Quantifying Efficient Tax Reform

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Question

- How large are welfare gains from efficient tax reform?
  - Baseline:
    - Positive economy matched to administrative data
  - Reform:
    - Pareto improvements on efficient frontier (full)
    - Optima given set of policy tools (restricted)

Idea in a Picture

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- Start with baseline OLG economy:
  - Incomplete markets
  - Heterogeneous households
    - Differing in education levels by individual
    - Facing productivity, marital, unemployment risks
    - Deciding on consumption, saving, hours
  - Technology parameters and tax policies
- Compute remaining lifetime utilities (v_j)

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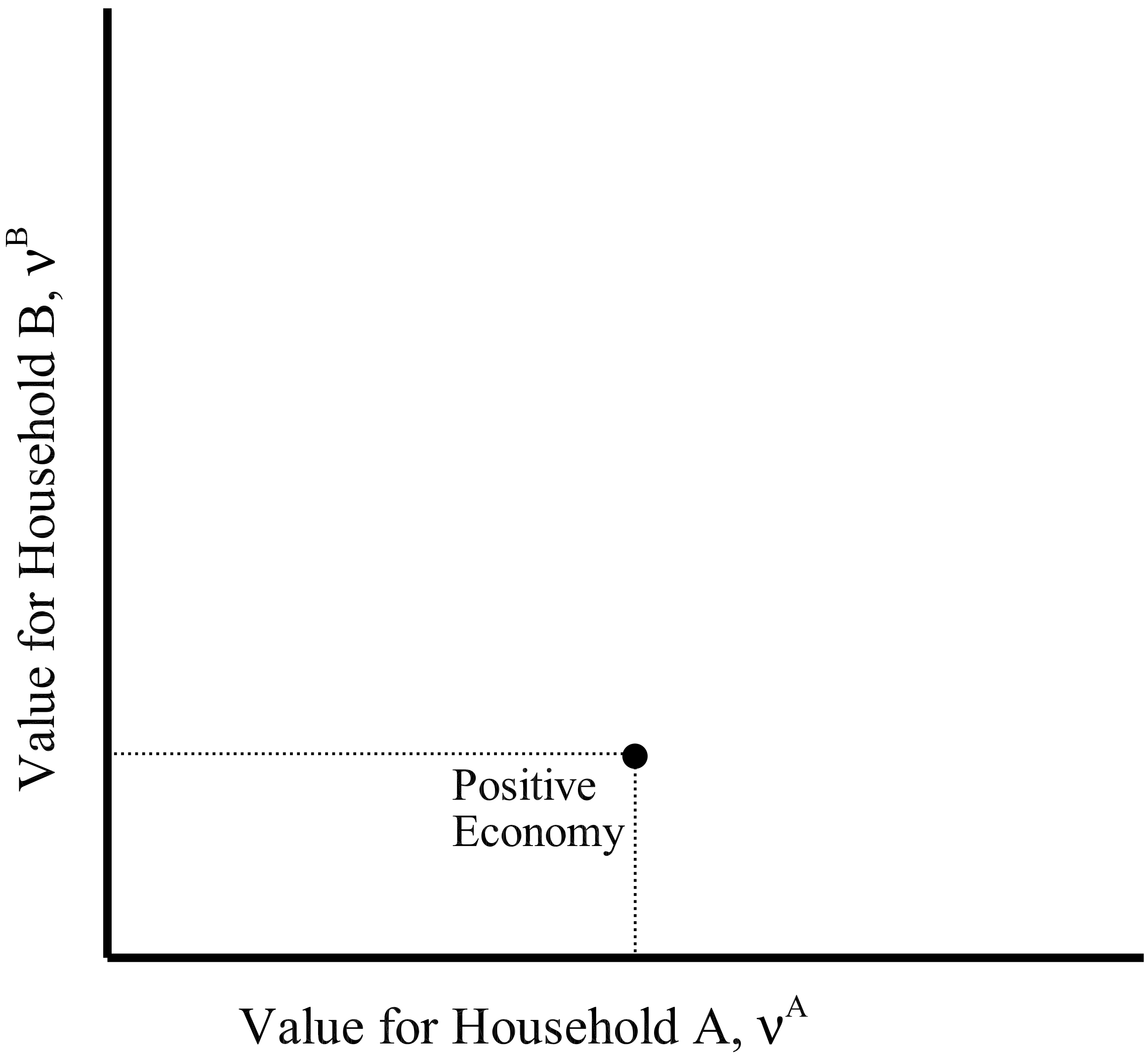
- Compute remaining lifetime utilities (v_j)
- Let's draw this for 2 households...

Idea in a Picture

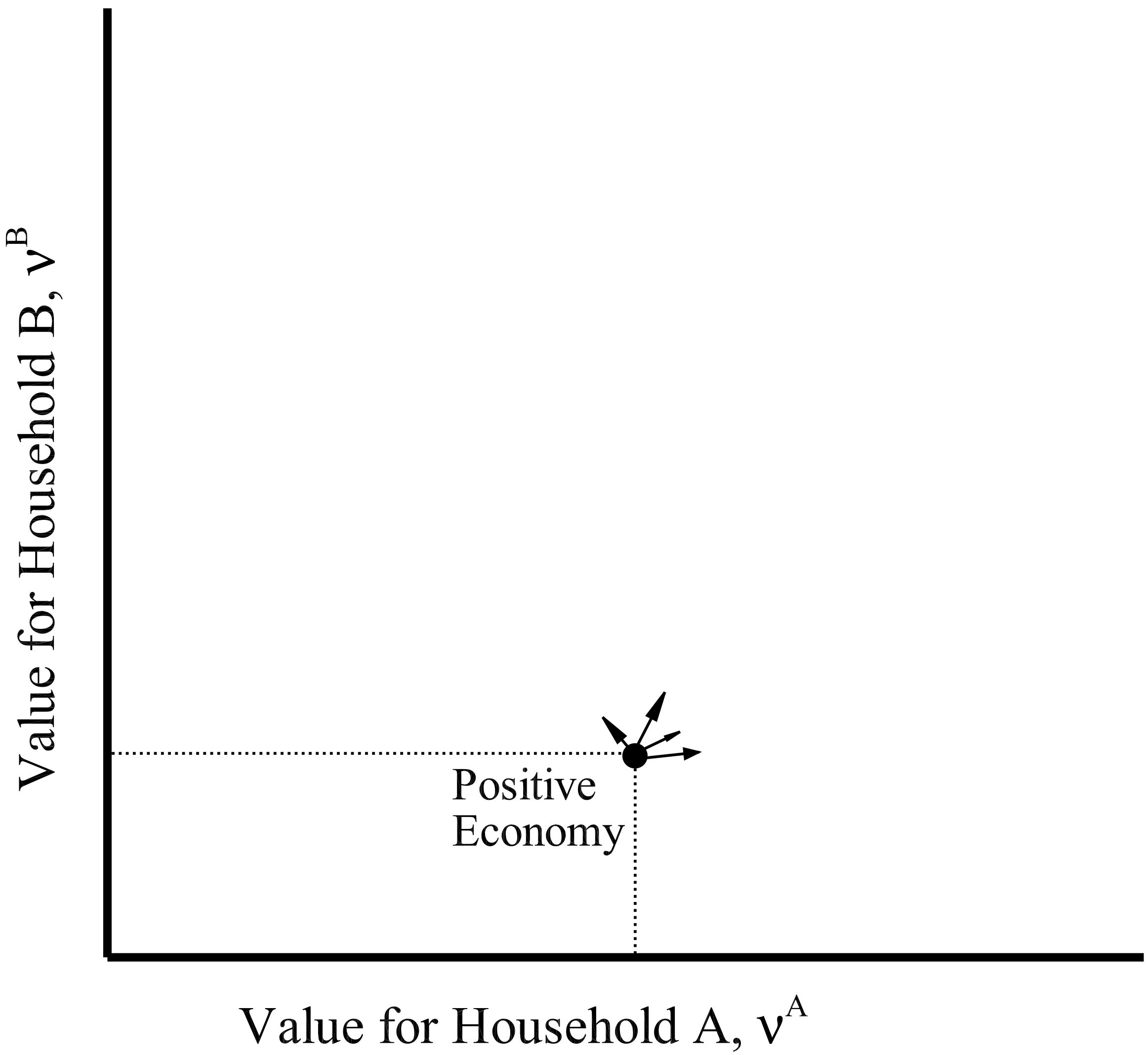


Idea in a Picture

- Typical starting point for most analyses
  - With constraints on policy instruments
  - Do counterfactuals or restricted optimal ("Ramsey")

- Let's draw this in the picture

Idea in a Picture

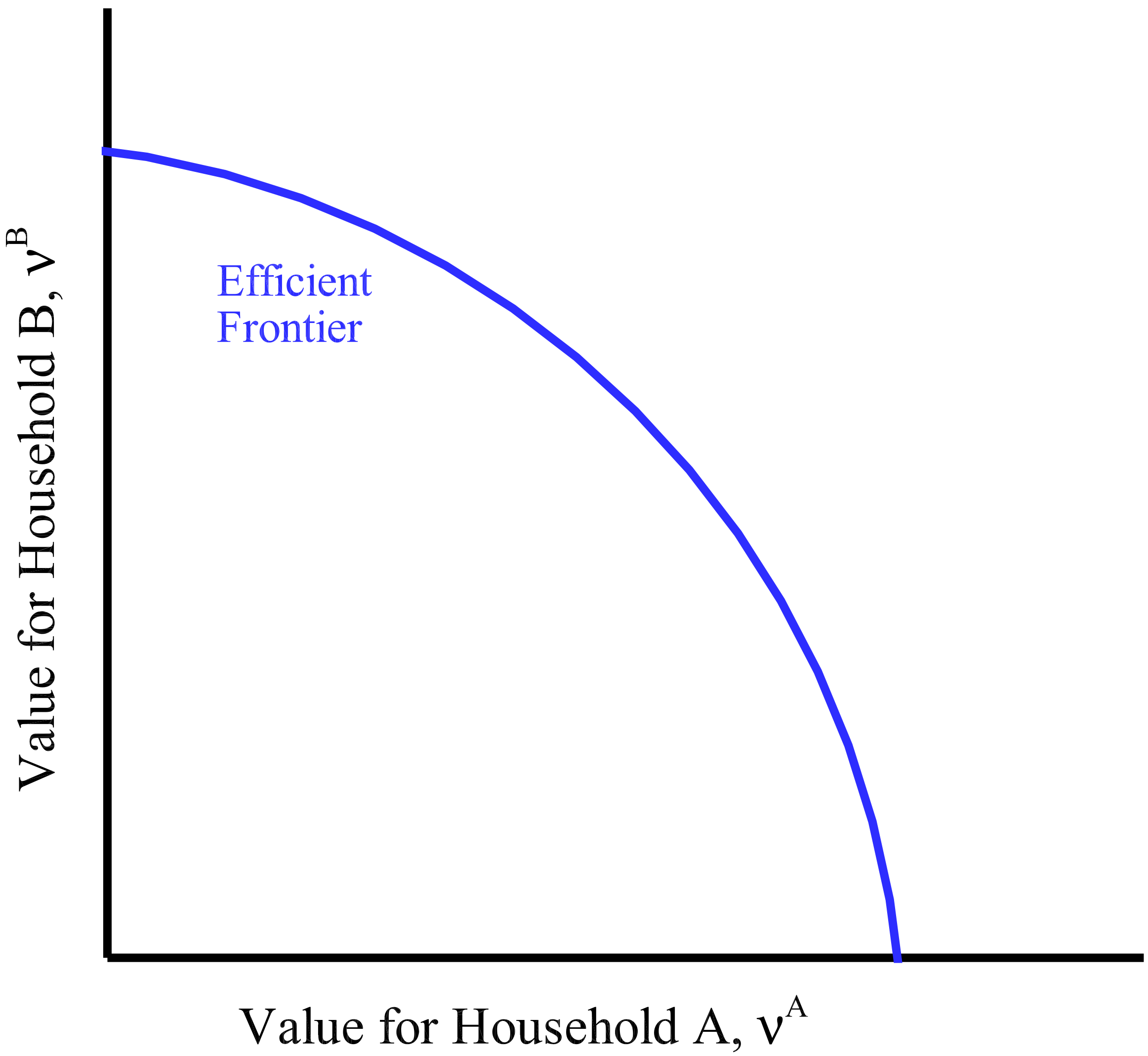


Idea in a Picture

- Not typical starting point for studies in Mirrlees tradition
  - With constraints on information sets
  - Characterize efficient allocations and policy “wedges”

- Let’s draw this in the picture

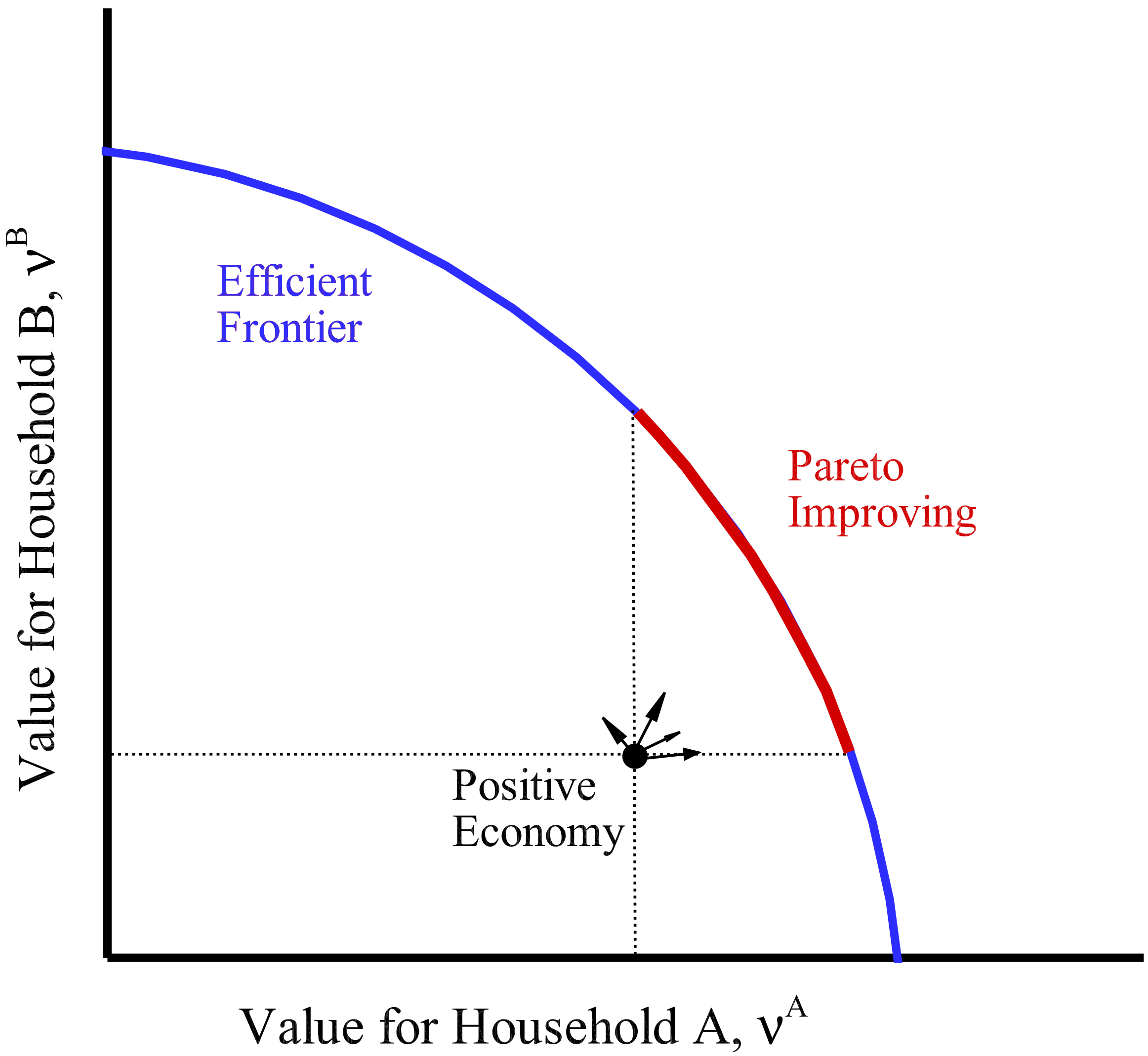
Idea in a Picture



Idea in a Picture

- This paper quantifies gains from:
    - Full Pareto-improving reform a la Mirrlees
    - Partial Pareto-improving reform a la Ramsey
    - Adding early-life transfer informed by Mirrlees
- Let's draw this in the picture

Idea in a Picture



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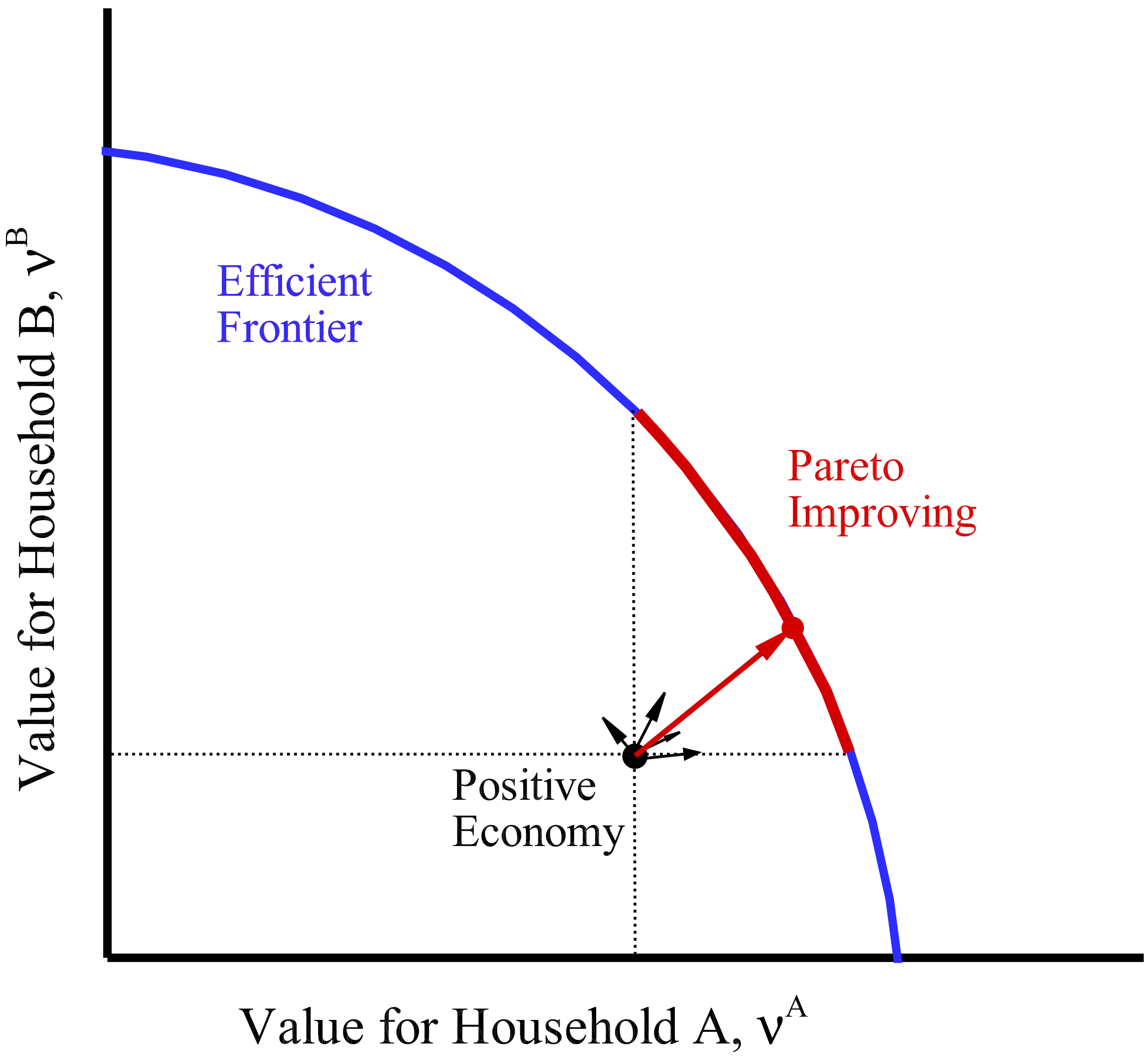

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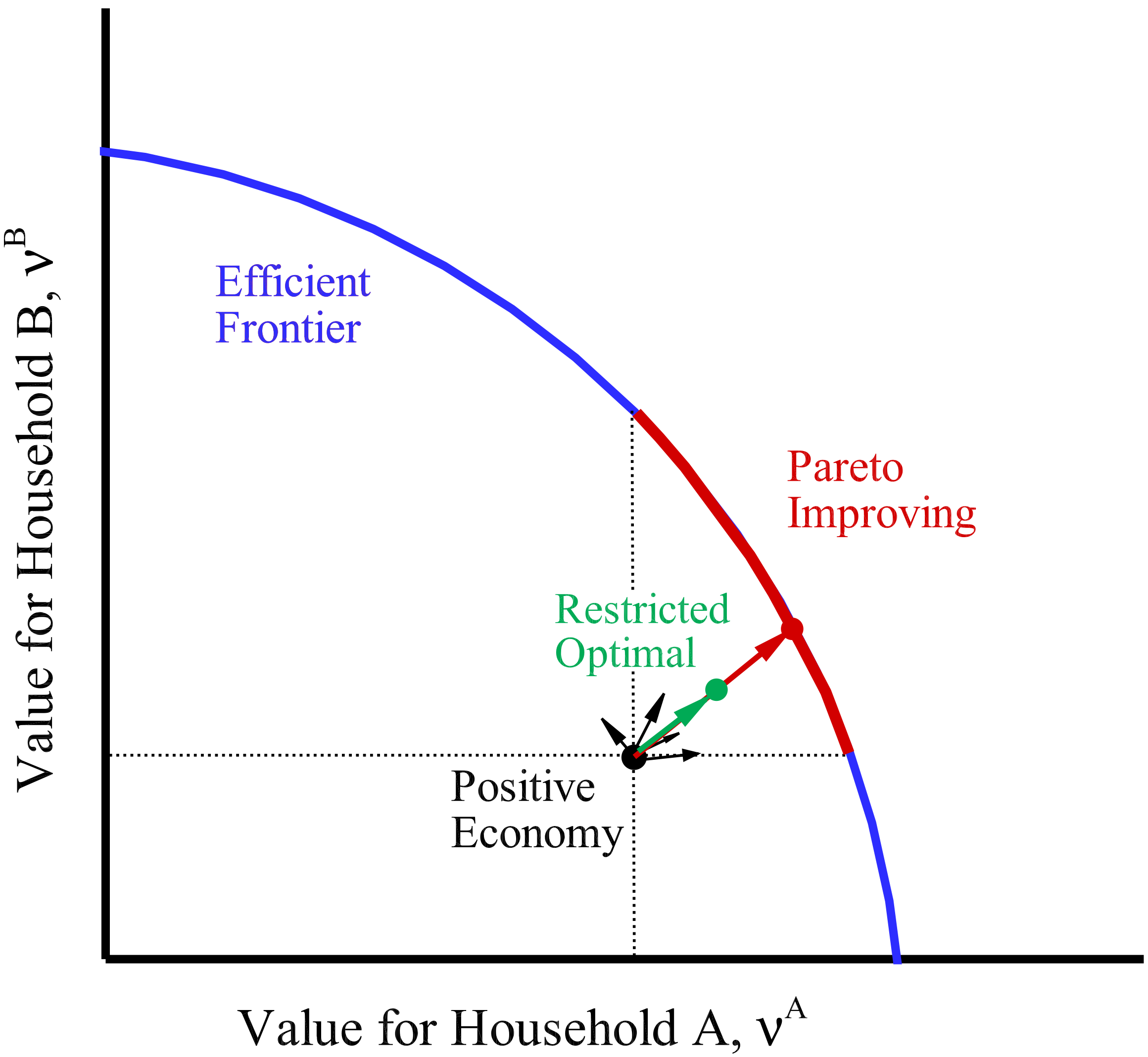


Our Approach

- Solve equilibrium for positive economy (●)
  - Inputs: fiscal policy and wage processes
  - Outputs: values under current policy
- Solve planner problem next (●)
  - Inputs: values under current policy
  - Outputs: labor and savings wedges and welfare gains
- Use results to inform current policy and reforms (●)

Main Findings ($\bullet \rightarrow \bullet$)

- Maximum consumption equivalent gains (future cohorts):
  - 21% starting at age 25
  - Comparisons made to utilitarian planner

- Decompose by comparing allocations:
  - Consumption: level $\uparrow$ and variance $\downarrow$ for all groups
  - Leisure: level $\downarrow$ and variance $\uparrow$ for all groups

Note: Currently computing transitions

Main Findings (●→●)

- Informed by comparison of baseline (●) and full reform (●)
  - Most gains in lifting consumption levels for young

$\Rightarrow$ Exploring early-life transfers

Note: Computer is still hillclimbing

Contributions to Literature

- Theory and application of income tax design (●→)

  ⇒ Using administrative data from NL, go to (●)

- Pareto-improving reforms with fixed types

  Hosseini-Shourideh (2019)

  ⇒ Extend analysis to add dynamic risks

- Theory behind dynamic taxation and redistribution (●)

  Kapicka (2013), Farhi-Werning (2013), Golosov et al. (2016)

  ⇒ Link OLG (●) to planner (●) in full GE

Positive Economy

Positive Economy (•)

- Open OLG economy a la Bewley
- Household heterogeneity in:
  - Age
  - Education (observed, permanent)
  - Productivity (private, stochastic)
  - Marital risk
  - Divorce risk (in progress)
  - Unemployment risk (in progress)
- Transfers and taxes on consumption, labor income, assets

Positive Economy (•)

- Household problem

$$v_j(a,\epsilon;\Omega) = \max_{c,n,a'} \{U(c,\ell) + \beta E[v_{j+1}(a',\epsilon';\Omega)|\epsilon]\}$$

$$\text{s.t. } a' = (1+r)a - T_a(ra) + w\epsilon n - T_n(j, w\epsilon n) - (1+\tau_c)c$$

where

  - j= age
  - a= financial assets
  - ϵ= productivity shock
  - Ω= factor prices and tax policies
  - c= consumption
  - n= labor supply ($n + \ell = 1$)

Positive Economy (•)

- Firms:
  - Technology: $F(K, N) = K^{\alpha} N^{1-\alpha}$
  - Prices: r, w set internationally

- Government:
  - Taxes: consumption, incomes, assets
  - Borrows: at home and abroad

In Equilibrium

- Add it up:

$$C_t + I_t + G_t + B_{t+1} = F(K_t, N_t) + RB_t$$

$$\lim_{T \to \infty} \frac{1}{R^{T-1}} (B_T + K_T) \geq 0$$

- Then use answers as inputs into planner's problem

Data from Netherlands

- Merged administrative data, 2006-2014
  - Earnings from tax authority
  - Hours from employer provided data
  - Education from population survey
- National accounts
- Tax schedules

$\Rightarrow$ Big data advantage for estimating elasticities & shocks

Estimation of Wage Processes

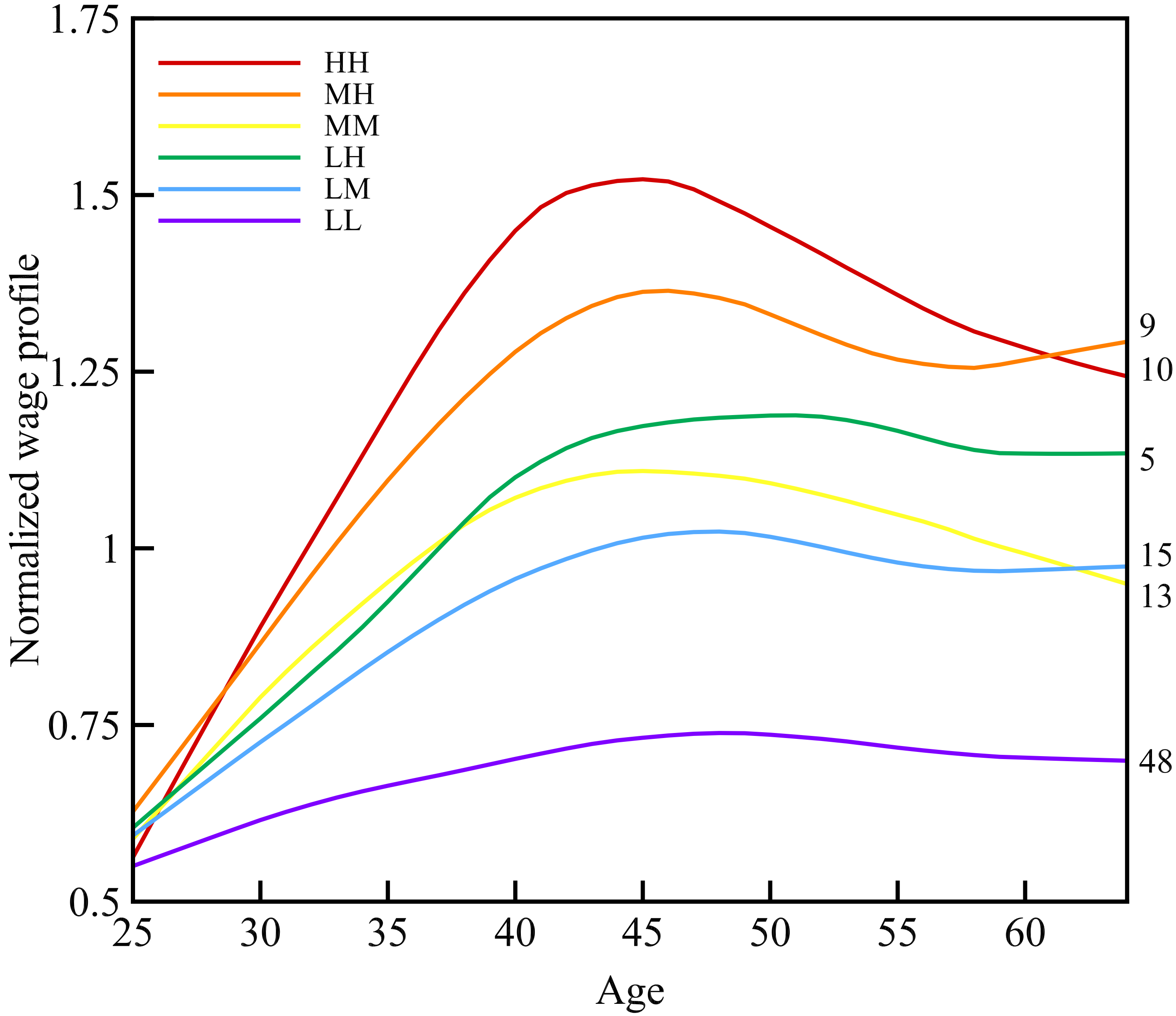
- Construct hourly wages W_{ijt} (j=age, t=time)
- Classify degrees:
  - High school or practical (Low)
  - University of applied sciences (Medium)
  - University (High)
- Construct residual wages ω_{ijt}:
  - $\log W_{ijt} = A_t + X_{ijt} + \omega_{ijt}$
  - Estimate AR(1) process for idiosyncratic risk

Marriage and Household Structure

- In period 0, individuals are single
  - Different by education (L,M,H)
- After that, individuals either
  - Form a couple (LL,LM,LH,MM,MH,HH) or
  - Remain single (included with LL,MM,HH)

Note: Working on adding divorce risk

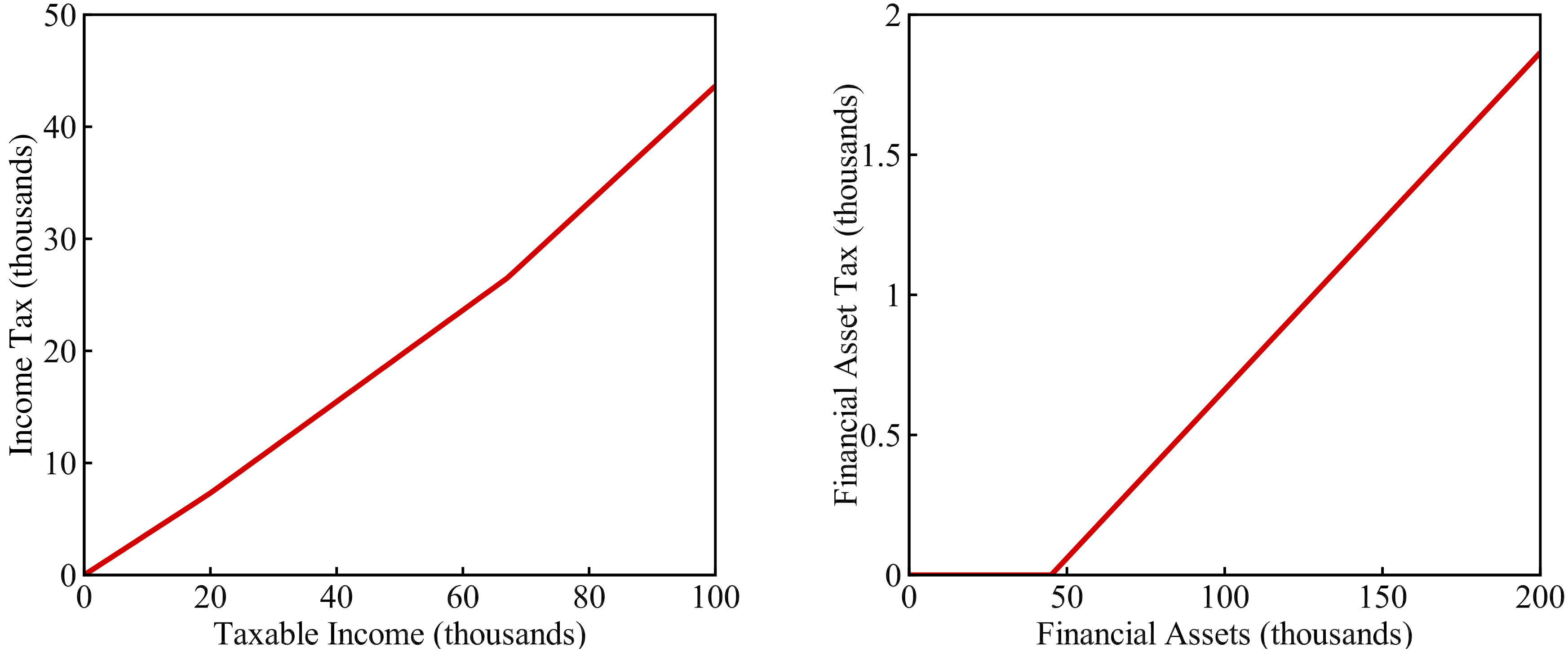
Wage Profiles



Wage Process Estimates

Group	$\hat{\rho}$	$\hat{\sigma}_u^2$
Low, Low	.9542	.0096
Low, Medium	.9660	.0087
Low, High	.9673	.0162
Medium, Medium	.9570	.0099
Medium, High	.9616	.0109
High, High	.9564	.0172

Income and Asset Tax Schedules



Reform Problem

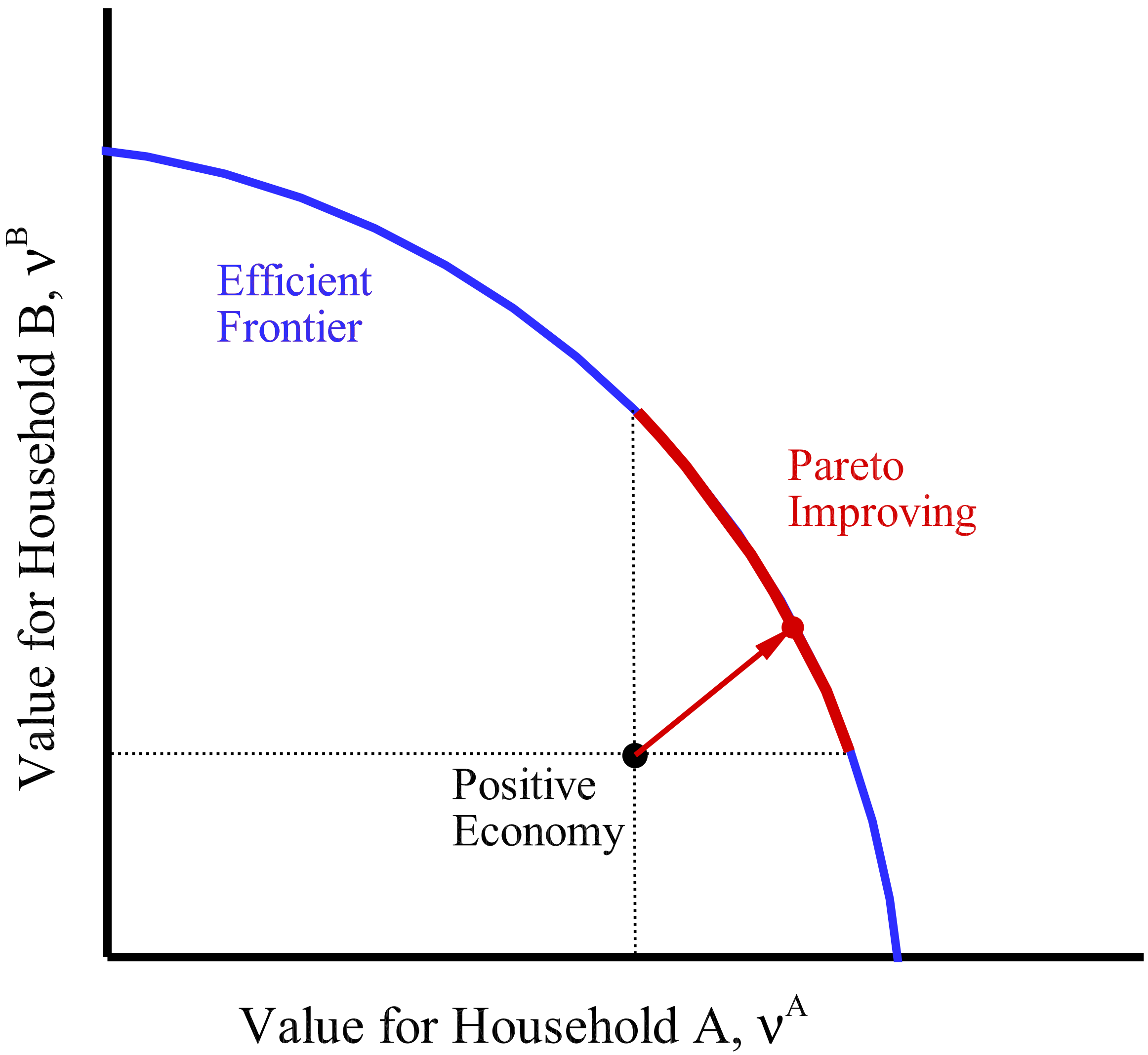
Reform Problem (•)

- Take inputs from positive economy:
  - Parameters for preferences and technologies
  - Wage profiles and shock processes
  - Values under current policy (v_A,v_B, . . .)
- Compute maximum consumption equivalent gain

Notion of Efficiency

- Our focus is Pareto-improving reforms:
  - There is no alternative allocation that is
    - Resource feasible
    - Incentive feasible
  - Making all better off and some strictly better off
- Will report gain assuming same percentage for all

Pareto-improving Reforms



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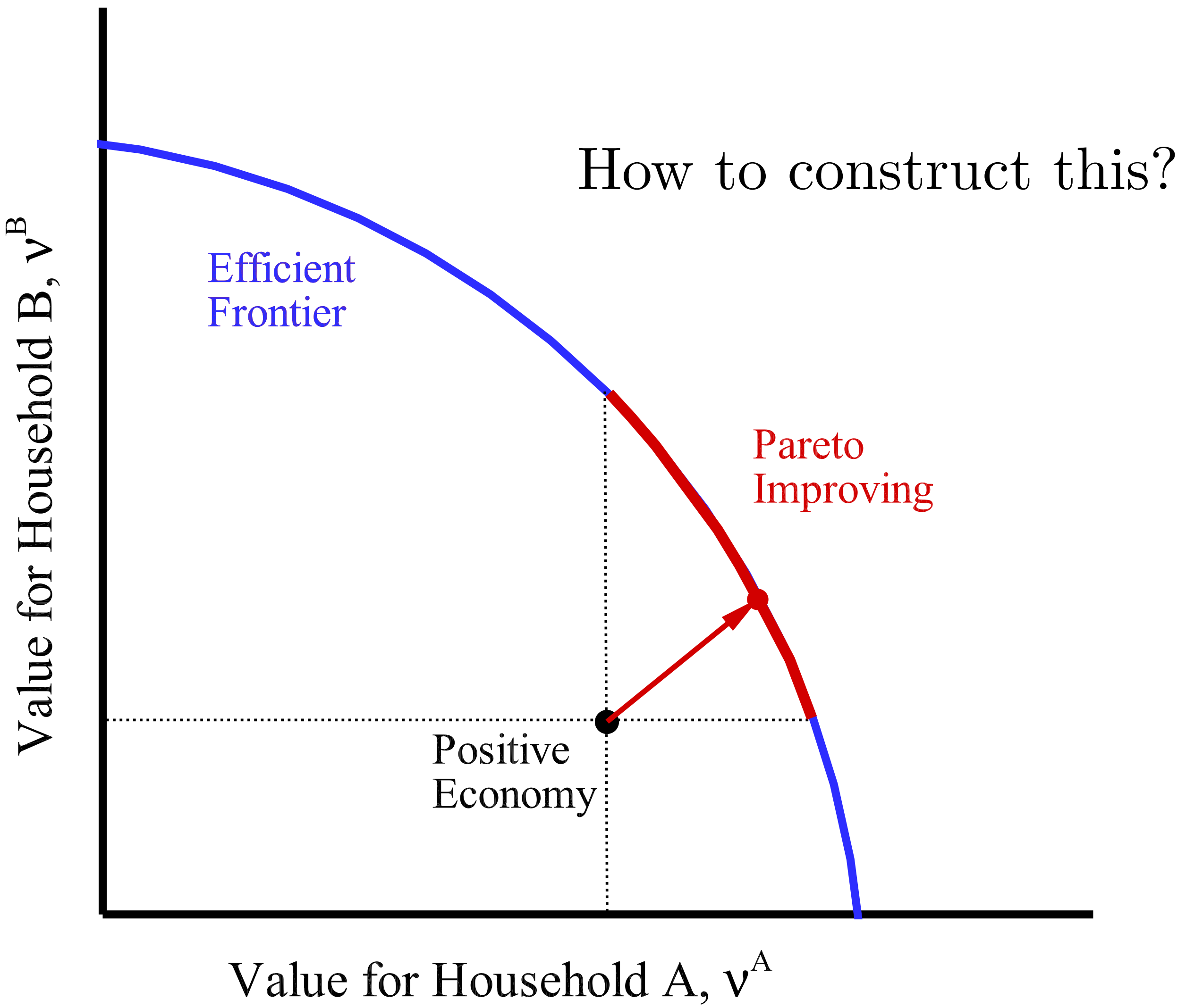

Planner Problem in Words (Primal)

- Maximize weighted sum of lifetime utilities
- subject to
  - Incentive constraints for every household and history
  - Resource constraints

Planner Problem in Words (Primal)

- Maximize weighted sum of lifetime utilities
- subject to
  - Incentive constraints for every household and history
  - Resource constraints

  - Computationally easier to solve dual problem

Planner Problem in Words (Dual)

- Maximize present value of aggregate resources
- subject to
  - Incentive constraints for every household and history
  - Value delivered exceeds that of positive economy

Planner Problem in Math (Dual)

$$\max \sum_h \pi_0(h)\, \Pi_0(V^h, -, \epsilon)$$

subject to

- Incentive constraints for all h
- $V^h \geq \vartheta^h$ for all h

Planner Problem in Math (Dual)

$\max \sum_h \pi_0(h)\, \Pi_0(V^h, -, \epsilon)$

subject to

- Incentive constraints for all h
- $V^h \geq \vartheta^h$ for all h

$\Rightarrow$ Exploit separability to solve household by household

Planner Problem in Practice

- Exploit separability to solve household by household
- Include only local downward incentive constraints
  - Verify numerically that constraints are satisfied
- Solve recursively by introducing additional states
  - Promised value for truth telling
  - Threat value for local lie

Planner Problem in Practice

- Exploit separability to solve household by household
- Include only local downward incentive constraints
  - Verify numerically that constraints are satisfied
- Solve recursively by introducing additional states
  - Promised value for truth telling (V)
  - Threat value for local lie ($\widetilde{V}$)

An Aside

- Government:
  - Can *ex-post* infer type from choices
  - Can't *ex-ante* observe type
- But, can design policy to *induce* truthful reporting of type

Planner Problem for a Household

Planner Problem for a Household

Max present value of resources

Planner Problem for a Household

$$\Pi_j(V,\widetilde{V},\epsilon) \equiv \max \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon)\big[w\epsilon_i n_j(\epsilon_i) - c_j(\epsilon_i)$$
$$+\text{future value}\big]$$

As in positive economy,

- j= age
- ϵ= productivity shock
- c= consumption
- n= labor supply

Planner Problem for a Household

$$\Pi_j(V, \widetilde{V}, \epsilon) \equiv \max \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon)\left[w\epsilon_i n_j(\epsilon_i) - c_j(\epsilon_i)\right.$$
$$\left. + \Pi_{j+1}(V_j(\epsilon_i), \widetilde{V}_j(\epsilon_{i+1}), \epsilon_i)/R\right]$$

Additionally, planner chooses

- V_j= promise value
- $\widetilde{V}_j$= threat value

Planner Problem for a Household

$$\Pi_j(V,\widetilde{V},\epsilon) \equiv \max \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon)\Big[w\epsilon_i n_j(\epsilon_i) - c_j(\epsilon_i) + \Pi_{j+1}(V_j(\epsilon_i),\widetilde{V}_j(\epsilon_{i+1}),\epsilon_i)/R\Big]$$

s.t. Local downward incentive constraints

Planner Problem for a Household

$$\Pi_j(V,\widetilde{V},\epsilon) \equiv \max \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon)\Big[w\epsilon_i n_j(\epsilon_i) - c_j(\epsilon_i)$$
$$+\Pi_{j+1}(V_j(\epsilon_i),\widetilde{V}_j(\epsilon_{i+1}),\epsilon_i)/R\Big]$$

$$\text{s.t.} \quad U(c_j(\epsilon_i),\ell_j(\epsilon_i)) + \beta V_j(\epsilon_i)$$
$$\geq U(c_j(\epsilon_{i-1}),\ell_j^{+}(\epsilon_{i-1})) + \beta \widetilde{V}_j(\epsilon_i), \;\; i \geq 2$$

where $\ell_j^{+}(\epsilon_{i-1}) = 1 - n_j(\epsilon_{i-1})\epsilon_{i-1}/\epsilon_i$

Planner Problem for a Household

$$\Pi_j(V,\widetilde{V},\epsilon) \equiv \max \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon)\Big[w\epsilon_i n_j(\epsilon_i) - c_j(\epsilon_i)$$
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$$\geq U(c_j(\epsilon_{i-1}),\ell_j^+(\epsilon_{i-1})) + \beta \widetilde{V}_j(\epsilon_i),\ i \geq 2$$

Deliver at least the promised value

Planner Problem for a Household

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$$V \leq \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon)\left[U(c_j(\epsilon_i),\ell_j(\epsilon_i)) + \beta V_j(\epsilon_i)\right]$$

Planner Problem for a Household

$$\Pi_j(V, \widetilde{V}, \epsilon) \equiv \max \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon) \Big[w\epsilon_i n_j(\epsilon_i) - c_j(\epsilon_i) + \Pi_{j+1}(V_j(\epsilon_i), \widetilde{V}_j(\epsilon_{i+1}), \epsilon_i)/R \Big]$$

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$$V \leq \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon) \left[U(c_j(\epsilon_i), \ell_j(\epsilon_i)) + \beta V_j(\epsilon_i) \right]$$

Deliver no more than the threat value

Planner Problem for a Household

$$\Pi_j(V,\widetilde{V},\epsilon) \equiv \max \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon)\Big[w\epsilon_i n_j(\epsilon_i) - c_j(\epsilon_i)$$
$$+\Pi_{j+1}(V_j(\epsilon_i),\widetilde{V}_j(\epsilon_{i+1}),\epsilon_i)/R\Big]$$

$$\text{s.t.} \quad U(c_j(\epsilon_i),\ell_j(\epsilon_i)) + \beta V_j(\epsilon_i)$$
$$\geq U(c_j(\epsilon_{i-1}),\ell_j^{+}(\epsilon_{i-1})) + \beta \widetilde{V}_j(\epsilon_i), \;\; i \geq 2$$

$$V \leq \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon)\left[U(c_j(\epsilon_i),\ell_j(\epsilon_i)) + \beta V_j(\epsilon_i)\right]$$

$$\widetilde{V} \geq \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon^{+})\left[U(c_j(\epsilon_i),\ell_j(\epsilon_i)) + \beta V_j(\epsilon_i)\right]$$

Planner Problem for Future Generation ($j = 1$)

$$\Pi_j(V, -, \epsilon) \equiv \max \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon) \Big[w\epsilon_i n_j(\epsilon_i) - c_j(\epsilon_i)$$

$$+ \Pi_{j+1}(V_j(\epsilon_i), \widetilde{V}_j(\epsilon_{i+1}), \epsilon_i)/R \Big]$$

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No threat value

Planner Problem for Future Generation ($j = 1$)

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$$V \leq \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon)\left[U(c_j(\epsilon_i), \ell_j(\epsilon_i)) + \beta V_j(\epsilon_i)\right]$$

Replace arbitrary V with $\vartheta(\epsilon_0) + \vartheta_\Delta$

General Equilibrium

- Solve planner problem for positive economy values

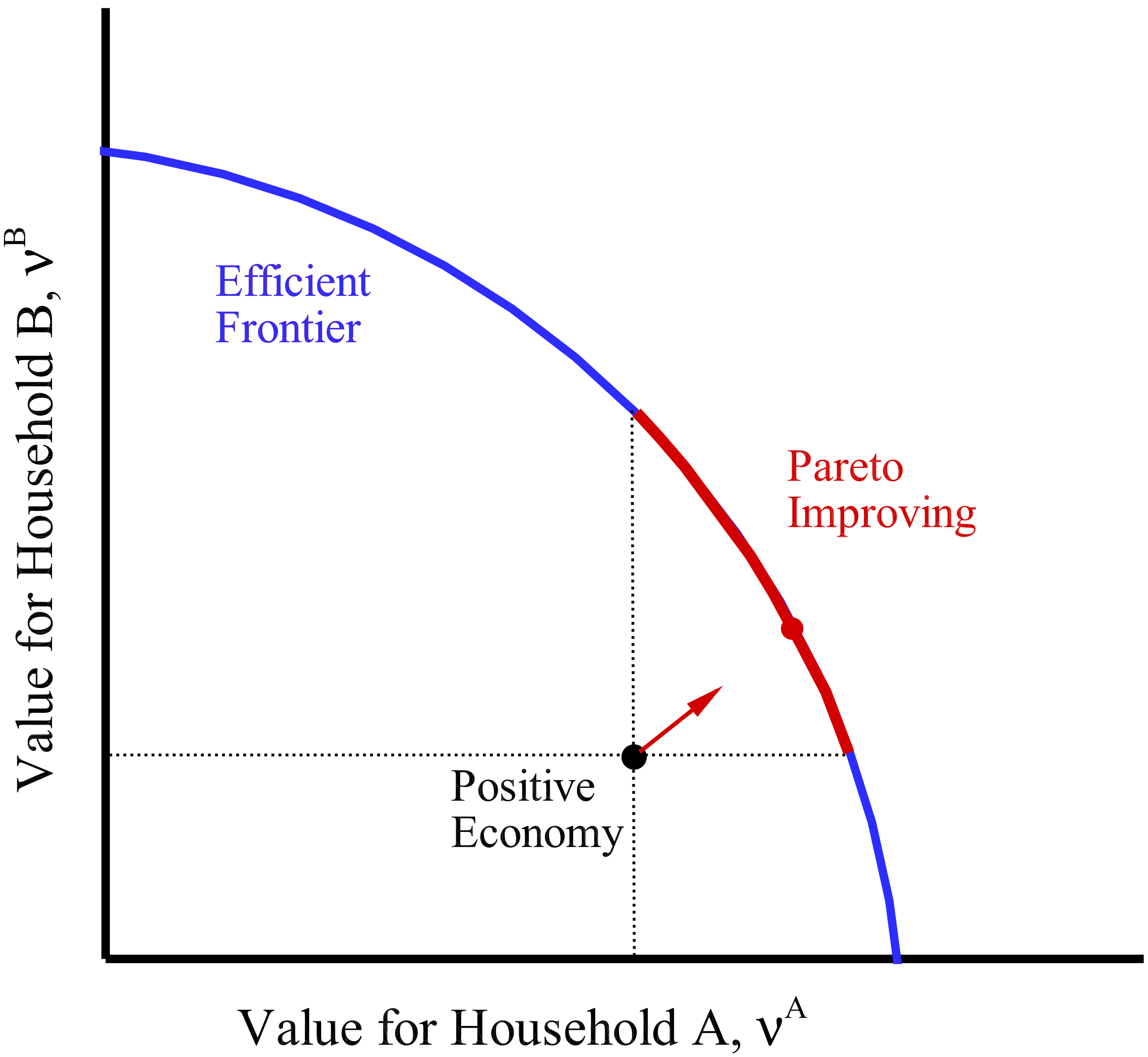
- Evaluate resource constraints

$$C_t + I_t + G_t + B_{t+1} = F(K_t, N_t) + RB_t$$

$$\lim_{T\to\infty} \frac{1}{R^{T-1}}(B_T + K_T) \geq 0$$

- Increase $\vartheta_{\triangle}$ until resources exhausted

Pareto-improving Reforms



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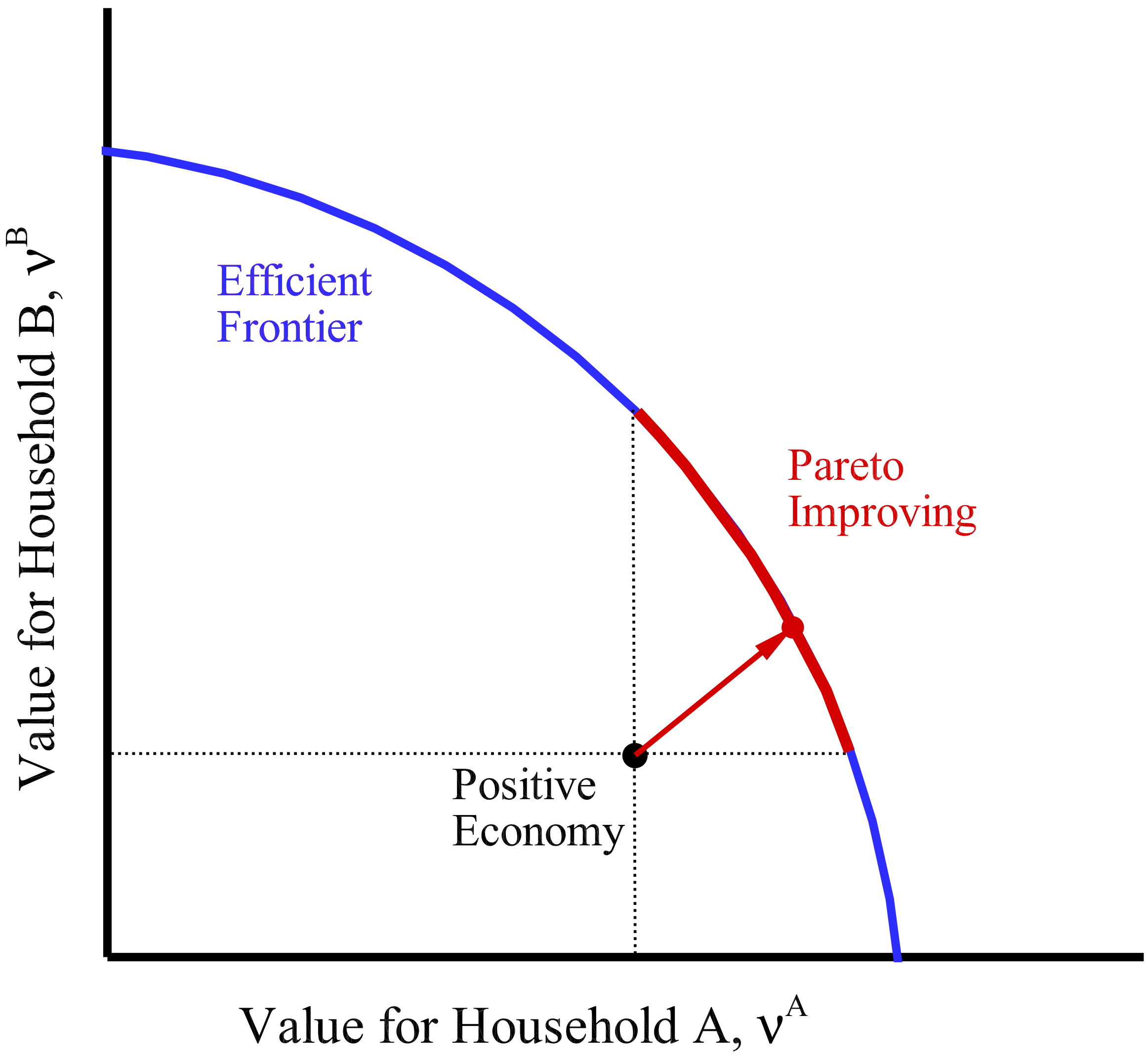


Putting this on the computer...

Next Quantitative Steps

1. Quantify efficient reform (● → ●)
2. Use answer to inform restricted reform (● → ●)

Other Key Parameters

- Number of productivity types
- Preferences
- Status quo policy

Baseline: 20 types, log preferences, NL wages & policy

Quantitative Deliverables

- Welfare gains
  - Total consumption equivalent (ϑ_Δ)
  - Decomposition
- Wedges

Wedges

- Labor wedge:

$$\tau_n(\epsilon^j) = 1 - \frac{1}{w}\frac{U_\ell(c(\epsilon^j),\ell(\epsilon^j))}{U_c(c(\epsilon^j),\ell(\epsilon^j))}$$

- Savings wedge:

$$\tau_a(\epsilon^j) = 1 - \frac{U_c(c(\epsilon^j),\ell(\epsilon^j))}{\beta R E[U_c(c(\epsilon^{j+1}),\ell(\epsilon^{j+1}))|\epsilon^j]}$$

Wedges

- Labor wedge:

$$\tau_n(\epsilon^j) = 1 - \frac{1}{w}\frac{U_\ell(c(\epsilon^j),\ell(\epsilon^j))}{U_c(c(\epsilon^j),\ell(\epsilon^j))}$$

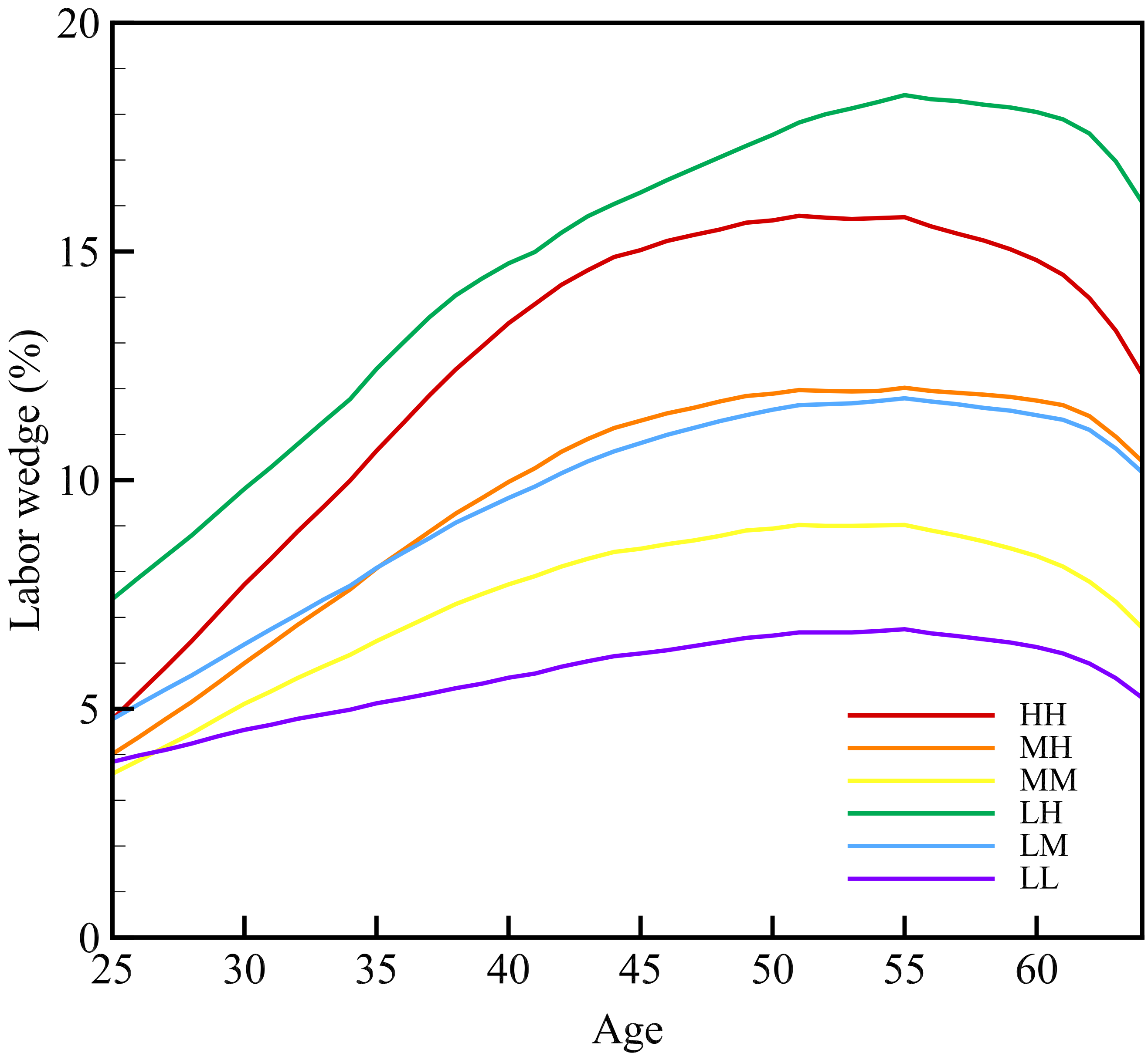
- Savings wedge:

$$\tau_a(\epsilon^j) = 1 - \frac{U_c(c(\epsilon^j),\ell(\epsilon^j))}{\beta RE[U_c(c(\epsilon^{j+1}),\ell(\epsilon^{j+1}))|\epsilon_j]}$$

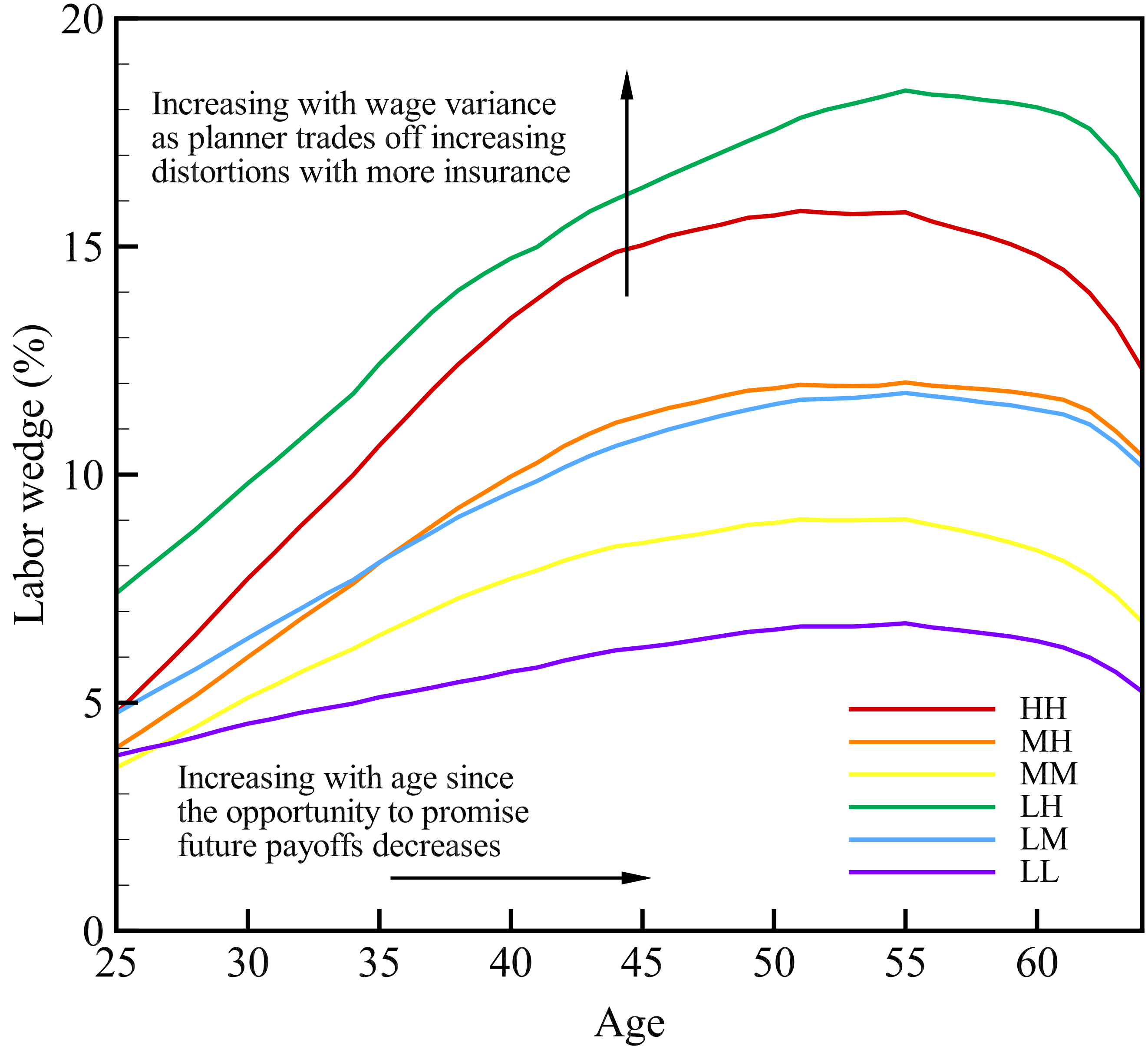
$\Rightarrow$ Hopefully informative for reforming current policy

Results

Labor Wedges



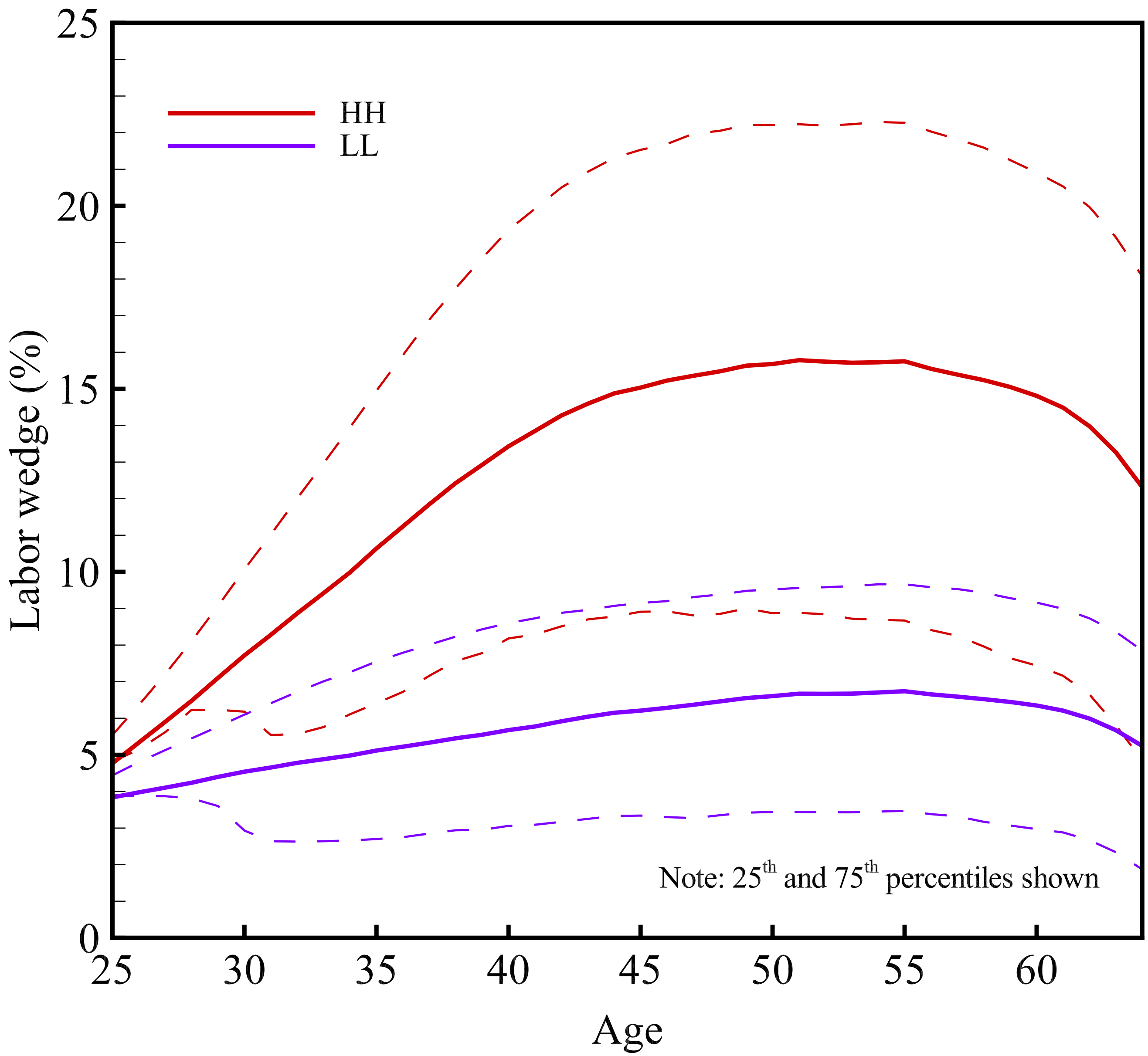
Labor Wedges



What We Learn

- Wedges are suggestive of
  - Informational frictions
  - Insurance needs
- But,
  - Wedges are not taxes
  - Averages mask significant variation

Labor Wedges for LL, HH



Welfare, (•) vs (•)

- Consumption equivalent gain of 21% for future cohorts
- Large but maybe not surprising given:
  - Tax rates in NL over 40%
  - Tax wedges of planner in 4% to 20% range

Welfare, (•) vs (•)

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Welfare, (•) vs (•)

- Consumption equivalent gain of 21% for future cohorts
- Large but maybe not surprising given:
  - Tax rates in NL over 40%
  - Tax wedges of planner in 4% to 20% range
- What are the implied Pareto weights?

Implied Pareto Weights

- Recall: could also have solved:
    - $\max \sum_i \pi_i \omega_i V^i$
    - subject to incentive and incentive constraints

Note: $\omega_i > 1 \Rightarrow$ overweight i relative to population share

Implied Pareto Weights

- Recall: could also have solved:
  - $\max \sum_i \pi_i \omega_i V^i$
  - subject to incentive and incentive constraints
- What are the implied ω_i's for L,M,H?

Pareto Weights and Welfare Gains

	Equal Gains		Equal Weights	
Education	ω_i	Δ_i	ω_i	Δ_i
Low	0.8	21		
Medium	1.0	21		
High	1.2	21		

Pareto Weights and Welfare Gains

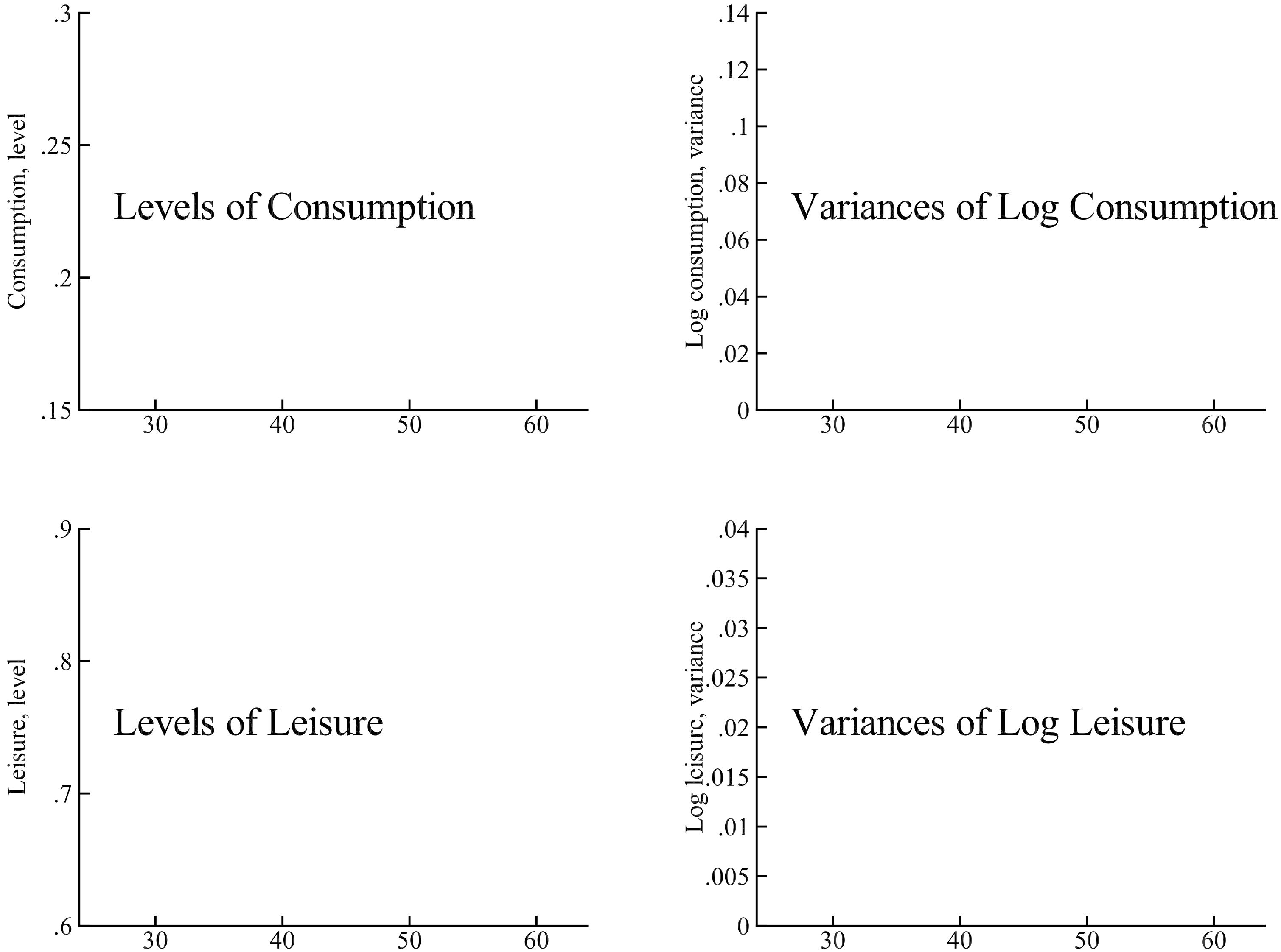
	Equal Gains		Equal Weights†	
Education	ω_i	Δ_i	ω_i	Δ_i
Low	0.8	21	1	32
Medium	1.0	21	1	18
High	1.2	21	1	2

† Utilitarian planner with $V^H \geq V^M \geq V^L$

Comparing Allocations, (•) vs (•)

- Consumption: level $\uparrow$ and variance $\downarrow$ for all groups
- Leisure: level $\downarrow$ and variance $\uparrow$ for all groups
- Intuition from simple static model:
  - No insurance: c varies, ℓ constant
  - Full insurance: c constant, ℓ varies
- What about magnitudes?

A Look Under the Hood: Group LL



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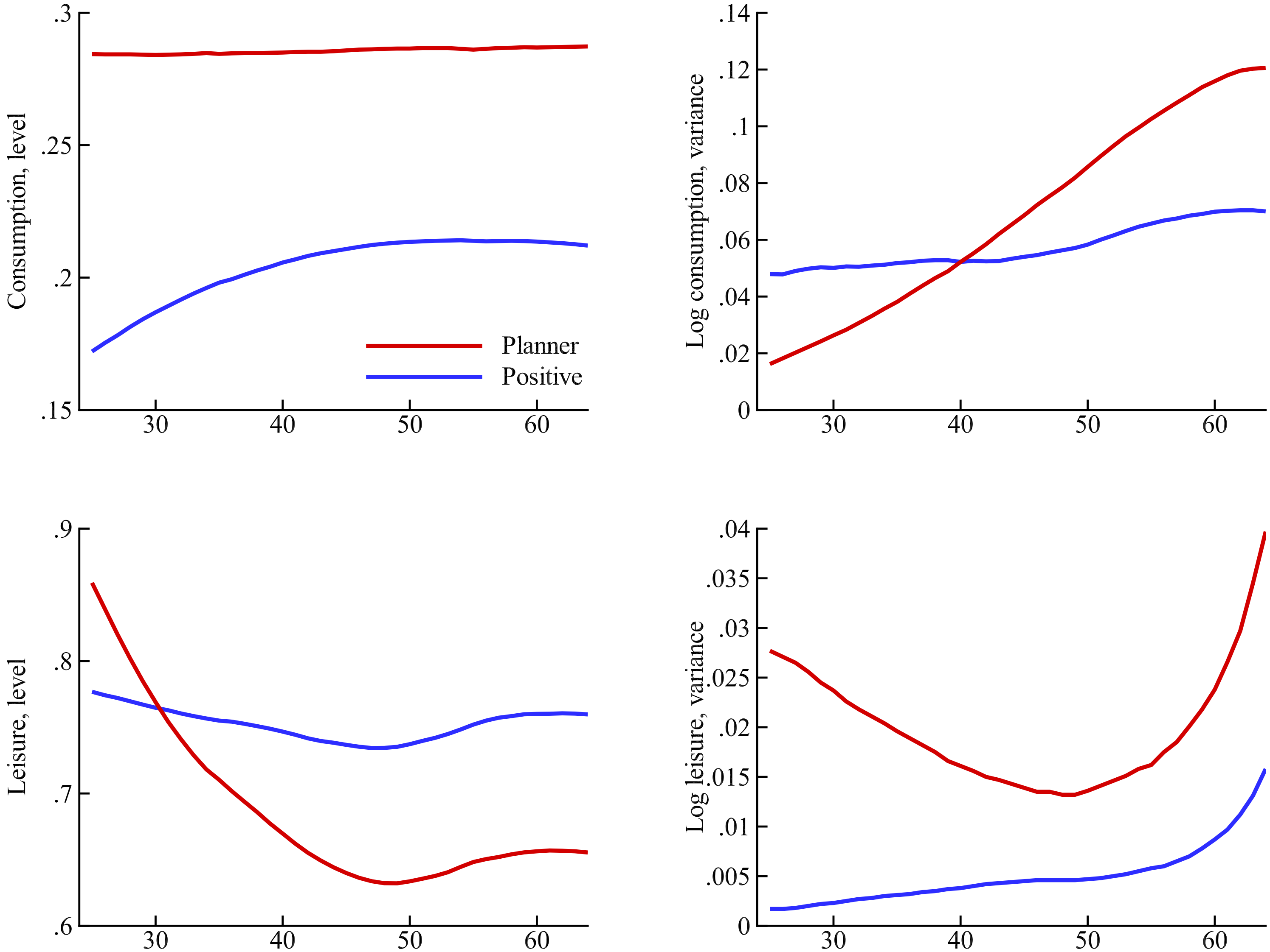
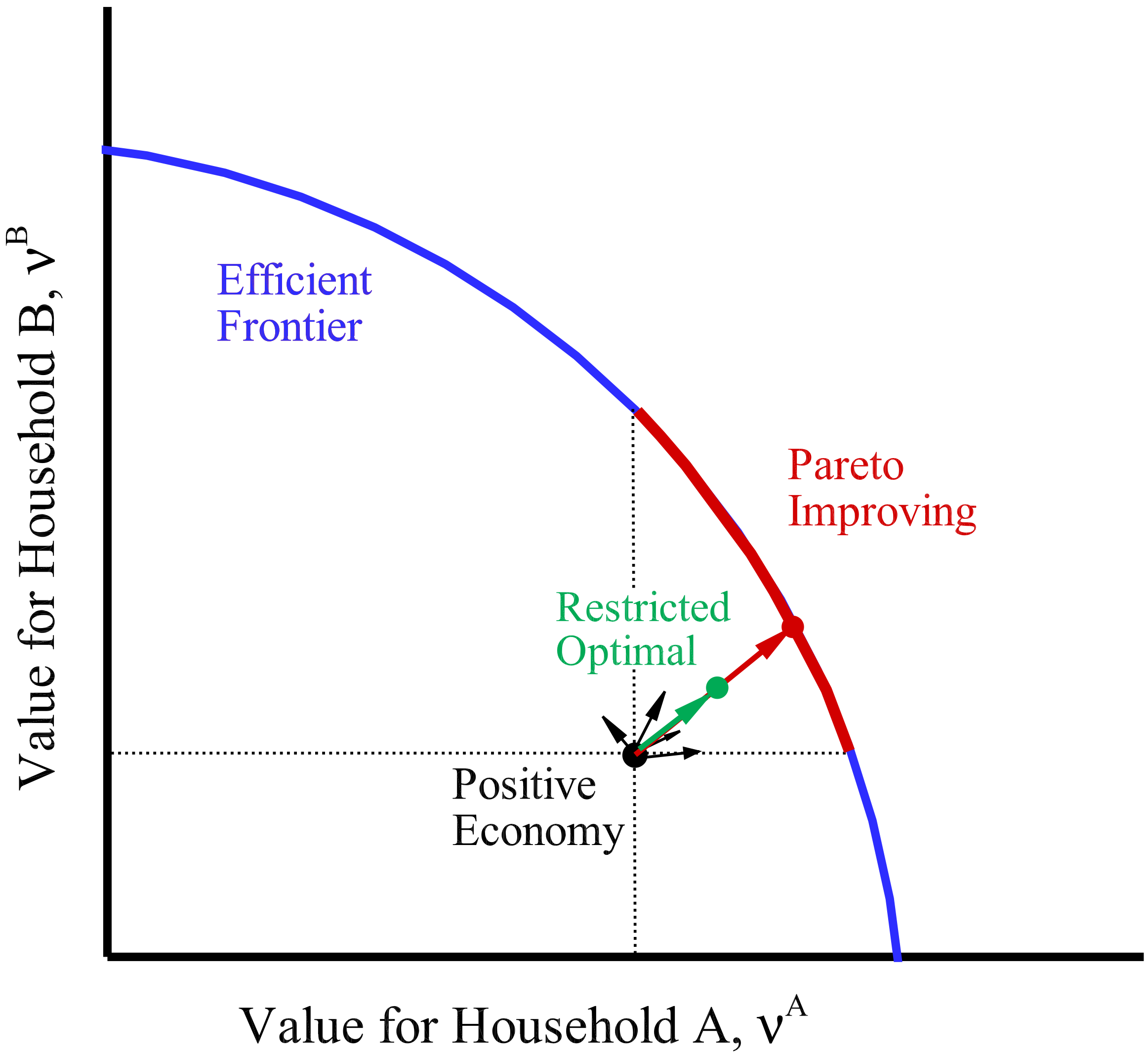


Informing Counterfactuals (●)

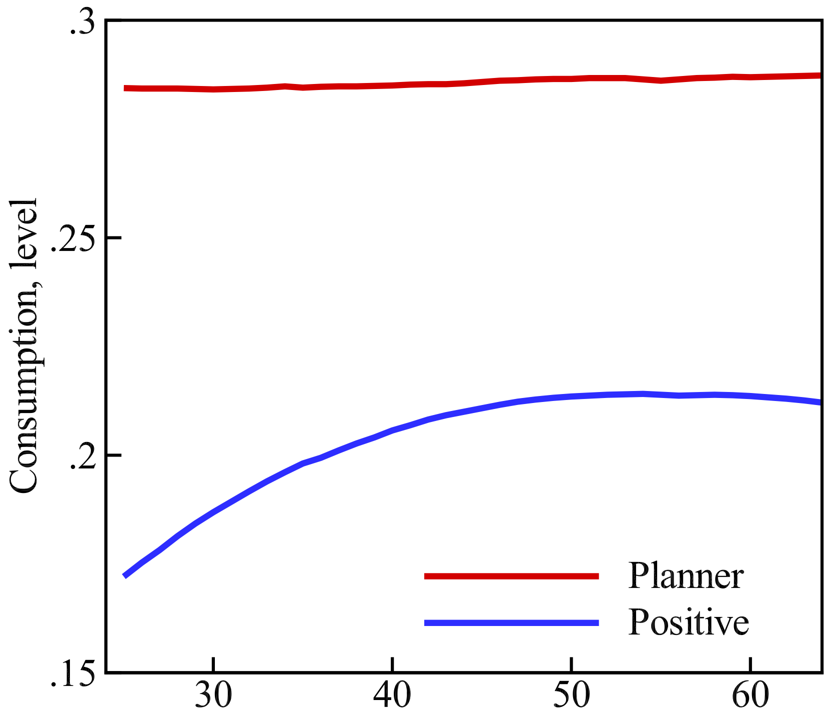


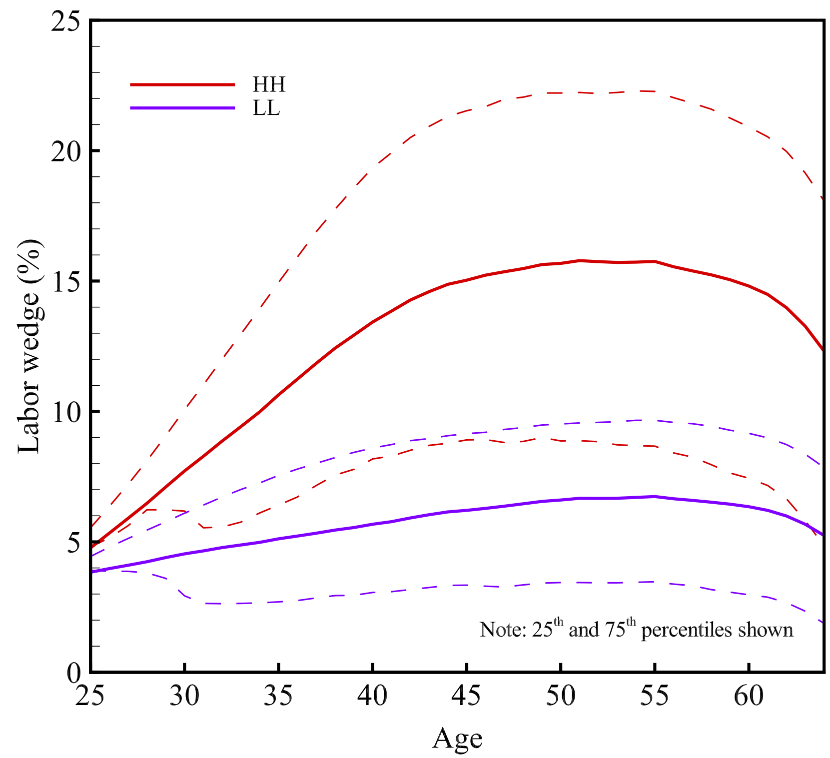
Informing Counterfactuals (•)

- Results of planner problem suggest large gains to
  - Lower average marginal tax rates
  - Early life transfers
  - Income-tested transfers

Note: our results on restricted gains still tentative

Informing Counterfactuals (●)





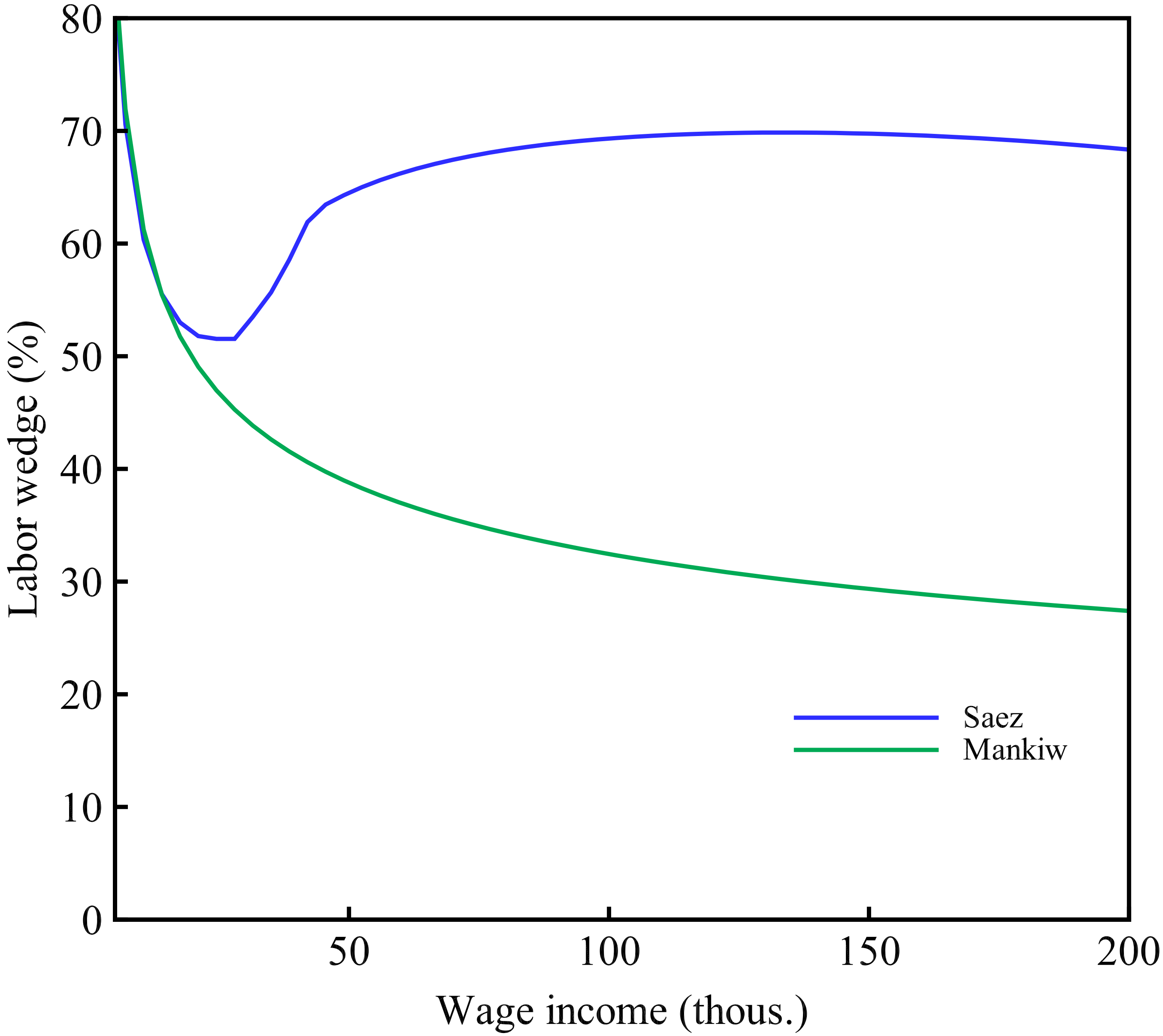
- Points to certain:
    - Early life transfers
    - Income-tested transfers

Summary

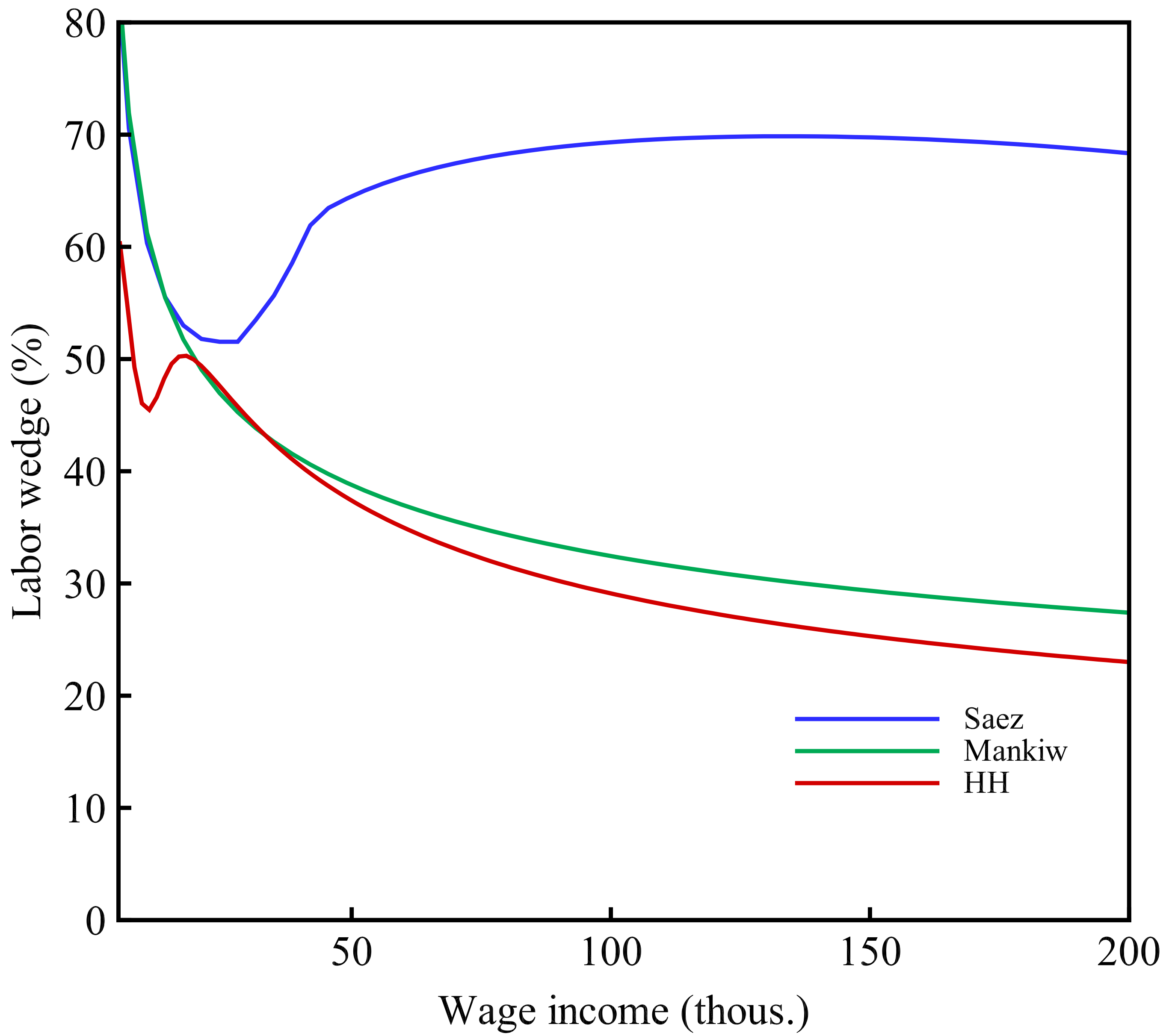
- Ultimate deliverables of project:
  - Estimates of gains for efficient reform
  - Identification of sources of gains
  - Ideas for new policy instruments
  - Prototype for future analyses
- Stay tuned...

Appendix Slides

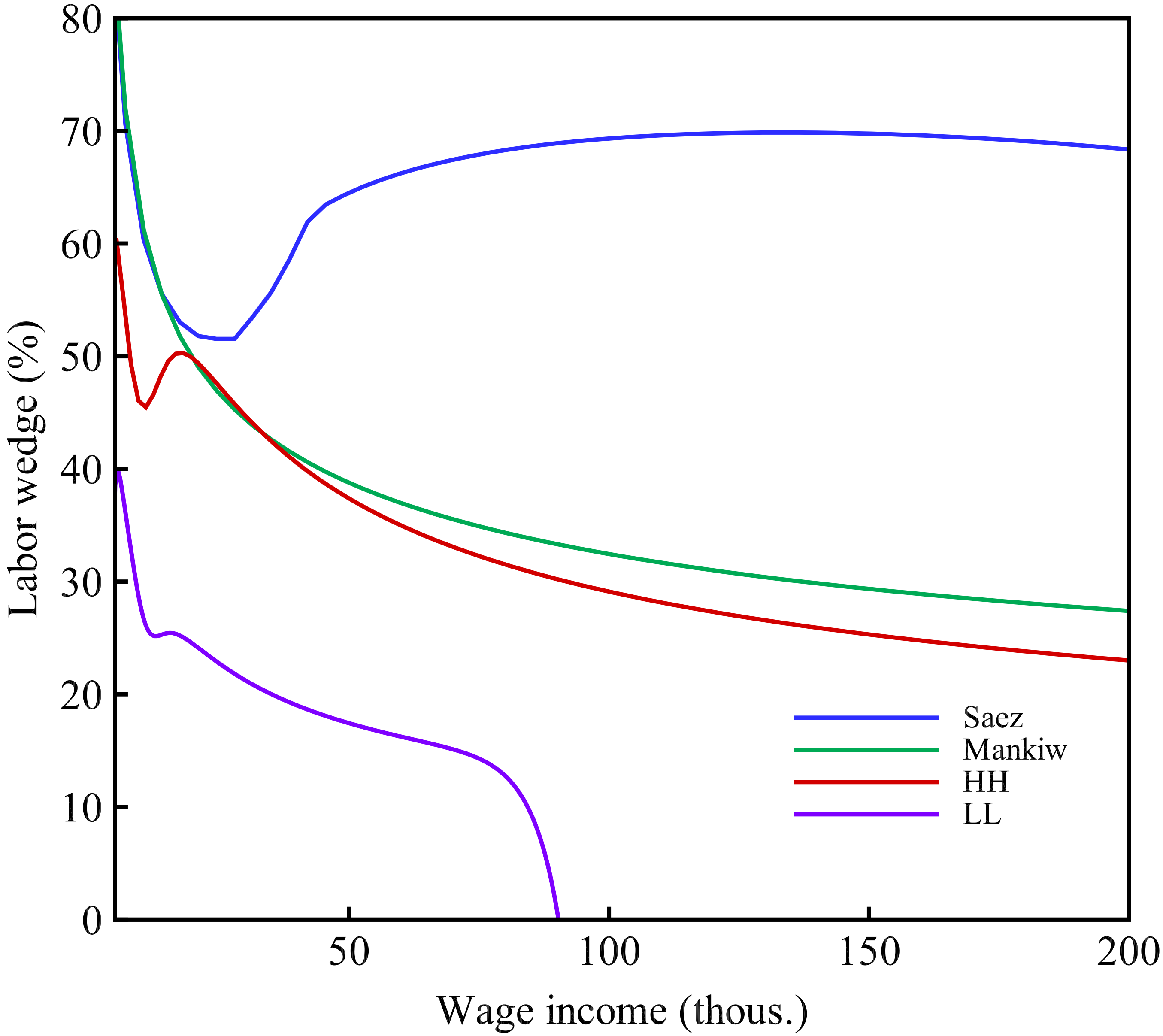
Comparison to Static Problem



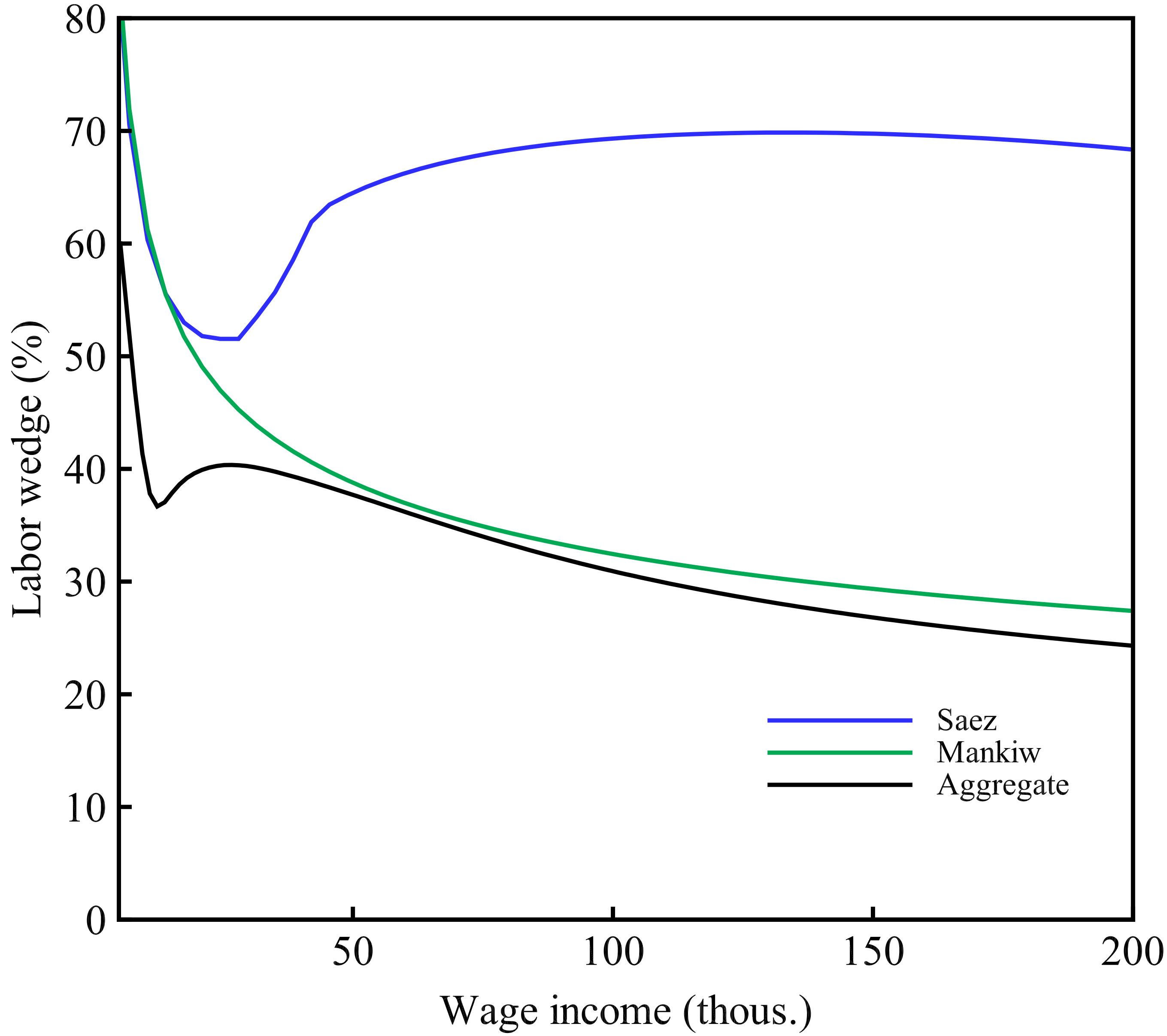
Comparison to Static Problem



Comparison to Static Problem



Comparison to Static Problem



Scaled Planner Problem ($\beta_j = 1 + \beta + \ldots + \beta^{J-j}$)

$$\hat{\Pi}_j(\hat{V}, \hat{\tilde{V}}, \epsilon) \equiv \max \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon) \Big(\frac{1}{\beta_j} \big(w\epsilon_i n_j(\epsilon_i) - c_j(\epsilon_i) \big) + \frac{\beta_{j+1}}{\beta_j} \hat{\Pi}_{j+1}(\hat{V}_j(\epsilon_i), \hat{\tilde{V}}_j(\epsilon_{i+1}), \epsilon_i)/R \Big)$$

$$\text{s.t.} \quad U(c_j(\epsilon_i), \ell_j(\epsilon_i)) + \beta\beta_{j+1}\hat{V}_j(\epsilon_i) \geq U(c_j(\epsilon_{i-1}), \ell_j^{+}(\epsilon_{i-1})) + \beta\beta_{j+1}\hat{\tilde{V}}_j(\epsilon_i), \;\; i \geq 2$$

$$\hat{V} \leq \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon) \left[\frac{1}{\beta_j} U(c_j(\epsilon_i), \ell_j(\epsilon_i)) + \beta \frac{\beta_{j+1}}{\beta_j} \hat{V}_j(\epsilon_i) \right]$$

$$\hat{\tilde{V}} \geq \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon^{+}) \left[\frac{1}{\beta_j} U(c_j(\epsilon_i), \ell_j(\epsilon_i)) + \beta \frac{\beta_{j+1}}{\beta_j} \hat{V}_j(\epsilon_i) \right]$$

Modify State Space

- Map to multiplier grid

- Envelope conditions

$$-\nu_j = \hat{\Pi}_{j,1}(\hat{V}_{-}, \hat{\tilde{V}}_{-}, \epsilon_{-})$$

$$\mu_j = \hat{\Pi}_{j,2}(\hat{V}_{-}, \hat{\tilde{V}}_{-}, \epsilon_{-})$$

- $\hat{V}_{-}$ and $\hat{\tilde{V}}_{-}$ residually determined by FOC

Optimality Conditions and Unknowns $(5I-2)$

$$\pi_j(\epsilon_i|\epsilon_-) = \left(\nu_j\pi_j(\epsilon_i|\epsilon_-) + q_j(\epsilon_i) - \mu_j\pi_j(\epsilon_i|\epsilon_-^+)\right) u_c(c_j(\epsilon_i))$$

$$- q_j(\epsilon_{i+1})u_c(c_j(\epsilon_i))$$

$$w\pi_j(\epsilon_i|\epsilon_-) = \left(\nu_j\pi_j(\epsilon_i|\epsilon_-) + q_j(\epsilon_i) - \mu_j\pi_j(\epsilon_i|\epsilon_-^+)\right) \frac{v_\ell(\ell_j(\epsilon_i))}{\epsilon_i}$$

$$- q_j(\epsilon_{i+1})\frac{v_\ell(\ell_j^+(\epsilon_i))}{\epsilon_{i+1}}$$

$$\nu_{j+1}(\epsilon_i) = \beta R(\nu_j\pi_j(\epsilon_i|\epsilon_-) - \mu_j\pi_j(\epsilon_i|\epsilon_-^+) + q_j(\epsilon_i))/\pi_j(\epsilon_i|\epsilon_-)$$

$$\mu_{j+1}(\epsilon_i) = \beta R q_j(\epsilon_{i+1})/\pi_j(\epsilon_i|\epsilon_-)$$

and the incentive constraints

Newton-Raphson Algorithm

- Guess consumption $\{c_i\}_1^{I-1}$
- Optimality condition $\{c_i\} \to c_N, \{q_i\}$
- Optimality condition $\{\hat{V}_j(\epsilon_i), \hat{\tilde{\tilde{V}}}_j(\epsilon_i)\} \to \{\nu_i, \mu_i\}$
- Optimality condition y_I and incentive constraints $\to \{y_i\}$
- Residual equations are optimality conditions $\{y_i\}_1^{I-1}$

Observe guess in terms of consumption and parallelizable