation, $\partial S/\partial x_i$ against beta for each security i. Notice that the points lie on a straight line through the origin, i.e., beta of a security does measure the incremental risk. Figure 3 plots $\partial S/\partial x_i$ against S_i. Notice that this relationship has no particular pattern.

4. Empirical tests of the CAPM

In theory, empirical tests of the CAPM are tests of equation (10) or (11). If expected returns and betas are known, then all we have to do is plot expected returns against betas. Unfortunately neither is known. We have to form estimates of them. We do this by assuming that sample analogues correspond to population values (if such values exist) plus some random noise. The noise is typically very large for individual securities but is less for portfolios. To understand why noise creates problems, notice that two portfolios with measured betas that are very different could well have the same population betas if the measurement error is very large. The objective is to have sufficient dispersion in asset betas and to measure this dispersion sufficiently precisely.

Black, Jensen and Scholes (1972) (or BJS) came up with a clever strategy that enables creation of portfolios that have very different betas. They estimate betas based on past history (by regressing historical returns on a proxy for the market portfolio), sort securities based on historical betas, group them into portfolios with increasing historical betas, hold them for a given number of years, and change the portfolio composition periodically. So long as historical betas contain information about population betas, this procedure will create portfolios with sufficient dispersion in cross-sectional betas.

Because estimates of the expected return and beta are used, the relation being tested is

$$r_p = \gamma_0 + \gamma_1 b_p + \epsilon_p, \tag{15}$$

where r_p is an estimate of the expected excess return on portfolio p (i.e., the return on the portfolio less the return on a risk-free asset), b_p is an estimate of beta for portfolio p, and ϵ_p is a random disturbance term in the regression equation. BJS (1972) use time series data on returns to construct a sample average for r_p (e.g., $r_p = \sum_{t=1}^{T} r_{p,t}/T$, where $r_{p,t}$ is the excess return at date t). However, there are problems with the standard errors on γ_0 and γ_1 obtained by a least squares regression of average excess returns on estimated betas. Therefore, BJS suggested computing the standard errors of the parameters in the cross-sectional regression in the following way. Run a cross-sectional regression for each period for which data on returns is available. This procedure generates a time series of parameter estimates. Use the standard deviation of the estimated time series of parameters as the standard error of the parameter in the cross-sectional regression.[4]

Estimates and standard errors are obtained for the coefficients γ_0 and γ_1 of (15). For the Sharpe-Lintner version of the CAPM, γ_0 should be equal to zero and γ_1 should be equal to the risk premium for the market portfolio. For the Black version of CAPM, γ_0 is not necessarily equal to zero. If we take a parameter estimate and divide by its standard error, we can construct a t-statistic. If the absolute value of the t-statistic is large (e.g., greater than 2), then the coefficient is said to be statistically different than zero. In most cases, empirical tests of the CAPM are based on the t-statistics of the coefficients in the regression equation (15).

In some tests to be described below, additional terms are added to the equation in (15), e.g.,

$$r_p = \gamma_0 + \gamma_1 b_p + \gamma_2' \psi_p + \epsilon_p. \tag{16}$$

The vector ψ_p in (16) corresponds to additional factors assumed to be relevant for asset pricing. In tests of the CAPM, researchers are interested to know if $\gamma_2 = 0$ holds.

Black, Jensen, and Scholes (1972) in support of CAPM

BJS use all of the stocks in the NYSE during the period 1931-65 to form 10 portfolios with different historical beta estimates. They regress average monthly excess returns on beta. Figure 4 which is reproduced from BJS gives the fitted relation between average excess monthly return (where the risk-free asset is the 30-day treasury bill) and beta. The average monthly excess return on the market proxy used in the study was 1.42%. The estimated slope is 1.08% instead of 1.42% as predicted by the CAPM. The estimated intercept is 0.519% instead of zero as predicted by the CAPM. The t-statistics that BJS report indicate that the slope and intercept are significantly different from their theoretical values.

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[4] See Shanken (1992) and Jagannathan and Wang (1995) for a discussion on computing the sampling errors associated with the estimates of the coefficients in the cross-sectional regressions. See Gibbons (1982), Shanken (1985), Stambaugh (1982), Mackinlay and Richardson (1991), and Jagannathan and Wang (1993) for a description of better alternatives for econometric evaluation of the CAPM that rely on either the method of maximum likelihood or the generalized method of moments.

As Black (1972,1993) points out, there are two plausible explanations for these results. First, there is measurement and model specification error that arises due to the use of a proxy instead of the market portfolio. This error causes the estimated slope to be biased towards zero and the estimated intercept to be biased away from zero.[5] Second, if no risk-free asset exists, the CAPM does not predict an intercept of zero. In fact, BJS conclude that the data are consistent with Black's (1972) version of the model (i.e., Eq. (11)).

Suppose we use data over a longer horizon. In Figure 5, we plot the average returns of the four funds described in Section 2 for the period 1926:1 through 1991:12 against their estimated betas. The estimates of beta for alternative subperiods are reported in Table 3. We also fit a straight line to the data by running a linear regression. Points on this line are marked 'o' whereas the actual returns are marked 'x'. Notice that the relation between average return and beta is very close to linear and portfolios with high (low) betas have high (low) average returns. This positive relationship is consistent with the CAPM prediction.

Banz (1981) and the size effect

In 1981, Banz published his influential paper in which he showed that during the 1936 – 1975 period, the average return to stocks of small firms (i.e., firms with low values of market equity) was substantially higher than that of large firms after adjusting for risk using the CAPM. The observation has since been referred to as the "size effect."

Banz (1981) followed a procedure similar to the portfolio-grouping procedure of Black, Jensen, and Scholes (1972). The securities were first assigned to one of five subgroups based on their historical betas. Stocks in each of the subgroups were then assigned to five further subgroups based on the market value of the firms' equities. Thus, there were a total of twenty-five portfolios. Portfolios were updated at the end of each year. He used firms on the NYSE and estimated the cross-sectional relation between return, beta, and relative size, i.e., equation (16) with ψ_p equal to the relative size of the pth portfolio. The coefficients were interpreted as follows: γ_0 is the rate of return for a portfolio with beta equal zero, and γ_1 and γ_2 are risk premia for beta and size risks, respectively.

Banz (1981) reports estimates for $\gamma_0 - R_0$ and $\gamma_1 - (R_m - R_0)$ where R_0 and $R_m - R_0$ are the intercept and slope predicted by the CAPM. The idea is to report deviations from theory. Theory predicts $\gamma_0 = R_0$, $\gamma_1 = R_m - R_0$, and $\gamma_2 = 0$. If the deviations are statistically significant (i.e., if the t-statistics are large in absolute value), then he would conclude that the CAPM is misspecified. For the entire period, 1936 – 1975, Banz (1981) obtains the following estimates: $\hat{\gamma}_0 - R_0 = 0.0045$ (2.76); $\hat{\gamma}_1 - (R_m - R_0) =$ -0.00092 (-1.0); and $\hat{\gamma}_2$=-0.00052 (-2.92) where R_m is a measure of the market

[5] Suppose the relation being estimated is $y_t = x_t\gamma + u_t$. If we observe $X_t = x_t + v_t$ rather than x_t where v_t is measurement error that is uncorrelated with x_t, then the least squares estimate for γ will be biased towards zero. The larger is the variance of v_t, the greater is the bias.

return and the numbers in parentheses are t-statistics. Because the t-statistic for γ_2 is large in absolute value, Banz (1981) concludes that the size effect is large and statistically significant. The fact that the estimate for γ_2 is negative implies that shares of firms with large market values have had smaller returns on average than shares of small firms. From these results, it appears that relative size is able to explain a larger fraction of the cross-sectional variation in average return than beta.

To assess the importance of these results, Banz (1981) conducted the following additional test. Construct two portfolios each having 20 securities. The first portfolio contains only small firms whereas the second contains only large firms. The portfolios are chosen in such a way that they both have the same beta. He found that the small firm portfolio earned on average 1.48% per month more than the large firm portfolio during the time period 1931 – 1975; and the differences in returns were statistically significant.

Fama and French (1992) and the death of beta

Fama and French (1992) estimated the relation in Eq. (16) for the period July 1963 to December 1990 with ψ_p equal to size. They grouped stocks for firms listed in the NYSE, the AMEX, and NASDAQ into 10 size classes and then into 10 beta classes, i.e., 100 portfolios. They obtain an estimate of γ_1 equal to -0.37 with a t-statistic of -1.21 and and an estimate of γ_2 equal to -0.17 with a t-statistic of -3.41. Furthermore, even when they include only beta in the regression equation (i.e., equation (15)), they do not find a significantly positive slope; their estimate for γ_1 is -0.15 with a standard error of 0.46. On the other hand, the size effect is significant with or without betas included. Thus, their estimates indicate that, for a large collection of stocks, beta has no ability to explain the cross sectional variation in average returns whereas size has substantial explanatory power.

Fama and French (1992) also considered the ability of a number of other attributes to account for the cross-sectional dispersion in average returns. When they include the ratio of the book value of common equity to its market value as an explanatory variable in addition to size, they find that the book to market value of the firm is able to account for a substantial portion of the cross-sectional variation in average returns. In fact, book to market equity appears to be even better than size.

What is so surprising about the Fama-French (1992) results is that they use the same methodology as Fama and MacBeth (1973) but reach a very different conclusion; Fama and MacBeth (1973) find a positive relation between return and risk and Fama and French (1992) find that there is no relation between return and risk. Fama and French (1992) attribute the different conclusions to the different sample periods used in the two studies. Fama and MacBeth (1973) use stock returns traded on the NYSE during the period 1926 – 1968. If Fama and French (1992) rerun their regressions for 1941 – 1965, they find a positive relationship between average return and beta. Therefore, if beta died, it died recently.

We can also see differences across sample periods when we use our data.

Suppose we repeat the exercise of Figure 5 for several subperiods. In Figure 6, we plot the average returns of the four funds described in Section 2 for the period 1926:1 through 1975:12 against their estimated betas. A straight line is fit to the data by running a linear regression. Points on this line are marked 'o' whereas the actual returns are marked 'x'. Notice that Figure 6 is very similar to Figure 5, which includes the entire sample period. In both, we see a positive and linear relation between beta and average return. For the subperiods 1976 – 1980 and 1981 – 1991, however, we do not see a perfectly linear relation between beta and average return. Consider first the plot in Figure 7 for the period 1976 – 1980. Small stocks gave a very large return of 35.6% while the S&P portfolio yielded only 14.2%. Another difference from the earlier period was the return on treasury bills which did much better in average return than treasury bonds. Consider next the plot in Figure 8 for the period 1981 – 1991. Notice that the small firm effect disappeared in this period. The S&P Composite index yielded an average return of 15.7% with the return on small stocks being only 13.3%. The fact that the two portfolios have approximately the same beta value is counter to the prediction of the CAPM. Thus, although we find empirical support for the CAPM over a long horizon (namely 1926 – 1991 or 1926 – 1975), there are periods in which we do not find a linear relation between betas and average returns.

The negative evidence for the CAPM can be summarized as follows. First, for some sample periods, the relation between beta and average return is completely flat. Second, other explanatory variables such as market equity (size) and book-to-market equity seem to do better than beta in explaining cross-sectional variation in average asset returns.

Some challenges to the Fama-French results

There have been a number of challenges to the Fama and French (1992) study. Most have challenged one of the following claims: (a) that betas have no role for explaining cross-sectional variation in returns; (b) that size is an important factor for returns; or (c) that book-to-market equity is an important factor for returns.

Kothari, Shanken, and Sloan (1995) reexamine the Fama and French (1992) analysis and argue that the findings depend critically on how one interprets the statistical tests. They focus on the Fama-French estimates for the coefficient on beta (i.e., γ_1 in Eq. (15)) which have high standard errors and therefore imply that a wide range of economically plausible risk premia cannot be rejected statistically. For example, if the estimate of γ_1 is 0.24% per month with a standard error of 0.23%, then 0 basis points per month is as statistically plausible as 50 basis points per month (i.e., a risk premium of 6% per year).[6] This view is supported by Amihud, Christensen, and Mendelson (1992) and Black (1993). Amihud, Christensen, and Mendelson (1992) find that when a more efficient statistical method is used, the estimated relation

[6] These figures are obtained from Fama and French's (1992) regressions of individual NYSE stocks on beta for 1941-1990.

between average return and beta is positive and significant.

Black (1993) has noted that the size effect could simply be a sample period effect; in other words, the size effect is observed in some periods and not others. To make his point, Black (1993) points to some findings of Fama and French (1992). Fama and French (1992) find that their estimate of γ_2 in Eq. (16) is not significantly different from 0 for the 1981 – 1990 period. In other words, size does not appear to have any power in explaining cross sectional variation in average returns for the period following publication of Banz's paper. This is also evident from the statistics of Table 1. Note that in the 1981 – 1991 sample, the return on small stocks was 13.27% whereas the return on the S&P Composite stocks was 15.71%. One troubling aspect of the Fama-French result is that their point estimate for the coefficient on beta (γ_1) for the 1981 – 1990 sample is statistically significant but negative rather than positive as the CAPM predicts risk premia to be. This is evidence against the CAPM although it is also evidence in favor of the view that the size effect may be spurious and period specific.

But even if there is a size effect, there is still a question about its importance given the relative number of small firms used in these studies. Jagannathan and Wang (1993) report the average market value of firms in each of 100 groups. Firms in the largest 40% of the groups account for more than 90% of the market value of all stocks on the NYSE and AMEX. Thus, for a large enough collection of assets, the CAPM may still be empirically valid.

Another variable that Fama and French (1992) find to be important for explaining cross-sectional variation in returns was book-to-market equity. However, Kothari, Shanken, and Sloan (1995) point to a problem with the data (from COMPUSTAT) used by Fama and French (1992).[7] The problem is the treatment of stocks that are added late in the COMPUSTAT data and then "back-filled." Firms that had high book-to-market equity early in the sample were less likely to survive and less likely to be included in COMPUSTAT. Those that do survive and are added later show high returns. Thus, there is a potential bias. Breen and Korajczyk (1993) follow up on this conjecture by using a COMPUSTAT sample that has the same set of firms for all years. No back-filled data is used. They find that the effect of book-to-market equity is much weaker than that reported by Fama and French (1992).[8]

Jagannathan and Wang (1993) and a possible resurrection

Most empirical studies of the CAPM assume that that the return on broad stock market indexes like the New York Stock Exchange Composite is a reasonable proxy for the return on the true market portfolio of all assets in the

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[7] Chari, Jagannathan and Ofer (1986) also pointed out this bias in the COMPUSTAT data.

[8] COMPUSTAT claims that they rarely add more than two years of back data when they add a firm to their list. In view of this, in their follow-up article, Fama and French (1993) omit the first two years of data but still find that average returns are strongly related to the book-to-market in the cross section. Hence, the effect of book-to-market equity is still in question.

economy. In the U.S., only one-third of non-government tangible assets are owned by the corporate sector and only one-third of corporate assets are financed by equity. Furthermore, intangible assets, like human capital, are another component of wealth not captured by stock market indexes.

Some recent attempts to resurrect the CAPM have abandoned the assumption that the return on stocks alone is a reasonable proxy for the return on the true market portfolio. For example, Jagannathan and Wang (1993) follow the suggestion of Mayers (1973) and include human capital in their measure of wealth. Since human capital is not directly observable, Jagannathan and Wang (1993) use the growth of labor income as a proxy. Let R_{vw} be the return to the value-weighted portfolio of all stocks traded on the NYSE and AMEX and let R_{labor} be the growth rate in per-capita labor income. Then Jagannathan and Wang's (1993) version of the CAPM is given by

$$ER_p = \alpha_0 + \alpha_1 \beta_p^{vw} + \alpha_2 \beta_p^{labor}, \tag{17}$$

where ER_i is the expected return on portfolio p, β_p^{vw} is the risk of portfolio p relative to the value-weighted portfolio of all stocks traded on NYSE and AMEX (i.e., $\text{cov}(R_p, R_{vw})/\text{var}(R_{vw})$), and β_p^{labor} is the risk of portfolio p relative to wealth due to human capital (i.e., $\text{cov}(R_p, R_{labor})/\ \text{var}(R_{labor})$). With human capital included, Jagannathan and Wang show that the CAPM is able to explain 28 percent of the cross-sectional variation in average returns in the 100 portfolios studied by Fama and French. Since only 1.4 percent of the cross-sectional variation can be explained by a traditional market portfolio that includes only stocks on the New York and American Stock Exchanges, the addition of human capital makes a significant difference.

Several studies[9] have pointed out that betas of assets vary over the business cycle in a systematic way. When Jagannathan and Wang (1993) also allow for time-varying betas, they show that the CAPM is able to explain 57 percent of the cross-sectional variation in average returns. They show that not all time variations in beta matter – only that part that comoves with the expected risk premium on the market portfolio is relevant for explaining the cross section of expected returns on financial markets. Since the market risk premium is highly correlated with other macroeconomic aggregates that vary over the business cycle, only the part of the time variation in the security's beta that can be predicted using variables that help forecast the business cycle matters for explaining expected returns. Let R_{prem} be the yield on low grade bonds minus the yield on high grad bonds. Jagannathan and Wang assume that R_{prem} is a good indicator of the business cycle and, hence, of cyclical movements in beta. They show that the CAPM with time-varying betas and human capital implies the following three-beta model for unconditional expected returns:

$$ER_p = \alpha_0 + \alpha_1 \beta_p^{vw} + \alpha_2 \beta_p^{labor} + \alpha_3 \beta_p^{prem}, \tag{18}$$

[9] See, for example, Harvey (1989), Ferson and Harvey (1991,1993), and Ferson and Korajczyk (1993).

where β_p^{prem} is equal to $\text{cov}(R_{p,t}, R_{prem,t-1})/\text{var}(R_{p,t})$. In (18) β_p^{prem} provides a measure of the instability of the beta of portfolio p. Other things being equal, assets with larger beta-instability should earn a higher expected return. When Jagannathan and Wang include size in (18), they find that size has little ability to explain what is left unexplained by the three-beta model.

In Figures 9 – 11, we reproduce Figures 2, 4, and 5 of Jagannathan and Wang (1993) which illustrate the performance of the various versions of the model. Figure 9 is a plot of the fitted expected return using estimated parameters against the realized average returns. In this case, only β_p^{vw} is included in the regression equation. If the predictions of the model are consistent with the data, the points should lie on the 45-degree line. With only stock betas included, the fitted expected returns are all about the same despite the variation in realized average returns. Figure 10 shows the fitted expected returns using estimated parameters in (18) against the realized average returns. In this case, the cluster of points comes close to lining up on the 45-degree line. In other words, the performance of the model improves significantly with β_p^{labor} and β_p^{prem} included. Finally, Figure 11 shows the fitted expected returns for the model that includes size. The fact that there is little difference between Figures 10 and 11 is consistent with the prediction that beta alone is the relevant variable for explaining average returns.

Thus, Jagannathan and Wang (1993) directly respond to the challenge of Fama and French who note that "resuscitation of the Sharpe-Lintner-Black model requires that a better proxy for the market portfolio (a) overturns our evidence that the simple relation between beta and average stock returns is flat and (b) leaves beta as the only variable relevant for explaining average returns." The version of the CAPM that Jagannathan and Wang use has three betas and hence does not meet condition (b) laid down by Fama and French. Therefore, one could argue that their results should not be viewed as a resurrection of the CAPM but constitutes the development of a new asset-pricing model. However, Jagannathan and Wang note that (*i*) when the use of a better proxy (i.e., including human capital) for the market portfolio results in a two-beta model instead of a single-beta model and (*ii*) when the CAPM holds in a conditional sense (i.e., period by period with betas and expected returns varying over time), unconditional expected returns will be linear in market beta as well as a measure of beta instability over business cycles. One could therefore argue that the CAPM really implies that more than "beta" is needed to explain the cross-section of expected returns on financial assets.

5. Conclusions

The goal of this paper was to provide an overview of one of the most intensely debated subjects in Financial Economics – the debate over the capital asset pricing model. The debate has been intense because there is a lot at stake. The CAPM is a widely used model in both the classrooms and the board rooms. But its empirical validity is now, more than ever, in question.

From the evidence, we would conclude that the debate is not yet over.

Although Fama and French (1992) make a persuasive case against the theory, there have been a number of recent studies that challenge their results. There have also been variations of the Sharpe-Lintner-Black CAPM not considered by Fama and French (1992) that are consistent with the data. Whether or not these alternative models will themselves withstand further scrutiny is yet to be determined.

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Table 1.

Monthly Returns on S&P Composite Index Stocks, Small Firm Stocks, Treasury Bonds, and Treasury Bills and Monthly Inflation Rates.

Period	S&P 500	Small	T-bond	T-bill	Inflation	
26/1-91/12	11.94%	16.05%	4.94%	3.64%	3.11%	Avg. monthly return $\times$ 12
	20.22%	31.02%	7.62%	0.94%	2.01%	Std. dev. $\times\sqrt{12}$
	675.59	1847.63	20.95	11.01	7.67	Value of $1 at beginning
	10.38%	12.07%	4.72%	3.70%	3.14%	(Value of \$1)$^{1/66}$-1
26/1-75/12	10.89%	14.71%	3.14%	2.30%	2.29%	Avg. monthly return $\times$ 12
	21.46%	33.59%	5.38%	0.61%	2.15%	Std. dev. $\times\sqrt{12}$
	73.86	109.34	4.29	3.14	3.08	Value of $1 at beginning
	8.99%	9.84%	2.95%	2.32%	2.22%	(Value of \$1)$^{1/50}$-1
76/1-80/12	14.17%	35.55%	2.27%	7.51%	8.85%	Avg. monthly return $\times$ 12
	14.48%	25.87%	11.16%	0.82%	1.14%	Std. dev. $\times\sqrt{12}$
	1.92	4.89	1.09	1.45	1.55	Value of $1 at beginning
	13.95%	37.35%	1.68%	7.77%	9.21%	(Value of \$1)$^{1/5}$-1
81/1-91/12	15.71%	13.27%	14.14%	7.98%	4.25%	Avg. monthly return $\times$ 12
	16.31%	18.25%	12.41%	0.75%	0.97%	Std. dev. $\times\sqrt{12}$
	4.81	3.53	4.33	2.40	1.59	Value of $1 at beginning
	15.34%	12.14%	14.24%	8.28%	4.33%	(Value of \$1)$^{1/11}$-1

Table 2.

Summary statistics

The second and the third columns give the mean and the standard deviation of the monthly percentage rates of return, and the betas of selected securities for the period January 31, 1972 through December 31, 1991. The third column gives the beta computed with respect to the value-weighted index of all stocks traded on the NYSE and AMEX. The last column gives the incremental standard deviation of the monthly rate of return to a portfolio with a beginning of month value of \$1000, invested in the value-weighted index of stocks in the NYSE and AMEX when an additional \$1 is borrowed and invested in the stock of the firm given in the first column.

Name/CUSIP (i)	Mean	Std. dev. (S_i)	Beta (β_i)	$\partial S/\partial x_i$
AT&T/03017710	1.19	5.36	0.552	2.63
Bristol/11012210	1.56	7.08	0.986	4.70
Coca Cola/19121610	1.40	6.75	0.917	4.37
ConEdison/20911110	1.61	7.38	0.566	2.70
Dayton/23975310	1.53	9.69	1.191	5.68
Digital/25384910	1.13	10.25	1.278	6.09
Exxon/30229010	1.47	5.26	0.729	3.47
Ford/34537010	1.15	8.32	0.968	4.61
IBM/45920010	0.61	6.03	0.769	3.66
McDonald's/58013510	1.37	8.15	1.129	5.38
McGraw/58064510	1.41	8.15	1.075	5.12

Table 3.

Estimates of Betas for S&P Composite Index Stocks, Small Firm Stocks, Treasury Bonds, and Treasury Bills.

Period	S&P 500	Small	T-bond	T-bill
26/1-91/12	1.03	1.39	0.07	0.00
26/1-75/12	1.03	1.44	0.03	0.00
76/1-80/12	0.94	1.46	0.22	0.00
81/1-91/12	1.01	0.99	0.31	-0.01

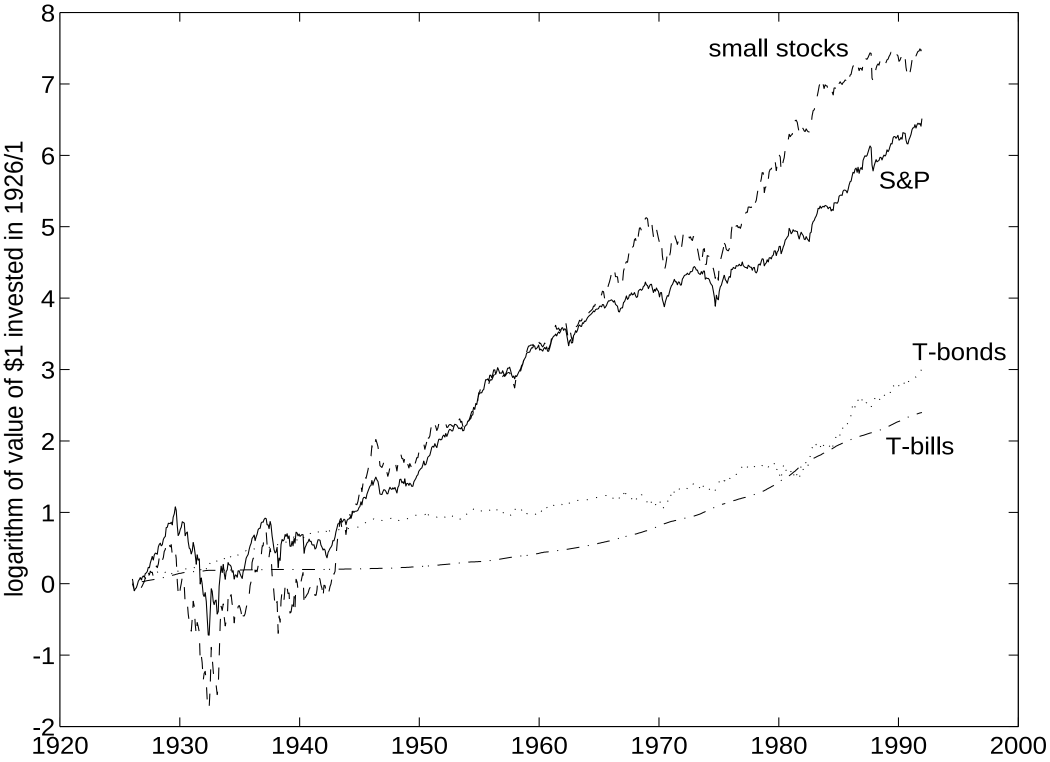

Figure 1. Logarithm of the value of \$1 invested in 1926/1 in four funds: small stocks, S&P composite, treasury bonds, and treasury bills.

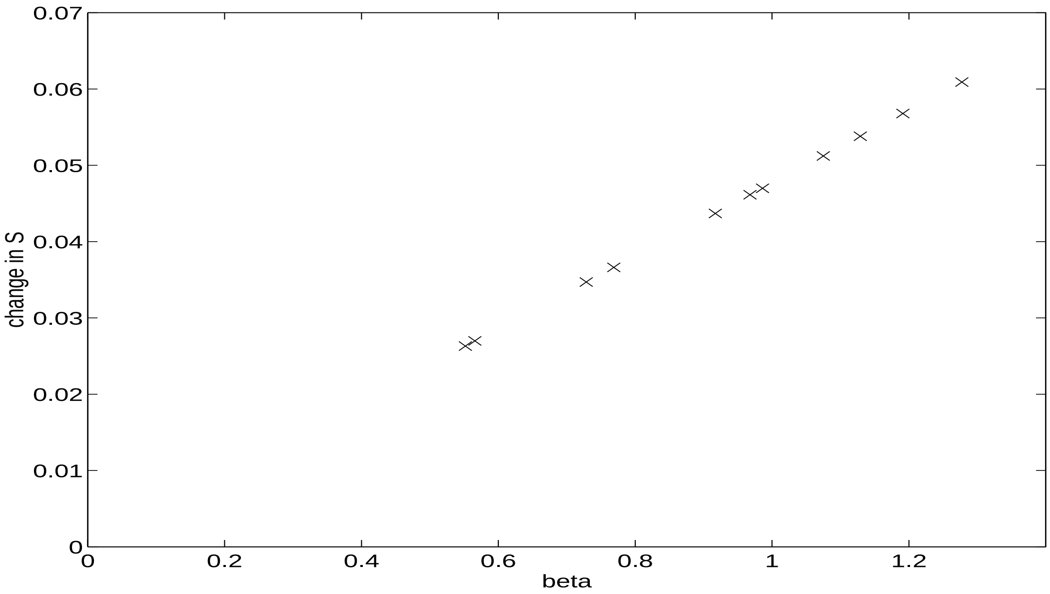

Figure 2. Estimated change in the standard deviation of the portfolio with a \$1 increase ($\partial S/\partial x_i$) versus beta (β_i) for the 11 securities listed in Table 2.

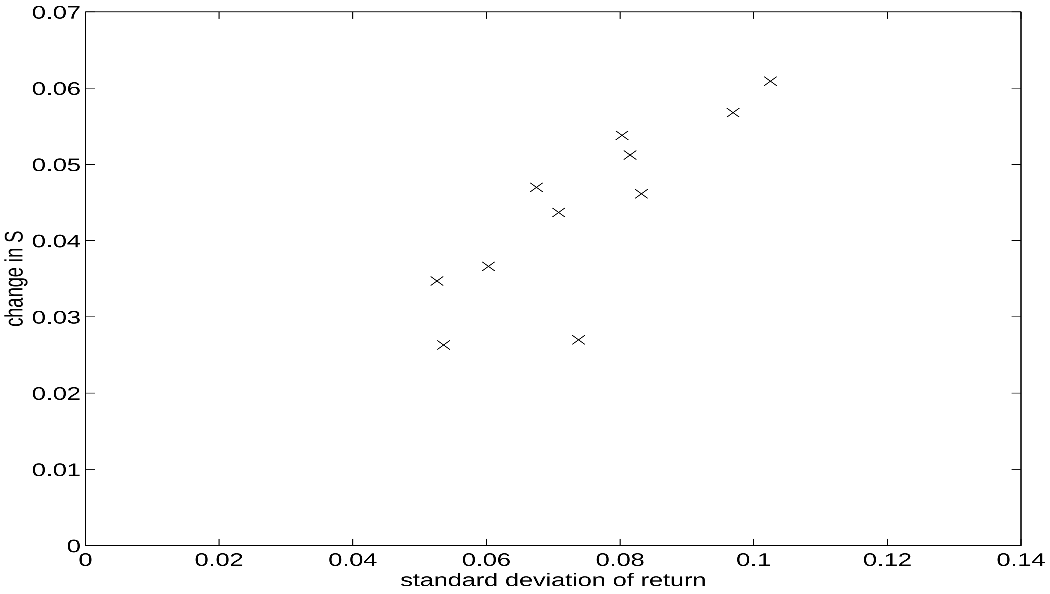

Figure 3. Estimated change in the standard deviation of the portfolio with a \$1 increase ($\partial S/\partial x_i$) versus the standard deviation of the security (S_i) for the 11 securities listed in Table 2.

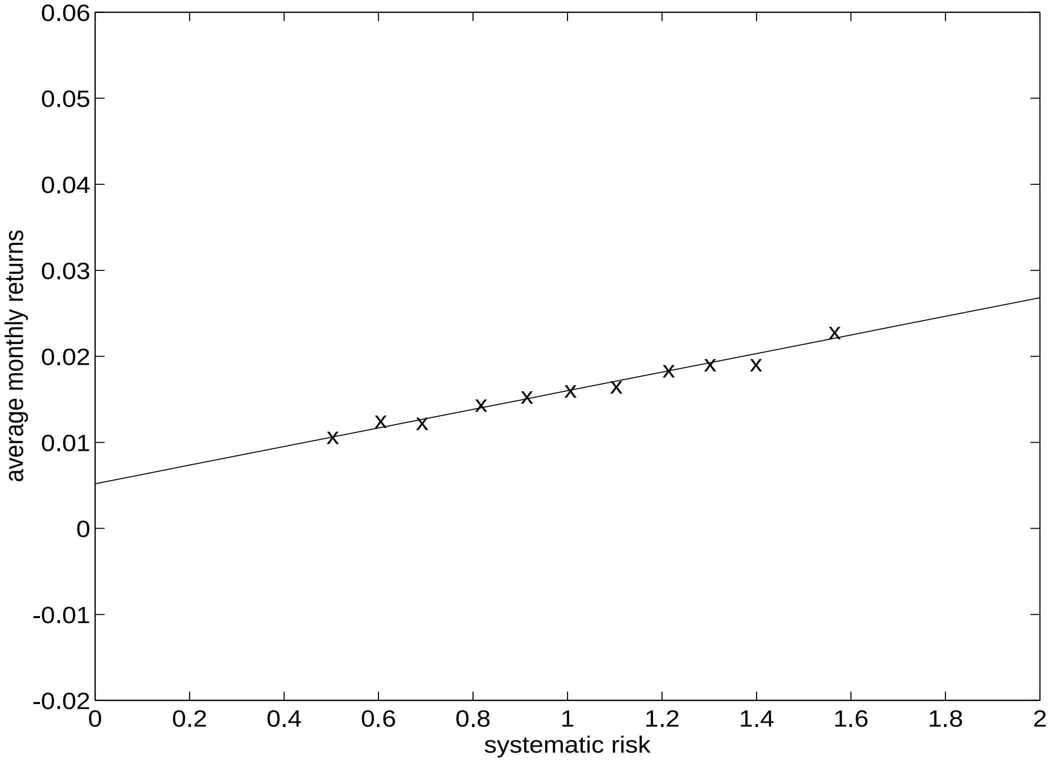

Figure 4. Average monthly rates of return versus systematic risk (beta) for the 35-year period 1931-1965 for the ten portfolios and the market portfolio. *Source*: Black, Jensen, and Scholes (1972).

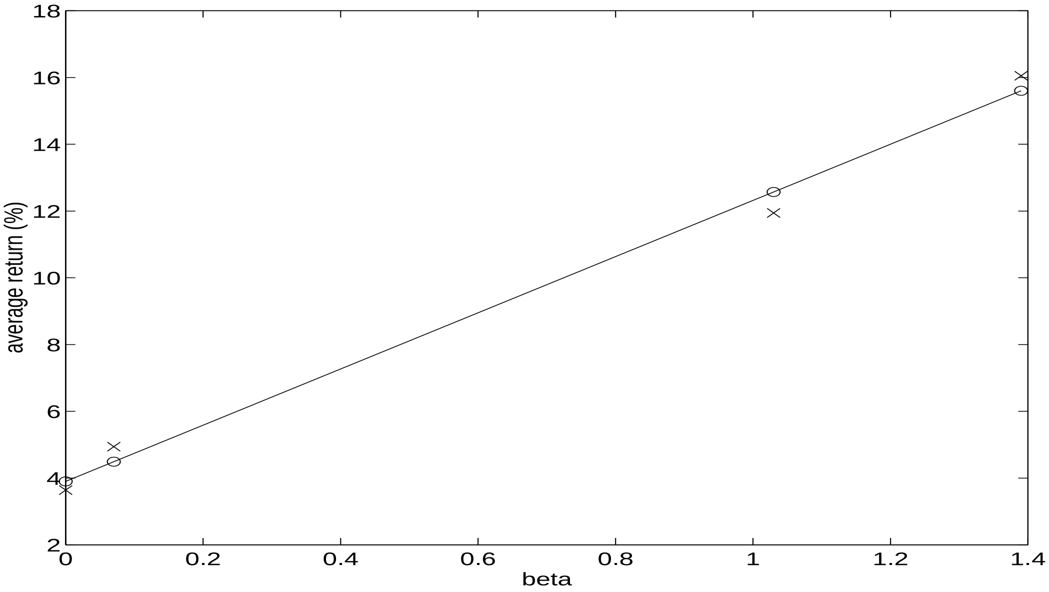

Figure 5. Average rate of return versus beta for 1926/1-1991/12 for small stocks, S&P composite stocks, treasury bonds, and treasury bills. ('x' indicates actual returns and 'o' indicates predicted returns.)

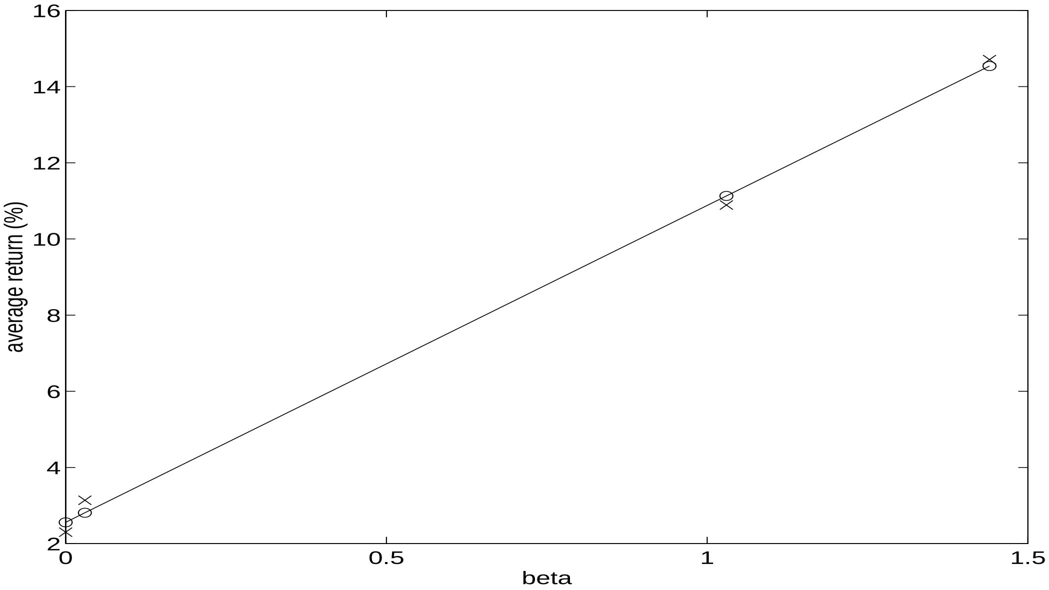

Figure 6. Average rate of return versus beta for 1926/1-1975/12 for small stocks, S&P composite stocks, treasury bonds, and treasury bills. ('x' indicates actual returns and 'o' indicates predicted returns.)

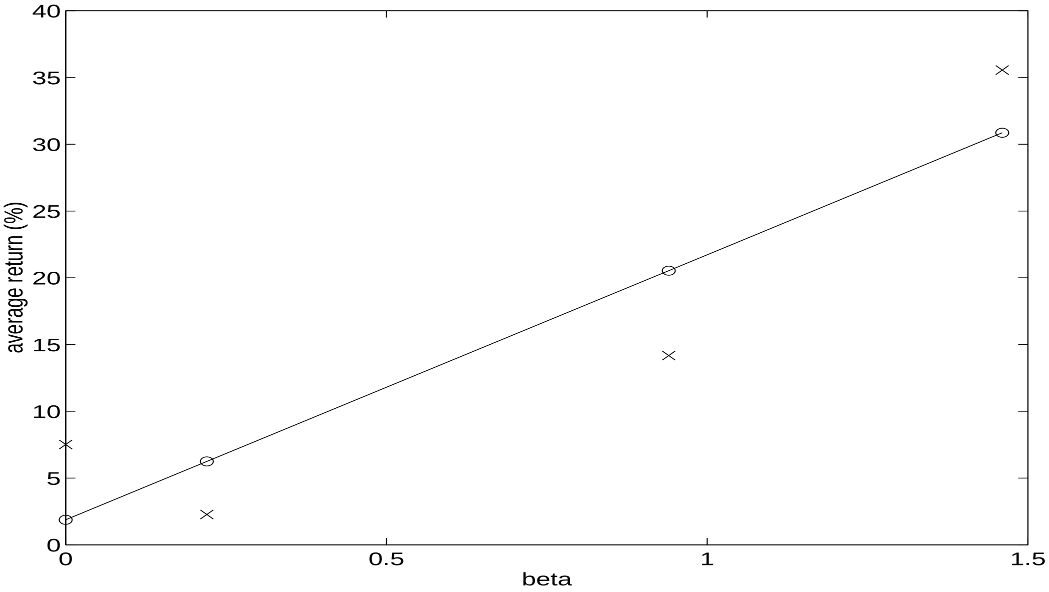

Figure 7. Average rate of return versus beta for 1976/1-1980/12 for small stocks, S&P composite stocks, treasury bonds, and treasury bills. ('x' indicates actual returns and 'o' indicates predicted returns.)

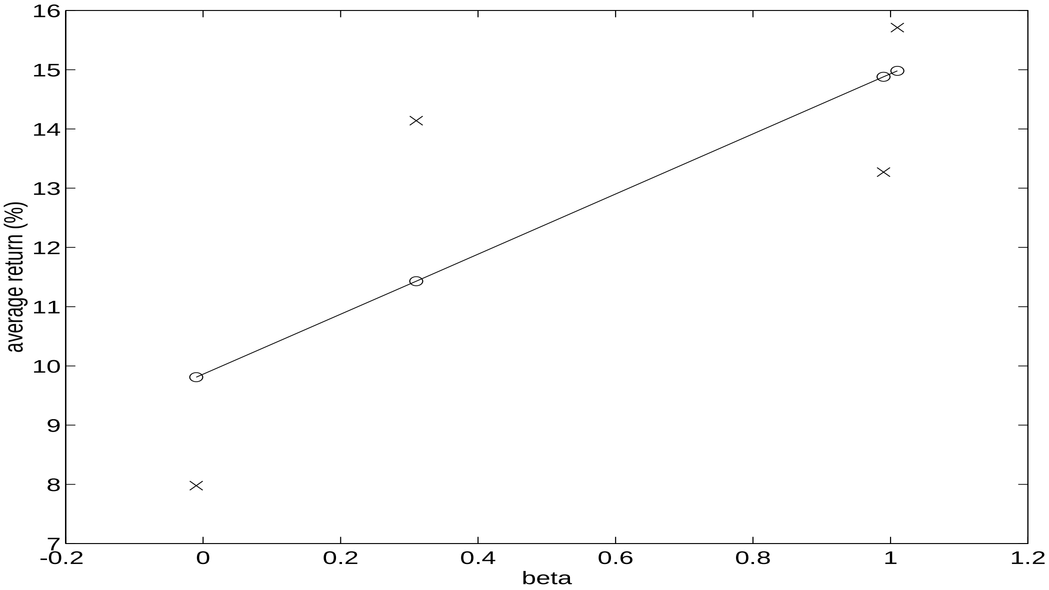

Figure 8. Average rate of return versus beta for 1981/1-1991/12 for small stocks, S&P composite stocks, treasury bonds, and treasury bills. ('x' indicates actual returns and 'o' indicates predicted returns.)