An Equilibrium Model of the Business Cycle With Household Production and Fiscal Policy

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Abstract

We estimate a dynamic general equilibrium model of the U.S. economy that includes an explicit household production sector and stochastic fiscal variables. We use our estimates to investigate two issues. First, we analyze how well the model accounts for aggregate fluctuations. We find that household production has a significant impact and reject a nested specification in which changes in the home production technology do not matter for market variables. Second, we study the effects of some simple fiscal policy experiments and show that the model generates different predictions for the effects of tax changes than similar models without home production.

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1 Introduction

In this paper, we construct a dynamic general equilibrium model that integrates several formulations from the recent literature, and obtain maximum likelihood estimates of the parameter values using aggregate time series. We then use the model to study business cycles and to analyze the effects of fiscal policy changes. Key features included in the specification are household production, government spending, and distortionary taxation. While all of these features have been shown to be important in isolation, there is good reason to expect that there may be interesting interactions between home production and the fiscal variables. This is the first attempt to analyze, and certainly the first attempt to estimate, a model that combines these features.

In related work, Benhabib et al. (1991) and Greenwood and Hercowitz (1991) show that real business cycle models with explicit household production sectors perform better than the standard real business cycle model (e.g., the base model in Hansen 1985) along several dimensions. Christiano and Eichenbaum (1992) show that introducing government spending also improves the ability of the base model to match certain key observations. Braun (1994) and McGrattan (1994) show how distortionary taxation affects the basic real business cycle model. We think it is important to include taxation and household production in the same model for the following reason. The extent to which shocks to or permanent changes in fiscal variables matter depends on the extent of individuals' opportunities and willingness to substitute in and out of market (i.e., taxed) activities. Explicit modeling of a household or nonmarket sector is an attempt to provide details concerning the nature of this substitution.

Moreover, although previous work shows that adding home production can improve the performance of business cycle models, the extent of the improvement depends critically on several parameters, including the elasticities of substitution between household and market variables in utility and production functions as well as the stochastic properties of the household and market technologies. As emphasized by Kydland (1995), little independent information is available regarding these parameters. When existing studies calibrate their models, they are forced to set some crucial parameters more or less arbitrarily.[1] We therefore estimate these parameters.[2] When we do so, we find that the overall fit of the model is very good and that the estimates are robust to different ways of detrending the data. Also, our specification nests a model without home production that is very similar to the one in McGrattan (1994), and we find that the nested model is rejected.

We ask of our estimated model the same question that economists who calibrate business cycle models typically ask: How well does it account for a set of statistics that is meant to capture the basic business cycle facts? It is by no means a foregone conclusion that our estimated model will do well along this dimension, since the estimation procedure is designed to fit the time series at all frequencies and not just at business cycle frequencies. It turns out that the estimated model performs well. It captures most of the standard business cycle statistics at least as well as other models in the literature, and it captures some of them better. Based on these findings, we would argue that a model with home production should be the benchmark by which other models in this class are compared.

We then use the model to predict the effects of tax changes. We consider several policy experiments to illustrate how our model differs from standard models, including eliminating the capital income tax, eliminating distortionary taxation entirely, and changing the tax treatment of residential capital. We compute changes in steady state levels of consumption, output, capital, labor hours, and welfare that result from these policy experiments. It turns out that for our estimated parameter values, the model predicts that these experiments have very different effects from those predicted by similar models without home production.

The inclusion of home production affects the response of individuals to tax changes to the extent that they are willing to substitute between market and nonmarket activity. Similarly, the business cycle implications of technology or fiscal policy shocks depend on this same willingness. The reason for the above results—that the model accounts for the business cycle better than

[1] Rupert et al. (1995) recently attempted to measure some of the elasticity parameters using micro data.

[2] Previous attempts to estimate similar models using maximum likelihood methods include Christiano (1988), Altug (1989), Leeper and Sims (1994), and McGrattan (1994), but none of these studies use models that include household production.

a model without home production and that it predicts different tax effects than a model without home production—is that our estimates imply that individuals have a fairly high elasticity of substitution between market and household consumption goods.

The rest of the paper is organized as follows. In Section 2, we present the model and define equilibrium. In Section 3, we describe the estimation procedure and data. In Section 4, we discuss the empirical results. In Section 5, we focus on business cycle implications. In Section 6, we run some experiments with the home production model and with a non-home production model and compare the results. Section 7 contains some brief summary remarks.

2 The Model

The framework is a discrete-time stochastic growth model with a [0,1] continuum of identical infinitely-lived individuals. Preferences of individuals are defined over stochastic processes for consumption c_t and leisure ℓ_t, as described by the utility function

$$E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t u(c_t, \ell_t), \tag{1}$$

where E_0 is the expectation operator conditional on information available at date 0 and $\tilde{\beta}$ is the discount factor. Momentary utility is assumed to be a constant relative risk aversion transformation of a Cobb-Douglas function:

$$u(c_t, \ell_t) = \frac{(c_t^b \ell_t^{1-b})^{1-\sigma} - 1}{1 - \sigma}. \tag{2}$$

Leisure is given by

$$\ell_t = \bar{h} - h_{mt} - h_{nt}, \tag{3}$$

where $\bar{h}$ is total discretionary time, h_{mt} is hours of market work, and h_{nt} is hours of nonmarket work (home work). Consumption is an aggregate of private consumption c_{pt} and government consumption c_{gt}:

$$c_t = \{a_1 c_{pt}^{b_1} + (1 - a_1) c_{gt}^{b_1}\}^{1/b_1}. \tag{4}$$

Private consumption itself is an aggregate of market consumption c_{mt} and nonmarket consumption c_{nt}:

$$c_{pt} = \{a_2 c_{mt}^{b_2} + (1 - a_2) c_{nt}^{b_2}\}^{1/b_2}. \tag{5}$$

For notational convenience, we combine (2)–(5) to write the momentary utility function as

$$u(c_t, \ell_t) = U(c_{mt}, c_{nt}, c_{gt}, h_{mt}, h_{nt}). \tag{6}$$

The household maximizes utility subject to several constraints. First, it owns capital k_t that it can divide at a point in time between market capital k_{mt} and nonmarket capital k_{nt}:

$$k_t = k_{mt} + k_{nt}. \tag{7}$$

The household combines nonmarket capital with hours to produce the nonmarket good according to the home production function

$$c_{nt} = g(k_{nt}, \mu^t e^{s_{nt}} h_{nt}) = \{a_3 k_{nt}^{b_3} + (1 - a_3)(\mu^t e^{s_{nt}} h_{nt})^{b_3}\}^{1/b_3}, \tag{8}$$

where μ is a trend growth rate and s_{nt} is a stochastic shock, both of which are labor augmenting. Constraint (8) says that home consumption must be produced in the home (i.e., c_{nt} must be consumed by the household, and it cannot be bought or sold on the market).

The household's capital stock evolves according to the law of motion

$$k_{t+1} = (1 - \tilde{\delta}) k_t + i_t, \tag{9}$$

where i_t denotes investment and $\tilde{\delta}$ is the depreciation rate. Capital that is not used in home production is rented to firms in the market. The market variables are constrained according to a standard budget equation

$$c_{mt} + i_t = (1 - \tau_{ht}) w_t h_{mt} + (1 - \tau_{kt}) r_t k_{mt} + \tilde{\delta} \tau_{kt} k_{mt} + T_t, \tag{10}$$

where w_t and r_t are the real wage rate and the rental rate on capital, τ_{ht} and τ_{kt} are (stochastic) taxes on labor and capital income, and T_t is a lump-sum transfer. Note that depreciation of market capital is tax deductible, which is why $\tilde{\delta} \tau_{kt} k_{mt}$ appears on the right-hand side, and that the household has no dividend income, since profits will be zero in equilibrium. Also note that we can always decompose i_t into market plus nonmarket investment $i_t = i_{mt} + i_{nt}$, where $i_{jt} = k_{jt+1} - k_{jt}(1-\tilde{\delta})$ for each sector j.[3]

Taxes in the model are determined by a fiscal authority that faces the budget constraint

$$c_{gt} = H_{mt}w_t\tau_{ht} + K_{mt}r_t\tau_{kt} - \tilde{\delta}K_{mt}\tau_{kt} - T_t, \tag{11}$$

where the upper case variables H_{mt} and K_{mt} denote aggregate (or, equivalently, per capita) values of the corresponding lower case variables. We assume that government consumption is a stochastic process given by

$$c_{gt} = \alpha_t Y_t, \tag{12}$$

where α_t is a random variable and Y_t is aggregate market output. We treat the transfer T_t as a residual that takes on whatever value is necessary to satisfy the government budget constraint at each point in time, given α_t, τ_{ht}, and τ_{kt}.[4] Notice that there is no tax on nonmarket capital. In the United States, residential capital is subject to a property tax, but this tax, as well as interest payments on home mortgages, is deductible against income taxes. We assume here that the net effect is approximately zero, although taxes on home capital are considered explicitly in Section 6.

Next we describe the technology in the market sector. All firms are assumed to have a constant returns to scale production function. With constant returns to scale, the number of firms is not determinate, and so we normalize this number to unity. The production function is given by

$$Y_t = f(K_{mt}, \mu^t e^{s_{mt}} H_{mt}) = \{a_4 K_{mt}^{b_4} + (1-a_4)(\mu^t e^{s_{mt}} H_{mt})^{b_4}\}^{1/b_4}, \tag{13}$$

where μ is the trend growth rate (which is the same as in the household production function) and s_{mt} is a technology shock. At each date, the firm

[3]This decomposition is simply an accounting identity; in one period, the household chooses investment in total capital, and in the next period, it allocates total capital to either market or nonmarket uses. Although we allow capital to move freely across sectors, it is rare that much capital is actually moved in equilibrium, since households typically want to add to both stocks to at least keep pace with depreciation and growth.

[4]We assume that the government balances the budget each period. Although unrealistic, this assumption greatly simplifies the analysis.

hires labor and capital in the market (taking w_t and r_t as given) to maximize instantaneous profit:

$$\Pi_t = Y_t - w_t H_{mt} - r_t K_{mt}. \tag{14}$$

Note that $\Pi_t = 0$ in equilibrium, because of constant returns to scale.

The final aspect of the environment we consider is the stochastic structure. At this point, we present a general specification that will be restricted later. The vector of exogenous stochastic variables at t is given by $s_t = (\alpha_t, s_{mt}, s_{nt}, \tau_{kt}, \tau_{ht})$. We assume that

$$\gamma(L)s_{t+1} = \gamma_0 + \gamma_\epsilon \epsilon_t, \tag{15}$$

where L denotes the lag operator, $\gamma(L) = I - \gamma_1 L \ldots - \gamma_q L^q$, and ϵ_t is a vector with $E\epsilon_t = 0$ and $E\epsilon_t \epsilon_t^{'} = I$. Let $S_t = (s_t, s_{t-1}, \ldots, s_{t-q})$. Then S_t contains all the information that is both available and relevant for forecasting future values of s_t. We write the law of motion for S_t as $S_{t+1} = S(S_t, \epsilon_t)$.

In an equilibrium for this economy, capital, output, consumption, and investment in each sector fluctuate around a trend with growth rate μ, while leisure and hours worked in each sector fluctuate around a constant level. It is important for our computational and econometric methods that we work with series that are stationary. Hence, we transform the economy by replacing μ with 1 in (8) and (13), replacing the discount factor $\tilde{\beta}$ with $\beta = \tilde{\beta}\mu^{b(1-\sigma)}$, and replacing the law of motion for capital in (9) with $k_{t+1} = (1-\delta)k_t + i_t/\mu$, where $\delta = 1 - (1-\tilde{\delta})/\mu$. In what follows, we treat the transformed economy as the fundamental unit of analysis, since it generates stationary outcomes. The behavior of capital, output, consumption, and investment in the original model can be obtained from the same series for the transformed economy simply by adding geometric growth at rate μ.

We now describe a recursive competitive equilibrium for the transformed economy. There are two aggregate state variables, K_t and S_t, and one individual state variable, k_t. The individual household chooses a decision vector $d_t = (h_{mt}, h_{nt}, k_{mt}, k_{nt}, i_t)$, taking as given the aggregate decision vector as a function of the aggregate state $D_t = D(K_t, S_t)$, where $D_t = (H_{mt}, H_{nt}, K_{mt}, K_{nt}, I_t)$. The individual household also takes as given the laws of motion $S_{t+1} = S(S_t, \epsilon_t)$, $K_{t+1} = K_t(1-\delta) + I_t/\mu$, and $k_{t+1} =$

$k_t(1-\delta) + i_t/\mu$ and the initial condition (k_0, K_0, S_0). Finally, it takes as given the market wage, rental rate, government consumption, and lump-sum transfer as functions of the aggregate state: $w_t = w(K_t, S_t)$, $r_t = r(K_t, S_t)$, $c_{gt} = c_g(K_t, S_t)$, and $T_t = T(K_t, S_t)$. Bellman's equation for the individual household's optimization problem is:

$$V(k_t, K_t, S_t) = \max_{d_t}\{U(c_{mt}, c_{nt}, c_{gt}, h_{mt}, h_{nt}) + \beta E_t V(k_{t+1}, K_{t+1}, S_{t+1})\}, \quad (16)$$

where the maximization is subject to the capital constraint (7), the home production constraint (8), and the budget constraint (10).

The solution to the maximization problem in (16) yields a stationary decision rule for the individual household: $d_t = d(k_t, K_t, S_t)$. This function implies an aggregate decision rule $D_t = D(K_t, S_t)$ which in equilibrium must be consistent with what the individual household takes as given. Also, we can use the necessary and sufficient conditions for the firm's problem,

$$r_t = f_k(K_{mt}, H_{mt}e^{s_{mt}}) \quad (17)$$

$$w_t = f_h(K_{mt}, H_{mt}e^{s_{mt}}), \quad (18)$$

to determine wage and rental rates as functions of (K_t, S_t), which must be consistent with what the individual household takes as given. Finally, we can use

$$c_{gt} = \alpha_t Y_t \quad (19)$$

$$T_t = H_{mt} w_t \tau_{ht} + K_{mt} r_t \tau_{kt} - \tilde{\delta} K_{mt} \tau_{kt} - c_{gt} \quad (20)$$

to determine fiscal policy as functions of (K_t, S_t), which must be consistent with what the individual household takes as given.

These considerations lead us to formally define an equilibrium for our economy as follows:

Definition. A recursive competitive equilibrium is given by a household value function $V(k, K, S)$ and decision rule $d(k, K, S)$, an aggregate decision rule $D(K, S)$, wage and rental rate functions $w = w(K, S)$ and $r = r(K, S)$, and policy functions $c_g(K, S)$ and $T(K, S)$, satisfying:

(a) the dynamic programming problem (16);

(b) the profit maximization conditions (17) and (18);
(c) the fiscal policy equations (19) and (20);
(d) the consistency requirement $D(K, S) = d(K, K, S)$.

3 Estimation Procedure and Data

We use a procedure motivated by Sargent (1989) and described in detail in Anderson et al. (1995) to solve and estimate the model. The reader is referred to those papers for details, and we provide only a brief overview here.

First, we use a deterministic version of the model in which $\epsilon_t \equiv 0$ to find the nonstochastic steady state. Then we substitute the individual household's home production function and budget constraint into its objective function, and we replace wages, rental rates and fiscal variables using conditions (17)–(20) in order to write the household's objective function as a function of (k, K, S, d, K_m, H_m).[5] Following Kydland and Prescott (1982), we then approximate this function using a second order Taylor series around the nonstochastic steady state values of the arguments. This yields a linear-quadratic version of the dynamic programming problem in (16), which generates linear Euler equations and linear decision rules. To find equilibria, we have to equate individual and aggregate variables in the Euler equations and solve for time paths of the variables of interest.[6]

This procedure generates the mapping

$$x_{t+1} = A_o x_t + C w_{t+1}, \tag{21}$$

where $x_t = (k_t, S_t), Ew_t = 0$, and $Ew_t w_t' = I$ for all t. From (21) and the decision rules, we can derive the paths for all the endogenous variables. The elements of A_o are nonlinear functions of the preference and technology parameter vector

$$\Gamma = (\beta, b, \sigma, a_1, b_1, a_2, b_2, a_3, b_3, a_4, b_4, \delta, \mu) \tag{22}$$

[5] Note that we replace wage, rental rate, and fiscal policy variables with functions of aggregate (not individual) values of K_m and H_m.

[6] Because equilibria in our model do not solve a social planning problem, we cannot guarantee ex ante either existence or uniqueness. However, our estimated parameter values satisfy the relevant stability conditions for the existence of a unique equilibrium.

as well as the parameters in the law of motion (15). The elements of C are linear functions of the elements of γ_ϵ. We allow variables to be measured with error.[7] This yields a measurement equation

$$z_t = Gx_t + \upsilon_t, \tag{23}$$

where $z_t = [k_t, h_{mt}, i_t, c_{gt}, y_t, k_{nt}, \tau_{kt}, \tau_{ht}]$ is a vector of observables and υ_t is a vector of measurement errors. We assume that

$$\upsilon_t = D\upsilon_{t-1} + \eta_t, \tag{24}$$

where $E\eta_t = 0$ and $E\eta_t\eta_t^{'} = R$.

As in Anderson et al. (1995), we specify our empirical model in state space form:

$$\begin{aligned} x_{t+1} &= A_o x_t + C\omega_{t+1} \\ \bar{z}_t &= \bar{G}x_t + GC\omega_{t+1} + \eta_{t+1}. \end{aligned} \tag{25}$$

System (25) is obtained by differencing (23) so that $\bar{z}_t = z_{t+1} - Dz_t$, and $\bar{G} = GA_o - DG$. If the disturbances, ω_t and η_t, are normal, then estimation involves maximizing the log-likelihood function (ignoring the constant term)

$$L(\Theta) = -\frac{1}{2}T \ln|\Omega| - \frac{1}{2}\sum_{t=1}^{T} u_t^{'}\Omega^{-1}u_t, \tag{26}$$

where $u_t = z_{t+1} - \hat{E}[z_{t+1}|z_t, z_{t-1}, \ldots, z_0, \hat{x}_0]$ is the innovation in z_{t+1}, $\Omega = Eu_t u_t^{'}$, $\hat{x}_0$ is an initial condition for x (assumed here to be the steady state of the vector of state variables), and Θ includes Γ plus the parameters of the processes in (15) and (24). Anderson et al. (1995) show how to construct the sequence of innovations by applying a Kalman filter to system (25). They also provide derivatives of the log-likelihood function.

Issues that arose during the course of estimation led us to place some additional restrictions on parameters. The first issue concerns the magnitude of measurement errors. Since there are many different estimates for effective marginal tax rates, we anticipated that our tax series were likely to be measured with the greatest error. Initial estimation, however, suggested that

[7] Given the large differences in marginal tax rates across studies, the possibility of measurement error seems particularly important for the tax series.

the tax rates were measured very precisely, whereas quantities (e.g., hours, investment, and capital) were measured with a lot of error.

In order to allow our priors about measurement error to influence the estimation procedure, we impose that the variance of the measurement error for all quantities is at most 5 percent of the total variance of each series. At least in the case of market hours, the data provide some guidance for this magnitude, since there are two series gathered for market hours (the Household Survey series and the Establishment Survey series). If we assume that each of these two series is equal to the true hours plus measurement error that is independent across the two series, then the covariance of the two series provides a measure of the true variance and, hence, an estimate of the measurement error variance. The 5 percent figure is based on this calculation. In contrast, we do not impose any bounds on the magnitude of measurement error in the tax series. While this procedure is obviously somewhat informal, it is a simple way to let the estimation procedure make use of the fact that the measures of marginal tax rates are very noisy.

The second issue concerns the fact that there are some near unit roots in the data for certain variables. We deal with this in two ways. First, we put restrictions on the eigenvalues of the system of exogenous states in (15) and on the eigenvalues of the matrix governing serial correlation in the measurement errors. The eigenvalues of γ are restricted to be less than 0.998 in absolute value. For the matrix D, we restrict its eigenvalues to be less than 0.8.[8] If we allow the eigenvalues of D to be close to 1, then we are essentially first-differencing the data and putting very little weight on sample means. The second way we address the unit root problem is to estimate the model using filtered data. Using filtered data greatly reduces the unit root problem and provides a check on the robustness of our key parameter estimates.

A final issue concerns the specification of the shock to the home production function. We have estimated the model imposing several different restrictions. The results reported in the next section are for the case in which the home technology shock is assumed to be uncorrelated with all other shocks. We have also studied two other cases: one in which we al-

[8]While this choice is arbitrary, our results are not sensitive to a wide range of alternative values.

low for a correlation between the home shock and all other shocks, and one in which we assume that there are no shocks to the home technology. As we describe below, our major conclusions are not affected by the choice of specification for the home technology shock.[9]

We now turn to a description of the data. All series are quarterly for the period 1947–1992, and are in real per capita terms. The series for investment, government consumption, private market consumption, and market output are taken from the National Income and Product Accounts. Investment is defined as fixed investment plus purchases of consumer durables. Private market consumption is defined as the consumption of nondurables and services, excluding the service flow attributed to the housing stock (since we interpret the latter component as nonmarket rather than market consumption). Government consumption is defined as government purchases of goods and services minus purchases of nonmilitary durables and structures. Market output is defined as the sum of investment, government consumption, and private market consumption (hence, it excludes net exports).

Market capital is the net stock of private nonresidential structures and equipment. Nonmarket capital is the net stock of private residential capital and consumer durables. These series are obtained from the U.S. Department of Commerce's *Survey of Current Business* and are annual data interpolated to quarterly data. The market hours series is defined as total manhours for all industries and all employees taken from the Bureau of Labor Statistics' Household Survey. This series is seasonally adjusted monthly data aggregated to quarterly data. The total hours parameter $\bar{h}$ is set to 1,134 hours per quarter (a little over 12 hours per day), which is the measure of discretionary time we took from Hill (1985).

[9]It is important to emphasize that, even if the home technology is nonstochastic, introducing a home production sector into an otherwise standard model leads to very different propagation mechanisms for other shocks. Hence, home production models do not need shocks to the home production function in order to generate different predictions from models without home production. One result in Benhabib et al. (1991) does require a shock to the home sector: the ability of that particular home production model to account for the low correlation between hours worked and productivity found in the data. It is well known that to account for this fact, a model needs to have more than one shock; but the model in this paper already has shocks to government spending and tax rates, so a stochastic home technology is not crucial in this regard.

The tax rates on labor and capital income are constructed using the definitions in Joines (1981). They are annual data interpolated to quarterly data. The Appendix contains the actual series. Sources for these measures are the National Income and Product Accounts and the *Statistics of Income* series of the Internal Revenue Service. The tax rate on labor corresponds to Joines' definition of MTRL1, and the tax rate on capital corresponds to Joines' definition of MTRK1 (excluding property tax revenue). We do not have data on nonmarket hours or nonmarket output. Nonetheless, it is still possible to draw inferences about the parameters of the preferences and technology for home production, because other observable variables depend on these parameters.

4 Empirical Results

In this section, we describe the empirical results and discuss some of their implications. We begin with preference and technology parameters. Estimates and standard errors for these parameters are shown in Table 1a.[10]

The estimates for a_1, b_1, σ, $\tilde{\delta}$, $\tilde{\beta}$, and μ are similar to estimates for the model without home production in McGrattan (1994), and so we discuss these estimates only briefly. The parameter a_1, which measures the weight on private (versus public) consumption in the household utility function, hits an upper bound of 1 and is therefore constrained during estimation. This means that b_1 is not identified and can be set arbitrarily. When $a_1 = 1$, government consumption does not affect the marginal utility of private consumption, as has been simply assumed in some related models (e.g., Christiano and Eichenbaum 1992). The risk aversion parameter σ has a point estimate of 6.783, but it has a large standard error. A log-linear specification for momentary utility is not inconsistent with the estimates. The estimate for the depreciation rate $\tilde{\delta}$ is 0.0214 with a small standard error, and is consistent with results found elsewhere. Table 1a shows an estimate for β of 0.985; the implied estimate of $\tilde{\beta}$ is 1.0018. The estimate of the growth rate μ is 1.0063

[10] In what follows, tables and figures labeled "a" are derived from estimation of the model assuming geometric growth in the raw data; tables and figures labeled "b" are derived from estimation after filtering the data as described below.

with a small standard error. This estimate is in the range of sample growth rates for our capital stock, output, and investment series.

As mentioned earlier, a key parameter governing the interaction between household and market activity is b_2, since the elasticity of substitution between the market and nonmarket consumption goods is $1/(1-b_2)$. Thus, b_2 measures the willingness of agents to substitute between the two goods. For example, if b_2 is equal to 1 then c_m and c_n are perfect substitutes. If b_2 is equal to 0 then c_p is a Cobb-Douglas function of c_m and c_n, so they are complements. Benhabib et al. (1991) show that, under certain assumptions, adding home production to their model has no effect on market variables when b_2 is equal to 0. Our point estimate of b_2 is 0.429, and it is significantly different from zero. It is also in the range of values found by Rupert et al. (1995) when they looked at microeconomic data on different demographic groups (single men, single women, and married couples). A value for b_2 of 0.429 implies a fairly high willingness to substitute between home and market goods, although lower than that assumed by Benhabib et al. (1991), who set b_2 to 0.8 in their preferred model.

We can conduct a formal test of the hypothesis that home production does not matter for market activity as follows. Consider a version of the model in which $b_2 = 0$, $b_3 = 0$, and $\sigma = 1$ (i.e., c_p is a Cobb-Douglas function of c_m and c_n, c_n is a Cobb-Douglas function of k_n and $h_n e^{s_n}$, and u is a log-linear function of c and ℓ). In this case, innovations in s_{nt} do not affect any of the variables in equilibrium except c_{nt}.[11] We test the hypothesis that these restrictions hold against the unconstrained alternative. The hypothesis is easily rejected using a likelihood ratio test: the probability that $\chi^2(3)$

[11]This result can be verified as follows. First, we substitute the equilibrium conditions for w, r, etc., into the first order conditions from (16). We then seek restrictions under which s_n does not affect $d = (h_m, h_n, k_m, k_n, i)$, but only affects c_n. If $b_2 = 0$, s_n drops out of the equation for h_m (because when c_p is a Cobb-Douglas function of c_m and c_n, c_n does not affect the marginal rate of substitution between c_m and h_m). If $b_3 = 0$, s_n drops out of the equations for h_n, k_n, and k_m (because when the home production function is Cobb-Douglas, the desired capital-labor ratios do not depend on s_n). If $\sigma = 1$, s_n drops out of the Euler equations for i (because when the utility function is log-linear, c_n does not affect the marginal utility of c_m), at least as long as s_n does not matter for the expectation of the future values of s_m or the fiscal variables, as we are assuming here. These are exactly the conditions described in the text.

falls below the computed likelihood ratio test statistic is essentially 1. We conclude that changes in relative productivity between the home and market sectors have a significant impact on market variables.

The remaining preference parameters shown in Table 1a are a_2 and b, which measure the weight on market (versus home) consumption and the weight on consumption (versus leisure) in momentary utility. These parameters have implications for the allocation of time between the home and the market. In fact, previous studies have used microeconomic data on time use to draw inferences on a_2 and b. For example, based on cross-sectional data described in Juster and Stafford (1991), Greenwood et al. (1995) report that market work and home work constitute 33 and 25 percent of discretionary time for a typical U.S. household, and they use these figures to calibrate a_2 and b. The mean of our market hours series is 27 percent of $\bar{h}$. We do not have data on home hours, but the model predicts that h_n is about 15 percent of $\bar{h}$ on average.

We next consider estimates of the production functions. First, note that b_4 is not significantly different from zero, so that a Cobb-Douglas market technology is consistent with the estimates. However, the estimate of b_3 is at least 1 standard deviation different from 0, which indicates that the home production function is not Cobb-Douglas. The weight on capital in the market technology, a_4, is 0.220. This weight is similar to estimates found by others (e.g., Christiano and Eichenbaum 1992) who ignore both household production and taxation. These two features tend to have offsetting effects on the estimated value of a_4. On the one hand, adding capital income taxation to a model causes the estimate of a_4 to rise, since capital has to be more productive in order to generate the observed stock if it is subject to taxation (e.g., McGrattan 1994 estimates a_4 to be 0.4). On the other hand, incorporating home production causes the estimate of a_4 to decrease, because a large fraction of observed capital is assumed to be allocated to home production. The net effect of these two considerations is an estimate close to that found by authors who neglect both taxation and household production.

We now turn to the stochastic process for the exogenous state variables, which was assumed to have two lags ($q = 2$) in the estimation exercise. Estimates and standard errors are shown in Figure 1a. The results indicate that the vector autoregressive process for the exogenous states is highly persistent. The estimation procedure attempts to capture low frequency movements in the data either through $\gamma(L)$ or through the measurement error process. Estimates of the measurement error process are shown in Figure 2a. Note that the measurement errors on output and government consumption are assumed to be zero, and that only hours of work and the tax rates are assumed to have serially correlated measurement errors.[12] Trends are particularly evident in the tax rate series, where τ_{ht} increases while τ_{kt} decreases over our sample. This trend causes a near unit root estimate in the labor tax rate process. There is also a trend in the series for h_{mt}. The result is that the eigenvalues of D are at their upper bounds.

Because of these considerations, we also estimate the model using data that were first transformed using the Hodrick-Prescott (HP) filter (see Prescott 1986), thereby eliminating low frequencies. We tried this filter not only because of trends in the market hours and tax rate series, but also because the theoretical model implies that all series that grow must grow at the same geometric rate. Although this assumption may be correct from some very long run perspective, the growth rates do not match perfectly in our relatively short sample. The filtered case provides a check on the robustness of our utility and production function parameter estimates in the sense that we can see if they are sensitive to the low frequency properties of the data. Note that in this estimation exercise, we do use information about sample means, i.e., the data are the filtered series added to their sample means.

Table 1b and Figures 1b and 2b show the results for this case. Notice that the utility and production function parameters are very similar to those shown in Table 1a: the differences in point estimates are less than one standard deviation for all parameters except δ.[13] The main differences between the filtered and unfiltered cases are the estimates of the autoregressive pro-

[12]We have estimated the model without setting the measurement errors on output and government consumption equal to zero. These estimates were small and we therefore set them equal to zero.

[13]For a particular class of loss functions, Diebold, Ohanian, and Berkowitz (1995) prove that parameter estimates are invariant to Hodrick-Prescott filtering the data. While our loss function is not covered by their theoretical results, we still find that the parameter estimates are not significantly affected.

cess and the measurement error equations. In particular, after filtering low frequencies in the data, we do not have difficulties with near unit roots, with the exception of the process for the home shock, which has an eigenvalue at the upper limit of 0.998.[14]

Lastly, we make a few remarks concerning the role of household productivity shocks. The above results show that if we estimate a model with a stochastic home production sector, then we find that home production matters. Two questions arise. The first question is: How important quantitatively are the home technology shocks? The second question is: Is the basic finding that home production matters sensitive to the presence of home technology shocks? Although we do not report the results here in any detail, home shocks are relatively insignificant, accounting for only a small fraction of fluctuations in output for any ordering of the elements of ϵ_t. Regarding the second question, the result that home production matters does not depend critically on the presence of home technology shocks. The reason is simple: what matters in the model is relative productivity differentials across the two sectors, and even if the home sector is deterministic, as long as the market sector is stochastic, there will be fluctuations in relative productivity.

Allowing home shocks to be correlated with other shocks also does not affect our main findings, although for details of this case, refer to McGrattan, Rogerson, and Wright (1993). An additional piece of information generated with this specification is the correlation between home and market technology shocks, a parameter that Benhabib et al. (1991) emphasize in their analysis. Our estimate of this correlation was basically zero, and hence this specification does not differ a lot from the case discussed above, where the correlation is assumed to be zero.

[14]With respect to measurement error variance, we note that in the unfiltered case, the five percent limits were binding for both investment and market hours, whereas in the filtered case the limit was binding for both capital and investment. For parameters at their bounds, we do not report standard errors.

5 Cyclical Implications

It is standard in the real business cycle literature to focus on a small set of summary statistics describing relative volatilities of a few series and their correlations with output. A common question that arises in this context is: To what extent are these particular features being matched at the expense of others that are not being reported? The formal estimation procedure used in the previous section obviously does not place all the weight on this small set of summary statistics, and it is of interest to see what the resulting parameter estimates imply for these commonly studied statistics. Table 2a is based on the estimates obtained for the model with geometric growth estimated on the raw data, and Table 2b is based on the estimates obtained for the HP filtered data. Both the data and the model series are logged before computation of the standard deviations, so these statistics are in percentage terms. The column headed "Model Forecast" indicates that $G\hat{E}[x_t|\bar{z}_{t-1}, \bar{z}_{t-2}, \ldots, \bar{z}_0, \hat{x}_0]$ is used to obtain the model's predictions, where $\hat{x}_0$ is an estimate of the initial state.[15]

Consider Table 2a, which compares the data and the model in the case of geometric growth. Overall, the match is quite good. There are two discrepancies worth noting. One is that the statistics related to the tax rates are off by a fair margin. This is perhaps not surprising since our assumptions lead to relatively large estimates for measurement error in these series. The other major discrepancy is the correlation between market hours and output. In the data, this number is basically zero, whereas in the model it is 0.75. The trend in market hours causes this correlation in the data. This trend in hours is absent from the model, and hence it cannot match this correlation. Note that the two investments are both positively correlated with output in the model, as they are in the data, a feature stressed by Greenwood and Hercowitz (1991).

Now consider Table 2b, which compares the data and the model in the case where estimation uses HP filtered data. Again, the fit seems to be quite good, although in the model, total investment is somewhat too smooth and,

[15] Although the tables show only standard deviations and correlations with output, note that the first moments all match very well.

therefore, market consumption is somewhat too volatile. Overall, we would say that the model does at least as well as a typical real business cycle model in matching the set of second moments on which real business cycle theorists tend to concentrate.

6 Comparison with Standard Model

In this section, we explore in more detail the sense in which a model with home production differs from one without it. We explore this in two ways. First, we compare impulse response functions for models with and without home production. Second, we compare implications of the two models for some standard fiscal policy experiments. In both exercises, the non-home production model is the one in McGrattan (1994), which also obtains maximum likelihood estimates.[16]

6.1 Impulse Responses

There are two reasons why the model with home production may perform better than one without home production. One reason is that the model with home production includes an additional shock (the home shock), and the other reason is that the model with home production may imply different propagation of shocks. In Section 4, we argued that the former is not the case – home shocks account for less than 1 percent of output fluctuations, given our estimates.

A natural comparison of propagation in the two models would be to compute impulse response functions. This comparison, however, turns out not to be very informative, since the estimated feedback effects differ across the two models. Moreover, the feedback effects are very imprecisely estimated. Instead, we compute the responses of both models to univariate forcing processes; i.e., we assume no feedback between the shocks. Table 3 shows the

[16]The model in McGrattan (1994) differs in some minor ways from the model in this paper. For example, it assumes a time-to-build technology and that past leisure affects current utility. These features have little effect on the model's output, which is why we have abstracted from them in this paper.

responses of several variables to one-percent innovations in the market technology, the labor and capital tax rates, and the home technology.[17] The forcing processes are assumed to be first-order autoregressive with a coefficient of 0.95.

An examination of Table 3 shows that the responses are qualitatively similar, but quantitatively different. For example, the home production model predicts larger responses of market output and market hours to innovations in the market technology. This prediction reflects the fact that a home production model allows resources to flow from the home sector to the market sector when market productivity increases. Note that home hours decrease in response to this shock.

For the innovation in labor taxes, market hours respond similarly in the two models, but the decrease in investment is smaller in the home production model. There is a simple explanation for the difference. Both home hours and home capital increase as activity moves from the market to the home sector. Because home capital increases, there is a smaller decrease in investment relative to the non-home production model. A similar pattern occurs with the response of investment to an innovation in the capital tax rate.

We also report the response to innovations in the home shock. Note that market output initially falls but subsequently rises. This response is not surprising, since there are opposing forces at work. On the one hand, the increase in the home technology relative to the market technology suggests that resources move out of the market sector and into the home sector. On the other hand, the improvement in technology causes aggregate capital to increase, and since capital can only be built in the market sector, there is an incentive for additional resources in the market. In any case, note that the implied changes in output are very small.

To summarize, the above analysis indicates that the models with and without home production predict qualitatively similar but quantitatively different responses of market variables to various types of shocks. In particular, the important difference between the home production and non-home production models is not the presence of additional shocks, but rather the

[17]Since the home technology shock has mean of zero, we use a 0.01 deviation for the innovation.

propagation mechanism.

6.2 Fiscal Policy Experiments

In this section, we analyze the effects of three fiscal experiments: eliminating all tax distortions by setting both the labor and the capital tax rates to zero, reducing the capital income tax rate to zero while increasing the labor income tax rate to keep revenue constant, and introducing a tax on nonmarket capital. We would not argue that these experiments are the only ones of interest; our point is simply to show how the predictions for a given policy experiment will differ between our model and a similar model without household production.

For the first two policy experiments, Table 4 shows percent changes in output, consumption, investment, hours worked, and capital in both the home and market sectors, plus a variable Δ that measures the welfare consequences in terms of market consumption. It is interpreted as follows: if, for example, $\Delta = 1$, then agents would be equally well off after the policy change if their market consumption were reduced by 1 percent of its new steady state value, all else being the same. Percentage changes are relative to a base case with $\tau_k = 0.55$ and $\tau_h = 0.24$ (the sample averages in the data). The first experiment sets $\tau_k = \tau_h = 0$, and the second sets $\tau_k = 0$ and τ_h at the level required to keep revenue constant. Government consumption is the same in each case and is set to balance the budget in the base case. In the experiment with $\tau_k = \tau_h = 0$, c_g is financed exclusively via lump-sum taxation.

Based on our parameter estimates, the effect of eliminating distorting taxation altogether is quite sizable: output increases 45.7 percent, market consumption increases 54.5 percent, total investment increases 82.2 percent, the stock of market capital more than doubles, and market hours increase 24.6 percent. In the home sector, consumption decreases 2.1 percent, capital increases 34.9 percent, and hours decrease 19.5 percent. Hence, there is a shift of labor from home to market production, but an increase in capital in both sectors. In terms of welfare, the policy change is worth 30.1 percent of market consumption. A model that ignores the home sector has very different predictions. As shown in the second column of Table 4, the model in McGrattan (1994) predicts for the same policy change an increase in both output and market consumption of about 55 percent and an increase in total capital of 123.5 percent, much larger than in our model. The response of market hours is roughly the same for the two models; but, because home hours change in our model, the implications for leisure are different. In terms of welfare, the policy is worth 27.1 percent of market consumption in the model without home production.[18]

Now consider the effect of eliminating the capital tax and raising the labor tax to keep revenue constant. Given our estimates, the labor tax rate would have to increase from 0.24 to 0.32 percent. This increase is accompanied by a 12.1 percent increase in output, a 6.6 percent increase in market consumption, a 41 percent increase in investment, an 85 percent increase in the market capital stock, and a 4.1 percent decrease in market hours. The welfare gain is 13.5 percent of market consumption. In contrast, the model without home production predicts that eliminating capital taxation requires increasing τ_h to 0.37 to keep revenue constant, that output increases 19.4 percent, that market consumption increases 7.3 percent, that investment and the capital stock both increase 71.9 percent, and that market hours decrease 6 percent. The welfare gain is 13.1 percent of consumption. Once again, there are big differences between the models with and without home production.

For the final experiment, we consider adding a tax on household capital (an experiment that cannot be conducted in a model without home production, of course). We replace the individual budget constraint with

$$c_m + i = (1 - \tau_h)wh_m + (1 - \tau_k)rk_m + \tilde{\delta}\tau_k k_m + T - \tau_p k_n$$

where τ_p is a property tax. Jorgenson and Yun (1991) argue that a value of $\tau_p = 0.01$ is realistic when it is interpreted as a residential property tax.[19]

[18] We emphasize that the above calculation is only illustrative in nature. It is meant to indicate that the same calculation in the two frameworks can lead to very different answers. Steady state comparisons measure how much one would be willing to pay to move to a country with a different policy, but not how much one would pay to change an existing policy. Obviously, if one were enacting changes in tax laws starting from an initial steady state, the relevant welfare comparison needs to take into account the transition path. Our point here is merely to show in a simple setting that the answer to a given question is significantly altered by the presence of a home sector.

[19] When we assume an interest rate of 4 percent annually, a tax of 0.01 on the value of

Table 5 shows percent changes for output, consumption, investment, capital, and hours worked in both the home and market sectors, plus our welfare measure for economies with $\tau_p = 0.01$, 0.02, and 0.03 relative to a base case with $\tau_p = 0$. In all cases, we set $\tau_h = 0.24$ and $\tau_k = 0.55$.

With the exception of market consumption, all variables are smaller when τ_p is positive than in the base case. Output produced at home falls 9.4 percent when τ_p is increased to 0.01. At $\tau_p = 0.03$, home production falls 22.1 percent. Not surprisingly, there is a significant decrease in home capital, which is the factor being taxed. Hours in home production decrease, but only slightly. Because home capital is produced in the market, market production also falls when τ_p is increased. Notice that inputs and outputs in the market fall by the same percentages. This follows from the fact that the property tax does not affect the market capital-labor ratio. The tax leads to a large fall in investment, and households increase their market consumption. The subsidy to market consumption needed to leave households indifferent between having $\tau_p > 0$ and $\tau_p = 0$ is given in the last row of Table 5. For $\tau_p = 0.01$, the subsidy required is 4.3 percent of market consumption. At $\tau_p = 0.03$, the subsidy required is 11.5 percent.

To summarize, the model can be used to analyze a wide variety of fiscal policy issues. Some, like the effects of a change in the tax treatment of housing, can only be analyzed seriously in a model with an explicit household sector. Others, like the effects of changes in capital taxation, can be analyzed in models without home production, but the predictions change once the household sector is incorporated.

7 Conclusion

In this paper, we evaluate the importance of home production in models of aggregate economic activity by obtaining maximum likelihood estimates of a stochastic growth model with an explicit household sector. The parameter that is most important for the hypothesis that household production affects market activity is the elasticity of substitution between output of the two sectors. Our estimates suggest that there is a significant elasticity of

capital corresponds to a 25 percent tax rate on the flow of services.

substitution between home and market goods. Furthermore, we can reject restrictions on the model that imply that market activity is not affected by changes in relative productivity across the two sectors. We also examine how well the estimated model accounts for the standard set of moments on which real business cycle theorists focus, and we use the model to study the effects of fiscal policy changes. We find that including home production significantly affects the model's predictions for these issues. There is a variety of other issues for which the addition of an explicit home production sector may make a difference. In such cases, the model and the parameter estimates in this paper may prove useful.

Appendix

The effective tax rates for labor and capital used to estimate the model are given in Table A. The data sources for these series are *Statistics of Income, Individual Income Tax Returns* (Sources of Income and Taxable Income, all returns) and *Social Security Bulletin* (Tables 2a and 4b). The rates are constructed using the definitions in Joines (1981), series MTRK1 and MTRL1. One important difference between $\tau_{k,t}$ in Table A and MTRK1 in Joines is the treatment of property taxes. MTRK1 is the sum of a proportional tax on income that is not specific to capital or labor, a proportional tax on income that is specific to capital, and a nonproportional tax on income that is specific to capital. The proportional tax on income that is specific to capital is simply tax receipts from capital divided by income from capital. We exclude property taxes from both tax receipts and Joines' measure of income, which includes indirect business taxes.

Table A. Effective Tax Rates for Capital and Labor

t	τ_{kt}	τ_{ht}	t	τ_{kt}	τ_{ht}	t	τ_{kt}	τ_{ht}
1947	62.0	20.0	1963	56.2	22.6	1979	53.4	27.3
1948	54.3	17.2	1964	54.2	21.4	1980	54.3	29.0
1949	51.7	17.2	1965	52.2	20.6	1981	51.5	29.5
1950	63.1	18.3	1966	52.3	21.4	1982	49.3	28.9
1951	66.2	19.8	1967	53.0	21.7	1983	46.9	28.2
1952	62.9	20.9	1968	58.4	23.3	1984	45.2	27.5
1953	63.7	21.0	1969	59.7	24.4	1985	46.2	27.4
1954	59.8	19.5	1970	54.4	24.9	1986	51.7	27.2
1955	58.7	19.6	1971	56.4	23.6	1987	50.5	26.9
1956	60.4	20.2	1972	56.8	24.0	1988	49.4	26.1
1957	59.5	21.0	1973	56.8	24.6	1989	48.6	25.8
1958	59.2	20.8	1974	59.4	25.9	1990	47.3	26.0
1959	59.4	21.4	1975	54.6	25.8	1991	46.7	26.2
1960	59.6	21.7	1976	56.7	25.8	1992	48.2	26.0
1961	59.9	22.0	1977	54.4	26.6			
1962	56.1	22.2	1978	53.3	26.5			

References

Altug, Sumru (1989), "Time-to-Build and Aggregate Fluctuations: Some New Evidence," *International Economic Review,* 30:889-920.

Anderson, Evan W., Lars P. Hansen, Ellen R. McGrattan, and Thomas J. Sargent (1995), "Mechanics of Forming and Estimating Dynamic Linear Economies," in H. Amman, D. Kendrick, and J. Rust, ed. *Handbook of Computational Economics*, North-Holland, forthcoming.

Benhabib, Jess, Richard Rogerson, and Randall Wright (1991), "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," *Journal of Political Economy,* 99:1166-1187.

Braun, R. Anton (1994), "Tax Disturbances and Real Economic Activity in the Postwar United States," *Journal of Monetary Economics,* 33:441-462.

Christiano, Lawrence J. (1988), "Why Does Inventory Investment Fluctuate So Much?" *Journal of Monetary Economics,* 21:247-280.

Christiano, Lawrence J. and Martin Eichenbaum (1992), "Current Real Business Cycle Theories and Aggregate Labor Market Fluctuations," *American Economic Review,* 82:430-450.

Diebold, Francis X., Lee E. Ohanian, and Jeremy Berkowitz (1995), "Dynamic Equilibrium Economies: A Framework for Comparing Models and Data," NBER Technical Working Paper Number 174.

Greenwood, Jeremy and Zvi Hercowitz (1991), "The Allocation of Capital and Time Over the Business Cycle," *Journal of Political Economy,* 99:1188-1214.

Greenwood, Jeremy, Richard Rogerson, and Randall Wright (1995), "Household Production in Real Business Cycle Theory," in *Frontiers of Business Cycle Research,* ed. T.F. Cooley, Princeton: Princeton University Press.

Hansen, Gary D. (1985), "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics,* 16:309-327.

Hill, M. (1985), "Patterns of Time Use," in *Time, Goods and Well-Being,* eds. F.T. Juster and F.P. Stafford, Ann Arbor: University of Michigan Press.

Joines, Douglas H. (1981), "Estimates of Effective Marginal Tax Rates on Factor Incomes," *Journal of Business,* 54:191-226.

Jorgenson, Dale W. and Kun-Young Yun (1991), *Tax Reform and the Cost of Capital,* Oxford: Clarendon Press.

Juster, F. Thomas and Frank P. Stafford (1991), "The Allocation of Time: Empirical Findings, Behavioral Models, and Problems of Measurement," *Journal of Economic Literature,* 29:471-522.

Kydland, Finn E. (1995), "Business Cycles and Aggregate Labor Market Fluctuations," in *Frontiers of Business Cycle Research,* ed. T.F. Cooley, Princeton: Princeton University Press.

Kydland, Finn E. and Edward C. Prescott (1982), "Time to Build and Aggregate Fluctuations," *Econometrica,* 50:1345-1370.

Leeper, Eric and Christopher Sims (1994), "Toward a Modern Macroeconomic Model Usable for Policy Analysis," *NBER Macroeconomics Annual 1994,* eds. S. Fischer and J. Rotemberg, Cambridge: MIT Press.

McGrattan, Ellen R. (1994), "The Macroeconomic Effects of Distortionary Taxation," *Journal of Monetary Economics,* 33:573-601.

McGrattan, Ellen R., Richard Rogerson, and Randall Wright (1993), "Household Production and Taxation in the Stochastic Growth Model," Staff Report 166, Federal Reserve Bank of Minneapolis.

Prescott, Edward C. (1986), "Theory Ahead of Business Cycle Measurement," *Carnegie-Rochester Conference Series on Public Policy,* 25:11-44.

Rupert, Peter, Richard Rogerson, and Randall Wright (1995), "Estimating Substitution Elasticities in Household Production Models," *Economic Theory,* 6:179-193.

Sargent, Thomas J. (1989), "Two Models of Measurements and the Investment Accelerator," *Journal of Political Economy,* 97:251-287.

Table 1a. Parameters of Preferences and Technology
(Geometric Detrending)

Function	Parameter Estimates	
$u(c,l) = \frac{(c^b l^{1-b})^{1-\sigma}}{1-\sigma}$	$b = \underset{(0.233)}{0.466}$	$\sigma = \underset{(5.985)}{6.783}$
$c = \{a_1 c_p^{b_1} + (1-a_1) c_g^{b_1}\}^{1/b_1}$	$a_1 = 1.0$	$b_1 = 0$
$c_p = \{a_2 c_m^{b_2} + (1-a_2) c_n^{b_2}\}^{1/b_2}$	$a_2 = \underset{(0.250)}{0.414}$	$b_2 = \underset{(0.116)}{0.429}$
$c_n = \{a_3 k_n^{b_3} + (1-a_3)(e^{s_n} h_n)^{b_3}\}^{1/b_3}$	$a_3 = \underset{(0.166)}{0.206}$	$b_3 = \underset{(0.123)}{0.189}$
$y = \{a_4 k_m^{b_4} + (1-a_4)(e^{s_m} h_m)^{b_4}\}^{1/b_4}$	$a_4 = \underset{(0.173)}{0.220}$	$b_4 = \underset{(0.270)}{0.053}$
$k' = (1-\tilde{\delta})k/\mu + i/\mu$	$\tilde{\delta} = \underset{(0.00003)}{0.0214}$	$\mu = \underset{(0.0001)}{1.0063}$
β	$\beta = \underset{(0.017)}{0.985}$	

Table 1b. Parameters of Preferences and Technology
(HP Detrending)

Function	Parameter Estimates	
$u(c,l) = \frac{(c^b l^{1-b})^{1-\sigma}}{1-\sigma}$	$b = \underset{(0.173)}{0.489}$	$\sigma = \underset{(4.037)}{5.799}$
$c = \{a_1 c_p^{b_1} + (1-a_1) c_g^{b_1}\}^{1/b_1}$	$a_1 = 1.0$	$b_1 = 0$
$c_p = \{a_2 c_m^{b_2} + (1-a_2) c_n^{b_2}\}^{1/b_2}$	$a_2 = \underset{(0.185)}{0.412}$	$b_2 = \underset{(0.082)}{0.326}$
$c_n = \{a_3 k_n^{b_3} + (1-a_3)(e^{s_n} h_n)^{b_3}\}^{1/b_3}$	$a_3 = \underset{(0.114)}{0.140}$	$b_3 = \underset{(0.066)}{0.269}$
$y = \{a_4 k_m^{b_4} + (1-a_4)(e^{s_m} h_m)^{b_4}\}^{1/b_4}$	$a_4 = \underset{(0.041)}{0.206}$	$b_4 = \underset{(0.083)}{0.054}$
$k' = (1-\tilde{\delta})k/\mu + i/\mu$	$\tilde{\delta} = \underset{(0.00003)}{0.0295}$	$\mu = 1.0$
β	$\beta = \underset{(0.004)}{0.987}$	

Figure 1a. Parameter Estimates of the Vector Autoregressive Process for Exogenous State Variables (Geometric Detrending)

$$s_{t+1} = (\alpha, s_m, s_n, t_k, t_h)'_{t+1} = \begin{pmatrix} \underset{(.0593)}{-.00343} & \underset{(.0954)}{.103} & 0 & \underset{(.0441)}{.0395} & \underset{(.04)}{.0174} \end{pmatrix}$$

$$+ \begin{pmatrix} \underset{(.262)}{1.51} & \underset{(.32)}{.0166} & 0 & \underset{(.603)}{-.0807} & \underset{(.355)}{.446} \\ \underset{(.522)}{.372} & \underset{(.0445)}{.893} & 0 & \underset{(.64)}{.14} & \underset{(.546)}{-.0787} \\ 0 & 0 & \underset{(.201)}{.627} & 0 & 0 \\ \underset{(.203)}{.0185} & \underset{(.208)}{-.0592} & 0 & \underset{(.33)}{1.67} & \underset{(.206)}{.122} \\ \underset{(.185)}{-.0735} & \underset{(.164)}{-.0241} & 0 & \underset{(.309)}{.0445} & \underset{(.112)}{1.33} \end{pmatrix} s_t$$

$$+ \begin{pmatrix} \underset{(.283)}{-.557} & \underset{(.342)}{-.00845} & 0 & \underset{(.612)}{.112} & \underset{(.372)}{-.515} \\ \underset{(.493)}{-.198} & \underset{(.0706)}{.0537} & 0 & \underset{(.672)}{-.158} & \underset{(.547)}{-.1} \\ 0 & 0 & \underset{(.146)}{.0502} & 0 & 0 \\ \underset{(.178)}{.00666} & \underset{(.211)}{.0372} & 0 & \underset{(.333)}{-.667} & \underset{(.218)}{-.155} \\ \underset{(.175)}{.093} & \underset{(.178)}{.0181} & 0 & \underset{(.31)}{-.0369} & \underset{(.124)}{-.392} \end{pmatrix} s_{t-1}$$

$$+ \begin{pmatrix} \underset{(.00304)}{.0107} & 0 & 0 & 0 & 0 \\ \underset{(.00875)}{.00827} & \underset{(.00498)}{.0145} & 0 & 0 & 0 \\ 0 & 0 & \underset{(.0824)}{.0507} & 0 & 0 \\ \underset{(.00471)}{.00469} & \underset{(.00362)}{.000102} & 0 & \underset{(.0032)}{.00716} & 0 \\ \underset{(.00302)}{.00518} & \underset{(.00379)}{.0000983} & 0 & \underset{(.00311)}{.00424} & \underset{(.00162)}{.00471} \end{pmatrix} \epsilon_t$$

Figure 2a. Parameter Estimates of the Measurement Error Equations (Geometric Detrending)

$$D_{ii} = (0,\ .800,\ 0,\ 0,\ 0,\ 0,\ .800,\ .800)$$

$$R_{ii} = \begin{pmatrix} \underset{(62.4)}{1380} & 2.09 & 195 & 0 & 0 & \underset{(105)}{39.7} & \underset{(5.92\times 10^{-5})}{2.16\times 10^{-4}} & \underset{(9.55\times 10^{-5})}{2.23\times 10^{-4}} \end{pmatrix}$$

Figure 1b. Parameter Estimates of the Vector Autoregressive Process for Exogenous State Variables (HP Detrending)

$$s_{t+1} = (\alpha, s_m, s_n, t_k, t_h)'_{t+1} = \begin{pmatrix} \underset{(.0306)}{-.00218} & \underset{(.0664)}{.073} & 0 & \underset{(.0516)}{.0453} & \underset{(.027)}{.0152} \end{pmatrix}$$

$$+ \begin{pmatrix} \underset{(.0947)}{1.31} & \underset{(.0322)}{.0145} & 0 & \underset{(.0775)}{-.0784} & \underset{(.294)}{.267} \\ \underset{(.282)}{.332} & \underset{(.0658)}{.945} & 0 & \underset{(.188)}{.13} & \underset{(.461)}{-.0792} \\ 0 & 0 & .95 & 0 & 0 \\ \underset{(.29)}{.0184} & \underset{(.0572)}{-.0517} & 0 & \underset{(.194)}{1.38} & \underset{(.507)}{.125} \\ \underset{(.127)}{-.0777} & \underset{(.0303)}{-.0211} & 0 & \underset{(.0979)}{.0406} & \underset{(.171)}{1.25} \end{pmatrix} s_t$$

$$+ \begin{pmatrix} \underset{(.0796)}{-.353} & \underset{(.034)}{-.0092} & 0 & \underset{(.0832)}{.127} & \underset{(.319)}{-.422} \\ \underset{(.26)}{-.182} & \underset{(.0612)}{.0436} & 0 & \underset{(.185)}{-.21} & \underset{(.534)}{-.113} \\ 0 & 0 & .0478 & 0 & 0 \\ \underset{(.244)}{.00665} & \underset{(.0565)}{.0443} & 0 & \underset{(.186)}{-.447} & \underset{(.513)}{-.113} \\ \underset{(.0973)}{.0739} & \underset{(.0301)}{.0214} & 0 & \underset{(.0895)}{-.0378} & \underset{(.176)}{-.342} \end{pmatrix} s_{t-1}$$

$$+ \begin{pmatrix} \underset{(.000428)}{.00209} & 0 & 0 & 0 & 0 \\ \underset{(.00132)}{-.00039} & \underset{(.0013)}{.00987} & 0 & 0 & 0 \\ 0 & 0 & \underset{(.0516)}{.0536} & 0 & 0 \\ \underset{(.00127)}{.000835} & \underset{(.000811)}{.0000974} & 0 & \underset{(.0009)}{.00436} & 0 \\ \underset{(.000499)}{.00108} & \underset{(.00037)}{.000125} & 0 & \underset{(.000325)}{.00112} & \underset{(.000233)}{.000866} \end{pmatrix} \epsilon_t$$

Figure 2b. Parameter Estimates of the Measurement Error Equations (HP Detrending)

$$D_{ii} = \begin{pmatrix} .638 & \underset{(.173)}{.401} & .673 & \underset{(1.32)}{.0005} & \underset{(.224)}{.535} & \underset{(25.7)}{.005} & \underset{(.0509)}{.800} & \underset{(.0424)}{.800} \end{pmatrix}$$

$$R_{ii} = \begin{pmatrix} 298 & \underset{(.353)}{.923} & 29.5 & \underset{(19.6)}{1.02} & \underset{(3.80)}{11.5} & \underset{(16.3)}{.0111} & \underset{(1.15\times 10^{-5})}{6.32 \times 10^{-5}} & \underset{(6.6\times 10^{-7})}{2.37 \times 10^{-6}} \end{pmatrix}$$

Table 2a. Standard Deviations and Correlations (Geometric Detrending)

	Data		Model Forecast	
Series (z)	std(ln(z))	corr(z,y)	std(ln(z))	corr(z,y)
Output	14.27	1.00	14.17	1.00
Market consumption	12.99	0.97	12.76	0.98
Total investment	10.25	0.80	10.37	0.80
Market investment	18.59	0.78	22.42	0.67
Home investment	19.27	0.78	19.03	0.77
Government consumption	26.06	0.93	26.03	0.93
Market hours	3.54	−0.01	3.17	0.75
Total capital	6.89	0.91	6.59	0.90
Market capital	6.61	0.91	6.08	0.86
Home capital	7.38	0.89	7.30	0.88
Capital tax rate	9.21	0.85	12.66	0.90
Labor tax rate	14.07	−0.85	17.69	0.69

Table 2b. Standard Deviations and Correlations (HP Detrending)

	Data		Model Forecast	
Series (z)	std(ln(z))	corr(z,y)	std(ln(z))	corr(z,y)
Output	1.87	1.00	1.88	1.00
Market consumption	1.26	0.90	1.56	0.91
Total investment	5.08	0.69	4.63	0.66
Market investment	5.35	0.45	7.73	0.72
Home investment	5.84	0.03	5.84	0.03
Government consumption	4.46	0.46	4.34	0.40
Market hours	1.49	0.69	1.40	0.70
Total capital	0.45	0.67	0.49	0.57
Market capital	0.48	0.44	0.71	0.23
Home capital	0.56	0.67	0.56	0.68
Capital tax rate	3.13	0.23	1.95	0.67
Labor tax rate	2.48	0.26	1.98	0.69

Table 3. Impulse Responses for Home Production (HP) and Non-Home Production (NHP) Model ($s_{t+1} = .95s_t + \epsilon_t$)

Responses to innovation in market technology								
	Output		Investment		Market Hours		Home Hours	
Period	HP	NHP	HP	NHP	HP	NHP	HP	NHP
1	2.22	1.15	3.92	3.16	0.93	0.25	−0.76	–
4	1.97	1.09	3.43	3.47	0.79	0.39	−0.66	–
8	1.69	0.93	2.87	2.27	0.64	0.10	−0.55	–
20	1.08	0.70	1.71	1.43	0.33	−0.01	−0.32	–
40	0.55	0.42	0.77	0.45	0.10	−0.14	−0.13	–
Responses to innovation in labor tax rate								
	Output		Investment		Market Hours		Home Hours	
Period	HP	NHP	HP	NHP	HP	NHP	HP	NHP
1	−0.23	−0.16	−0.21	−0.42	−0.28	−0.26	0.18	–
4	−0.20	−0.15	−0.19	−0.46	−0.24	−0.25	0.16	–
8	−0.17	−0.13	−0.16	−0.30	−0.20	−0.17	0.13	–
20	−0.10	−0.10	−0.09	−0.19	−0.11	−0.08	0.07	–
40	−0.04	−0.06	−0.04	−0.06	−0.04	−0.01	0.03	–
Responses to innovation in capital tax rate								
	Output		Investment		Market Hours		Home Hours	
Period	HP	NHP	HP	NHP	HP	NHP	HP	NHP
1	−0.17	−0.06	−0.29	−0.40	−0.08	−0.09	0.07	–
4	−0.15	−0.06	−0.25	−0.44	−0.07	−0.10	0.06	–
8	−0.13	−0.06	−0.21	−0.29	−0.06	−0.06	0.05	–
20	−0.08	−0.06	−0.13	−0.18	−0.03	−0.02	0.03	–
40	−0.04	−0.04	−0.06	−0.06	−0.01	0.01	0.01	–
Responses to innovation in home technology								
	Output		Investment		Market Hours		Home Hours	
Period	HP	NHP	HP	NHP	HP	NHP	HP	NHP
1	−0.02	–	0.65	–	−0.02	–	0.27	–
4	−0.01	–	0.57	–	−0.02	–	0.23	–
8	0.01	–	0.47	–	−0.01	–	0.18	–
20	0.03	–	0.28	–	−0.01	–	0.10	–
40	0.04	–	0.13	–	−0.01	–	0.03	–

Table 4. Comparison of Steady State Tax Experiments for the Home Production (HP) and Non-Home Production (NHP) Model[20]

	(a) Change to Lump-Sum Tax		(b) Eliminate Capital Tax	
Series	HP	NHP	HP	NHP
Output	45.7	55.3	12.1	19.4
Market consumption	54.5	54.8	6.6	7.3
Total investment	82.2	123.5	41.0	71.9
Market capital	140.3	123.5	85.0	71.9
Market hours	24.6	22.2	−4.1	−6.0
Home consumption	−2.1	0.0	2.3	0.0
Home capital	34.9	0.0	5.3	0.0
Home hours	−19.5	0.0	0.7	0.0
Δ	30.1	27.1	13.5	13.1

Table 5. Steady State Effects of Property Taxes[21]

	Tax on Residential Capital		
Series	.01	.02	.03
Output	−1.0	−1.5	−1.6
Market consumption	3.5	6.1	8.3
Total investment	−13.3	−21.9	−27.9
Market capital	−1.0	−1.5	−1.6
Market hours	−1.0	−1.5	−1.6
Home consumption	−9.4	−16.5	−22.1
Home capital	−23.3	−38.5	−49.1
Home hours	−0.5	−1.2	−1.8
Δ	4.3	8.1	11.5

[20] Numbers are percent changes relative to the steady state with $\tau_k = 0.55$ and $\tau_h = 0.24$.

[21] Numbers are percent changes relative to the base case with the tax on residential capital equal to 0.