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TECHNICAL APPENDIX: Does Neoclassical Theory Account for the Effects of Big Fiscal Shocks? Evidence from World War II*

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* The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

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1. Introduction

In this technical appendix we describe the details of the computation of the benchmark model (Section 2) and the variable capacity utilization model (Section 3). The Fortran codes that accompany the benchmark model are bench.f90/bench39.f90 (used for the perfect foresight examples) and sbench.f90/sbench39.f90 (used for the stochastic examples). The Fortran codes that accompany the variable capacity utilization model are caputil.f90/caputil39.f90 (used for the perfect foresight examples) and scaputil.f90/ scaputil39.f90 (used for the stochastic examples). The "39" in the name indicates simulations are assumed to begin in 1939. These codes along with the data are available at the website of the Minneapolis Federal Reserve and are listed under Staff Report 315. Users can edit these codes and simulate their own examples. Instructions are provided at the website.

The sections that follow describe experiments and results that are not included in the main text. In Section 4, we discuss results for the stochastic case with expectations based on earlier U.S. wars and results for the variable capacity utilization model. In Section 5, we discuss results of our sensitivity analyses. First, we compare the benchmark model starting in 1939 with the benchmark model starting in 1941. Second, we compare results of the benchmark model for alternative labor tax rate series during the war. Third, we discuss how the results change as we vary the postwar tax rates. Fourth, we discuss how the results change as we lower the elasticity of labor supply. Finally, we compare equilibrium outcomes for investment and hours as we vary households' expectations about the fiscal variables and TFP in the postwar period.

2. Benchmark model

In this section we analyze the maximization problem for a stand-in household with civilians and draftees. Total output in the model is the sum of nonmilitary output and military compensation. Nonmilitary output is produced with civilian hours, private capital, and public capital that does not include military equipment or structures. Private and public capital used to produce nonmilitary output are assumed to be perfect substitutes. Here, we do not distinguish the two margins for adjusting the labor input: hours per worker and the fraction employed. Later we will.

2.1. Households

The representative household has two types of family members, civilians and draftees. In period t, fraction $1-a$ are civilians and fraction a are drafted (i.e., in the "army"). Civilians can choose their level of hours but draftees cannot. The problem the household solves is given by[1]

$$\max_{\{c_{ct},c_{dt},i_{pt},l_{ct}\}} E\sum_{t=0}^{\infty}\beta^t \left\{(1-a_t)\,U(c_{ct},l_{ct})+a_tU\left(c_{dt},\bar{l}\right)\right\}(1+\gamma_n)^t$$

subject to

$$(1-a_t)\,c_{ct}+a_tc_{dt}+i_{pt}=(1-\tau_{kt})\,r_tk_{pt}+(1-\tau_{lt})\,(1-a_t)\,w_tl_{ct}+\tau_{kt}\delta k_{pt}+T_t$$

$$k_{pt+1}=[(1-\delta)\,k_{pt}+i_{pt}]/(1+\gamma_n)$$

$$i_{pt}\geq 0 \quad \text{in all states}$$

with processes for a_t, r_t, w_t, τ_{kt}, τ_{lt}, and T_t given. Quantities are in per capita terms and the population grows at rate γ_n. When computing, we assume non-negativity constraints on consumption, leisure, and labor do not bind. We also rely on Ricardian equivalence between transfers and government debt when writing the household budget constraint for the computer code.

From here on, we will assume that $U(c,l)=\log(c)+V(1-l)$ so that the marginal utilities of civilians and draftees are equated. The Lagrangian for the optimization problem in this case is:

$$\begin{aligned}\mathcal{L}=E\sum_t\tilde{\beta}^t\Bigg\{&(1-a_t)\left[\log(\hat{c}_{ct})+V(1-l_{ct})\right]+a_t\log(\hat{c}_{dt})+\frac{\zeta}{3}\min\left(\hat{i}_{pt},0\right)^3\\&+\mu_t\Big\{(1-\tau_{kt})\,r_t\hat{k}_{pt}+(1-\tau_{lt})\,(1-a_t)\,\hat{w}_tl_{ct}+\tau_{kt}\delta\hat{k}_{pt}+\hat{T}_t\\&\qquad-(1-a_t)\,\hat{c}_{ct}-a_t\hat{c}_{dt}-\hat{i}_{pt}\Big\}\\&+\lambda_t\Big\{(1-\delta)\,\hat{k}_{pt}+\hat{i}_{pt}-(1+\gamma_n)\,(1+\gamma_z)\,\hat{k}_{t+1}\Big\}\Bigg\},\end{aligned}$$

where $\tilde{\beta}=(1+\gamma_n)\beta$. Variables that grow over time are detrended and denoted with a hat (e.g., $\hat{c}_{ct}=c_{ct}/(1+\gamma_z)^t$). Note that we have added penalty functions to account for the non-negativity constraint on investment when computing equilibria.

[1] The codes also allow users to impose bounds on consumption as a form of rationing and bounds on labor as a crude way of introducing depression-era restrictions on labor. Interested readers should look at notes on our website.

The first-order conditions are thus:

$$1/\hat{c}_{ct} = \mu_t$$

$$1/\hat{c}_{dt} = \mu_t$$

$$V'\left(1 - l_{ct}\right) = \mu_t \left(1 - \tau_{lt}\right) \hat{w}_t$$

$$\zeta \min\left(\hat{i}_{pt}, 0\right)^2 + \lambda_t = \mu_t$$

$$\left(1+\gamma_n\right)\left(1+\gamma_z\right)\lambda_t = \tilde{\beta} E_t \left[\lambda_{t+1}\left(1-\delta\right) + \mu_{t+1}\left\{\left(1-\tau_{kt+1}\right) r_{t+1} + \tau_k \delta\right\}\right].$$

Equilibrium equations for computation can be summarized as follows:

$$\mu_t = 1/\hat{c}_{pt}$$

$$\mu_t - \zeta \min\left(\hat{i}_{pt}, 0\right)^2 = \hat{\beta} E_t \left[\mu_{t+1}\left\{\left(1-\tau_{kt+1}\right)\left(r_{t+1}-\delta\right)+1\right\} - \left(1-\delta\right)\zeta \min\left(\hat{i}_{pt+1}, 0\right)^2\right]$$

$$V'\left(1 - l_{ct}\right) = \mu_t \left(1 - \tau_{lt}\right) \hat{w}_t,$$

where $\hat{\beta} = \beta/(1+\gamma_z)$. Notice that $\hat{c}_{ct} = \hat{c}_{dt}$ in equilibrium. We let $\hat{c}_{pt}$ represent both later.

2.2. Firms

The firm's problem in t is:

$$\max_{\{K_t, L_{pt}\}} \; F\left(K_t, Z_t L_{pt}\right) - r_t K_t - w_t L_{pt},$$

where L_p is the total labor input (and equal to the fraction of civilians in the population $(1-a)$ times the ratio of total civilian hours to total civilians l_c times the population $(1+\gamma_n)^t$).

In equilibrium, the rental price and the wage rate are given by:

$$r_t = F_1\left(K_t, Z_t L_{pt}\right)$$

$$w_t = F_2\left(K_t, Z_t L_{pt}\right) Z_t.$$

Suppose $F(K,L) = K^\theta L^{1-\theta}$. Then,

$$r_t = \theta K_t^{\theta-1}\left(Z_t L_{pt}\right)^{1-\theta} = \theta \hat{k}_t^{\theta-1}\left(z_t l_{ct}\left(1-a_t\right)\right)^{1-\theta}$$

$$w_t = \left(1-\theta\right) K_t^\theta Z_t^{1-\theta} L_{pt}^{-\theta} = \left(1+\gamma_z\right)^t \left(1-\theta\right) \hat{k}_t^\theta \left(z_t l_{ct}\left(1-a_t\right)\right)^{-\theta}.$$

2.3. Government

The government's period t budget constraint is given by:

$$C_{gt} + I_{gt} + (1+\gamma_n)^t T_t = \tau_{kt}(r_t - \delta) K_{pt} + \tau_{lt} w_t L_{pt} + r_t K_{gt},$$

where we are assuming that wage payments to soldiers are included in transfers to draftees. Spending is therefore:

$$G_t = C_{gt} + I_{gt} + (1+\gamma_n)^t a_t w_t \bar{l},$$

which when detrended for technological growth is

$$\hat{g}_t = \hat{c}_{gt} + \hat{i}_{gt} + a_t \hat{w}_t \bar{l}.$$

Government capital (which is privately operated) is assumed to have the same rate of depreciation as private capital:

$$K_{gt+1} = (1-\delta) K_{gt} + I_{gt}.$$

When normalized this equation becomes

$$(1+\gamma_n)(1+\gamma_z)\hat{k}_{gt+1} = (1-\delta)\hat{k}_{gt} + \hat{i}_{gt}.$$

2.4. Aggregates

Total private consumption, total private hours (which includes nonmilitary government hours), total private investment, and total private capital are given by:

$$\begin{aligned}
C_{pt} &= (1+\gamma_n)^t \left[(1-a_t) c_{ct} + a_t c_{dt}\right] = (1+\gamma_n)^t c_{ct} \\
L_{pt} &= (1+\gamma_n)^t (1-a_t) l_{ct} \\
I_{pt} &= (1+\gamma_n)^t i_{pt} \\
K_{pt} &= (1+\gamma_n)^t k_{pt}.
\end{aligned}$$

The resource constraint for this economy is:

$$C_{pt} + I_{pt} + C_{gt} + I_{gt} = F(K_t, Z_t L_{pt})$$

or

$$\hat{c}_{pt} + \hat{i}_{pt} + \hat{c}_{gt} + \hat{i}_{gt} = F\left(\hat{k}_t, z_t l_{pt}\right).$$

2.5. Exogenous stochastic processes

The exogenous stochastic processes in this economy are $\{a_t, \hat{c}_{gt}, \hat{i}_{gt}, \tau_{kt}, \tau_{lt}, z_t\}$. Let s index the state, where s is determined by a nth-order Markov chain. We assume that at time t if the state is s, then $a_t = a(s)$, $\hat{c}_{gt} = \hat{c}_g(s)$, $\hat{i}_{gt} = \hat{i}_g(s)$, $\tau_{kt} = \tau_k(s)$, $\tau_{lt} = \tau_l(s)$, and $z_t = z(s)$. The process for s is intended to capture different stages of war and/or peace and different levels of technology.

2.6. Steady state

We will use the following functional forms for disutility and production:

$$V(1-l) = \psi[(1-l)^{\xi} - 1]/\xi \tag{2.1}$$

$$F(K, L) = K^{\theta} L^{1-\theta}. \tag{2.2}$$

In this case, the steady state solves

$$\begin{aligned}
r &= [(1+\gamma_z)/\beta - 1]/(1-\tau_k) + \delta \\
\hat{k}_g &= \hat{i}_g/[(1+\gamma_n)(1+\gamma_z) - 1 + \delta] \\
\hat{k}_p &= \hat{i}_p/[(1+\gamma_n)(1+\gamma_z) - 1 + \delta] \\
\hat{y} &= \left(\hat{k}_p + \hat{k}_g\right)^{\theta} (z(1-a) l_c)^{1-\theta} \\
\theta &= r\left(\hat{k}_p + \hat{k}_g\right)/\hat{y} \\
\hat{c}_p &= \hat{y} - \hat{c}_g - \hat{i}_p - \hat{i}_g \\
\psi &= (1-\tau_l)(1-\theta)(1-l_c)^{1-\xi}\, \hat{y}/[\hat{c}_p (1-a) l_c],
\end{aligned}$$

which is 7 equations in 7 unknowns (r, $\hat{k}_g$, $\hat{k}_p$, $\hat{y}$, $\hat{c}_p$, θ, ψ) with $\hat{c}_g$, $\hat{i}_g$, a, τ_k, τ_l, z, γ_n, γ_z, δ, β, ξ, i_p, and l_c given.

2.7. Computational algorithm

When writing the codes, we use x for total capital, i as the index for today's exogenous state, j as the index for tomorrow's exogenous state, I as the total number of exogenous states, and π_{ij} as the probability of transiting from i to j. Here, we describe how we compute the decision function $\hat{c}_p(x,i)$, which is represented as a weighted sum of known basis functions,

$$\hat{c}_p = \sum_{k=1}^{nnodes} \alpha_k^i N_k(x),$$

where the N_k is a "tent" function that takes on nonzero values in 2 elements on the grid over x surrounding node k, that is

$$N_k(x) = \begin{cases} \frac{x - x_{a-1}}{x_a - x_{a-1}} & x_{a-1} \le x \le x_a \\ \frac{x_{a+1} - x}{x_{a+1} - x_a} & x_a \le x \le x_{a+1} \\ 0 & \text{elsewhere.} \end{cases}$$

We apply a finite element method as in McGrattan (1996). This means that we find unknown coefficients α_k, $k = 1, \ldots nnodes$ that satisfy the following equations:

$$\int R(x,i;\alpha) N_k(x)\, dx = 0,$$

for all i and k where

$$R(x,i;\alpha) = \mu - \zeta \min\left(\hat{i}_p, 0\right)^2 + \hat{\beta}(1-\delta)\zeta \sum_j \pi_{i,j} \min\left(\hat{i}'_p, 0\right)^2$$
$$- \hat{\beta} \sum_j \pi_{i,j}\, \mu' \left\{(1 - \tau_k(j))\left[F_1(x', z(j)(1 - a(j)) l'_c) - \delta\right] + 1\right\}$$

and $\alpha = [\alpha_1^1, \ldots, \alpha_{nnodes}^1, \ldots \alpha_{nnodes}^I]'$. The multipliers μ and μ' are

$$\mu = \left(\sum_k \alpha_k^i N_k(x)\right)^{-1}$$

$$\mu' = \left(\sum_k \alpha_k^j N_k(x')\right)^{-1}$$

since $\hat{c}_p = \sum_k \alpha_k^i N_k(x)$. The private investments i_p and i'_p satisfy resource constraints:

$$\hat{i}_p = F(x, z(i)(1 - a(i)) l_c) - \sum_k \alpha_k^i N_k(x) - \hat{c}_g(i) - \hat{i}_g(i)$$

$$\hat{i}'_p = F(x', z(j)(1 - a(j)) l'_c) - \sum_k \alpha_k^j N_k(x') - \hat{c}_g(j) - \hat{i}_g(j).$$

The next period capital stock is given by:

$$x' = \left((1-\delta)x + \left(\hat{i}_p + \hat{i}_g(i)\right)\right) / [(1+\gamma_n)(1+\gamma_z)].$$

The labor inputs l_c and l_c' solve:

$$V'(1-l_c) = \mu(1-\tau_l(i)) F_2(x, z(i)(1-a(i)) l_c) z(i)$$

$$V'(1-l_c') = \mu'(1-\tau_l(j)) F_2(x', z(j)(1-a(j)) l_c') z(j).$$

Derivatives of the residual equation are as follows:

$$\begin{aligned}
\frac{\partial R(x,i;\alpha)}{\partial \alpha_k^i} &= \left(\frac{\partial \mu}{\partial \hat{c}_p} - 2\zeta \min\left(\hat{i}_p, 0\right) \frac{d\hat{i}_p}{d\hat{c}_p}\right) \frac{d\hat{c}_p}{\alpha_k^i} \\
&\quad + \hat{\beta} \sum_j \pi_{i,j} \left\{(1-\tau_k(j)) \left[F_1(x', z(j)(1-a(j)) l_c') - \delta\right] + 1\right\} \frac{d\mu'}{d\hat{c}_p'} \frac{d\hat{c}_p'}{d\alpha_k^i} \\
&\quad - \hat{\beta} \sum_j \pi_{i,j}\, \mu' (1-\tau_k(j)) \\
&\qquad \left(F_{11}(x', z(j)(1-a(j)) l_c') \frac{dx'}{d\alpha_k^i}\right. \\
&\qquad \left. + F_{12}(x', z(j)(1-a(j)) l_c') z(j)(1-a(j)) \frac{dl_c'}{d\alpha_k^i}\right) \\
&\quad + \hat{\beta}(1-\delta)\zeta \min\left(\hat{i}_p, 0\right) \frac{d\hat{i}_p'}{d\alpha_k^i}
\end{aligned}$$

and for $j \neq i$

$$\begin{aligned}
\frac{\partial R(x,i;\alpha)}{\partial \alpha_k^j} &= \hat{\beta}\pi_{i,j} \left\{(1-\tau_k(j)) \left[F_1(x', z(j)(1-a(j)) l_c') - \delta\right] + 1\right\} d\mu' d\hat{c}_p' \frac{d\hat{c}_p'}{d\alpha_k^j} \\
&\quad - \hat{\beta}\pi_{i,j}\, \mu' (1-\tau_k(j)) F_{12}(x', z(j)(1-a(j)) l_c') z(j)(1-a(j)) \frac{dl_c'}{d\alpha_k^j} \\
&\quad + \hat{\beta}(1-\delta)\zeta \min\left(\hat{i}_p, 0\right) \frac{d\hat{i}_p'}{d\alpha_k^j}.
\end{aligned}$$

The partial derivative of μ is

$$\frac{d\mu}{d\hat{c}_p} = -\frac{1}{\hat{c}_p^2}.$$

For the partial derivatives of l_c and l_c', totally differentiate the static first order condition for the consumer to get:

$$-V''(1-l_c)\, dl_c = d\mu(1-\tau_l) F_2 z + \mu(1-\tau_l)\left[F_{12} dx + F_{22} z(1-a)\, dl_c\right] z(1-a),$$

where the arguments of F are $(x, z(1-a)l_c)$. For the current period l_c, $dx = 0$ since x is a given state variable. For l'_c, dx' is given by:

$$\begin{aligned} dx' &= d\hat{i}_p / [(1+\gamma_n)(1+\gamma_z)] \\ &= \left(F_2\left(x, z\left(1-a\right)l_c\right) z\left(1-a\right) dl_c - d\hat{c}_p\right) / [(1+\gamma_n)(1+\gamma_z)]. \end{aligned}$$

Therefore, we have

$$\frac{dl_c}{d\hat{c}_p} = -\frac{\left(1-\tau_l\right) F_2 z}{V'' + \mu\left(1-\tau_l\right) F_{22} z^2 \left(1-a\right)^2} \frac{d\mu}{d\hat{c}_p},$$

where the argument of V is $1 - l_c$ and the arguments of F are $(x, z(i)(1-a(i))l_c)$, and in the next period we have

$$dl'_c = \frac{V'\, d\hat{c}'_p - \left(1-\tau_l\left(j\right)\right) F_{12} z\left(j\right)\left(1-a\left(j\right)\right) dx'}{V''\hat{c}'_p + \left(1-\tau_l\left(j\right)\right) F_{22} z\left(j\right)^2 \left(1-a\left(j\right)\right)^2},$$

where the argument of V is $1-l'_c$ and the arguments of F are $(x', z(j)(1-a(j))l'_c)$. Finally we need

$$\frac{d\hat{c}'_p}{d\alpha^i_k} = \left(\sum_l \alpha^j_l \frac{\partial N_k\left(x'\right)}{\partial x'}\right) \frac{dx'}{d\alpha^i_k}$$

$$\frac{d\hat{c}'_p}{d\alpha^j_k} = N_k\left(x'\right)$$

$$\begin{aligned} \frac{dx'}{d\alpha^i_k} &= \frac{1}{\left(1+\gamma_n\right)\left(1+\gamma_z\right)} \left(F_2\left(x, z\left(1-a\right)l_c\right) z\left(1-a\right) \frac{dl_c}{d\alpha^i_k} - \frac{d\hat{c}_p}{d\alpha^i_k}\right) \\ &= \frac{1}{\left(1+\gamma_n\right)\left(1+\gamma_z\right)} \left(F_2\left(x, z\left(1-a\right)l_c\right) z\left(1-a\right) \frac{dl_c}{d\hat{c}_p} - 1\right) \frac{d\hat{c}_p}{d\alpha^i_k} \\ &= \frac{1}{\left(1+\gamma_n\right)\left(1+\gamma_z\right)} \left(F_2\left(x, z\left(1-a\right)l_c\right) z\left(1-a\right) \frac{dl_c}{d\hat{c}_p} - 1\right) N_k\left(x\right) \end{aligned}$$

$$d\hat{i}'_p = F_1\left(x', z\left(j\right)\left(1-a\left(j\right)\right)l'_c\right) dx' + F_2\left(x', z\left(j\right)\left(1-a\left(j\right)\right)l'_c\right) z\left(j\right)\left(1-a\left(j\right)\right) dl'_c - d\hat{c}'_p.$$

3. A Version of the Model with Capacity Utilization

In this section, we consider the extension of the benchmark model considered by Braun and McGrattan (1993) in their Appendix B.

3.1. Households

The maximization problem of the stand-in household is now:

$$\max_{\{c_{1t},c_{0t},c_{dt},i_{pt},n_t,h_t\}} E\sum_{t=0}^{\infty}\beta^t \Big((1-a_t)\left\{n_t[\log c_{1t}+V(1-h_t)] + (1-n_t)[\log c_{0t}+V(1)] - p(n_t)\right\} + a_t[\log c_{dt}+V(1-\bar{h})]\Big)(1+\gamma_n)^t$$

$$\begin{aligned}
\text{subject to}\quad & (1-a_t)\{n_t c_{1t}+(1-n_t)c_{0t}\}+a_t c_{dt}+i_{pt} \\
&= (1-\tau_{kt})r_t k_{pt}+(1-\tau_{lt})(1-a_t)w_t(h_t)n_t + \tau_{kt}\delta k_{pt}+T_t \\
& k_{pt+1} = [(1-\delta)k_{pt}+i_{pt}]/(1+\gamma_n) \\
& i_{pt}\geq 0 \quad \text{in all states}
\end{aligned}$$

with processes for a_t, r_{pt}, $w_t(h_t)$, τ_{kt}, τ_{lt}, and T_t given. Quantities are in per capita terms. The household chooses consumption of those employed c_{1t}, consumption of those not employed c_{0t}, the consumption of the draftees c_{dt}, the number (or fraction) of those employed n_t, the length of the workweek h_t, and investment i_{pt}.[2] Each term in the budget constraint is in per capita terms. For example, the first term $(1-a_t)n_t c_{1t}$ is

$$\frac{\text{number of civilians}}{\text{total population}}\times\frac{\text{number of civilian workers}}{\text{number of civilians}}\times\frac{\text{consumption of civilian workers}}{\text{number of civilian workers}}.$$

The function $p(n)$ represents costs—in utility terms—of increasing employment or changing its level. If $p(n)=0$ then the equilibrium workweek is constant. (See Braun and McGrattan (1993).) The disutility of employment captured by $p(n_t)$ ensures that both the intensive margin and the extensive margin are used. The disutility of employment term

[2] Note that we could assume that c_{dt} is determined by the government as Braun and McGrattan (1993) do. Since we work with log preferences, whether it is given or not will not affect our results.

was motivated by Braun and McGrattan (1993) as follows. They assume that individual preferences are given by

$$E\sum_{t=0}^{\infty}\beta^t[U\left(c_t,1-h_t\right)-\eta\chi_{\{h_t>0\}}],$$

where η measures the disutility of entering the workforce and χ is an indicator function. If the utility costs of entering the workforce vary, then η will have a nondegenerate distribution. If civilians are aligned with points on the interval [0,1-a), then we can construct a cost function. For example, suppose that individuals are aligned so that costs are linear and increasing. Then, in the aggregate the costs of increasing employment are given by

$$-\left(1-a\right)\int_0^n\left(\zeta_0+2\zeta_1 x\right)dx=-\left(1-a\right)\left(\zeta_0 n+\zeta_1 n^2\right)$$

with $\zeta_1>0$.

The Lagrangian for the optimization problem in this case is:

$$\begin{aligned}\mathcal{L}=E\sum_t\tilde{\beta}^t\Big\{&\left(1-a_t\right)\left\{n_t[\log\left(\hat{c}_{1t}\right)+V\left(1-h_t\right)]+\left(1-n_t\right)\left[\log\hat{c}_{0t}+V\left(1\right)\right]\right\}\\&\qquad+a_t\log\left(\hat{c}_{dt}\right)-\left(1-a_t\right)p\left(n_t\right)+\frac{\zeta}{3}\min\left(\hat{i}_{pt},0\right)^3\\&+\mu_t\Big\{\left(1-\tau_{kt}\right)r_t\hat{k}_{pt}+\left(1-\tau_{lt}\right)\left(1-a_t\right)\hat{w}_t\left(h_t\right)n_t+\tau_{kt}\delta\hat{k}_{pt}+\hat{T}_t\\&\qquad-\left(1-a_t\right)\left\{n_t\hat{c}_{1t}-\left(1-n_t\right)c_{0t}\right\}-a_t\hat{c}_{dt}-\hat{i}_{pt}\Big\}\\&+\lambda_t\Big\{\left(1-\delta\right)\hat{k}_{pt}+\hat{i}_{pt}-\left(1+\gamma_n\right)\left(1+\gamma_z\right)\hat{k}_{t+1}\Big\}\Big\},\end{aligned}$$

where $\tilde{\beta}=(1+\gamma_n)\beta$. As before, variables that grow over time are detrended and denoted with a hat (e.g., $\hat{c}_{ct}=c_{ct}/(1+\gamma_z)^t$).

The first-order conditions are thus:

$$1/\hat{c}_{1t} = \mu_t$$

$$1/\hat{c}_{0t} = \mu_t$$

$$1/\hat{c}_{dt} = \mu_t$$

$$V'\left(1-h_t\right) = \mu_t\left(1-\tau_{lt}\right)\hat{w}_t'\left(h_t\right)$$

$$\begin{aligned} 0 = {} & \log\hat{c}_{1t} + V\left(1-h_t\right) - \log\hat{c}_{0t} - V\left(1\right) - p'\left(n_t\right) \\ & + \mu_t[\left(1-\tau_{lt}\right)\hat{w}_t\left(h_t\right) - \hat{c}_{1t} + \hat{c}_{0t}] \end{aligned}$$

$$\zeta \min\left(\hat{i}_{pt}, 0\right)^2 + \lambda_t = \mu_t$$

$$\left(1+\gamma_n\right)\left(1+\gamma_z\right)\lambda_t = \tilde{\beta}E_t\left[\lambda_{t+1}\left(1-\delta\right) + \mu_{t+1}\left\{\left(1-\tau_{kt+1}\right)r_{t+1} + \tau_k\delta\right\}\right].$$

Equilibrium equations for computation can be summarized as follows:

$$\mu_t = 1/\hat{c}_{pt}$$

$$\mu_t - \zeta\min\left(\hat{i}_{pt}, 0\right)^2 = \hat{\beta}E_t\left[\mu_{t+1}\left\{\left(1-\tau_{kt+1}\right)\left(r_{t+1}-\delta\right)+1\right\} - \left(1-\delta\right)\zeta\min\left(\hat{i}_{pt+1}, 0\right)^2\right]$$

$$V'\left(1-h_t\right) = \mu_t\left(1-\tau_{lt}\right)\hat{w}_t'\left(h_t\right) \tag{3.1}$$

$$V\left(1-h_t\right) - V\left(1\right) + V'\left(1-h_t\right)\hat{w}_t\left(h_t\right)/\hat{w}_t'\left(h_t\right) = p'\left(n_t\right), \tag{3.2}$$

where $\hat{\beta} = \beta/(1+\gamma_z)$ and $\hat{c}_{pt} = \hat{c}_{1t} = \hat{c}_{0t} = \hat{c}_{dt}$.

3.2. Firms

Let's turn next to the production technologies. There are different technologies each defined by length of the workweek, h,

$$Y_t = Z_t^{1-\theta}K_t^{\theta}N_t^{1-\theta}h_t^{\phi} = z_t^{1-\theta}\hat{k}_t^{\theta}\left(n_t\left(1-a_t\right)\right)^{1-\theta}h_t^{\phi}. \tag{3.3}$$

A firm of type h solves the following maximization problem

$$\max_{K_t, N_t} Y_t - r_tK_t - w_t\left(h_t\right)N_t$$

subject to (3.3). The rental and wage rates are therefore given by

$$r_t = \theta Z_t^{1-\theta} K_t^{\theta-1} N_t^{1-\theta} h_t^{\phi}$$
$$w_t\left(h_t\right) = \left(1-\theta\right) Z_t^{1-\theta} K_t^{\theta} N_t^{-\theta} h_t^{\phi}.$$

The total capital stock K_t and workforce N_t are

$$K_t = \left(1+\gamma_n\right)^t \left(1+\gamma_z\right)^t \hat{k}_t$$
$$N_t = \left(1+\gamma_n\right)^t n_t \left(1-a_t\right).$$

Thus, when we normalize rental rates and wage rates, we have

$$\begin{aligned} r_t &= \theta \hat{k}_t^{\theta-1} \left(z_t n_t \left(1-a_t\right)\right)^{1-\theta} h_t^{\phi} \\ \hat{w}_t\left(h_t\right) &= \left(1-\theta\right) \hat{k}_t^{\theta} z_t^{1-\theta} \left(n_t \left(1-a_t\right)\right)^{-\theta} h_t^{\phi} = w_t\left(h_t\right)\left(1+\gamma_z\right)^t \\ \hat{w}_t'\left(h_t\right) &= \phi \hat{k}_t^{\theta} z_t^{1-\theta} \left(n_t \left(1-a_t\right)\right)^{-\theta} h_t^{\phi-1} \\ &= \phi \hat{w}_t\left(h_t\right) / \left[\left(1-\theta\right) h_t\right] \end{aligned}$$

and $\hat{w}_t(h_t)/\hat{w}_t'(h_t) = (1-\theta)h_t/\phi$. From the perspective of the household, $\hat{w}_t$ is a function of both h_t which they are choosing and r_t which they are not. To see this, note that

$$\begin{aligned} \hat{w}_t\left(h_t; r_t\right) &= \left(1-\theta\right) \left(\frac{\hat{k}_t}{z_t n_t \left(1-a_t\right)}\right)^{\theta} z_t h_t^{\phi} \\ &= \left(1-\theta\right) \left(\frac{r_t}{\theta h_t^{\phi}}\right)^{\frac{\theta}{\theta-1}} z_t h_t^{\phi} \\ &= z_t \left(1-\theta\right) \theta^{\frac{\theta}{1-\theta}} r_t^{\frac{\theta}{\theta-1}} h_t^{\frac{\phi}{1-\theta}-1}. \end{aligned}$$

3.3. Government

The government's period t budget constraint is given by:

$$C_{gt} + I_{gt} + \left(1+\gamma_n\right)^t T_t = \tau_{kt}\left(r_t-\delta\right) K_{pt} + \tau_{lt} w_t\left(h_t\right) N_t + r_t K_{gt},$$

where we are assuming that wage payments to soldiers are included in transfers to draftees. Spending is therefore:

$$G_t = C_{gt} + I_{gt} + \text{military compensation}.$$

3.4. Aggregates

Total private consumption, total private labor input (which includes nonmilitary government), total private investment, and total private capital are given by:

$$\begin{aligned} C_{pt} &= (1+\gamma_n)^t \left[(1-a_t)(n_t c_{1t} + (1-n_t) c_{0t}) + a_t c_{dt}\right] \\ I_{pt} &= (1+\gamma_n)^t i_{pt} \\ K_{pt} &= (1+\gamma_n)^t k_{pt}. \end{aligned}$$

The resource constraint for this economy is:

$$C_{pt} + I_{pt} + C_{gt} + I_{gt} = Z_t^{1-\theta} K_t^{\theta} N_t^{1-\theta} h_t^{\phi}$$

or

$$\hat{c}_{pt} + \hat{i}_{pt} + \hat{c}_{gt} + \hat{i}_{gt} = \hat{k}_t^{\theta} \left(z_t n_t (1-a_t)\right)^{1-\theta} h_t^{\phi}.$$

3.5. Exogenous stochastic processes

The exogenous stochastic processes in this case are the same as in the benchmark model, except for the productivity shock. We use (3.3) in the variable capacity utilization model when constructing our U.S. estimates for productivity z.

3.6. Steady state

We will use the same functional form for V and F as before (see (2.1) and (2.2)), and we will try various functional forms for $p(n)$. At the start, we assume

$$p(n) = \eta[n^{\rho} - 1]/\rho.$$

The steady state for this problem is

$$\begin{aligned} r &= [(1+\gamma_z)/\beta - 1]/(1-\tau_k) + \delta \\ \hat{k}_g &= \hat{i}_g/[(1+\gamma_n)(1+\gamma_z) - 1 + \delta] \\ \hat{k}_p &= \hat{i}_p/[(1+\gamma_n)(1+\gamma_z) - 1 + \delta] \\ \hat{y} &= \left(\hat{k}_p + \hat{k}_g\right)^{\theta} (z(1-a)n)^{1-\theta} h^{\phi} \end{aligned}$$

$$
\begin{aligned}
\hat{c}_p &= \hat{y} - \hat{c}_g - \hat{i}_p - \hat{i}_g \\
\theta &= r\left(\hat{k}_p + \hat{k}_g\right)/\hat{y} \\
\hat{w} &= (1-\theta)\,\hat{y}/\left(n\,(1-a)\right) \\
\psi &= (1-\tau_l)\,\phi\,(1-h)^{1-\xi}\,\hat{w}/[\hat{c}_p h] \\
\eta &= \left(\psi\left((1-h)^{\xi}-1\right)/\xi + (1-\tau_l)\,\hat{w}/\hat{c}_p\right)/n^{\rho-1},
\end{aligned}
$$

which is 9 equations in 9 unknowns (r, $\hat{k}_g$, $\hat{k}_p$, $\hat{y}$, $\hat{c}_p$, θ, $\hat{w}$, ψ, η) with $\hat{c}_g$, $\hat{i}_g$, a, τ_k, τ_l, z, γ_n, γ_z, δ, β, ϕ, ξ, ρ, i_p, h, and n given.

3.7. Computational algorithm

As before, we are computing $c_p(x,i)$, which is represented as a weighted sum of known basis functions,

$$
\hat{c}_p = \sum_{k=1}^{nnodes} \alpha_k^i N_k(x).
$$

The residual in this case is given by:

$$
\begin{aligned}
R(x,i;\alpha) &= \mu - \zeta \min\left(\hat{i}_p, 0\right)^2 + \hat{\beta}(1-\delta)\,\zeta \sum_j \pi_{i,j} \min\left(\hat{i}'_p, 0\right)^2 \\
&\quad - \hat{\beta}\sum_j \pi_{i,j}\,\mu' \left\{(1-\tau_k(j))\left[\theta\,(x')^{\theta-1}\left(z(j)\,(1-a(j))\,n'\right)^{1-\theta}(h')^{\phi} - \delta\right] + 1\right\}
\end{aligned}
$$

and $\alpha = [\alpha_1^1, \ldots, \alpha_{nnodes}^1, \ldots \alpha_{nnodes}^I]'$. The multipliers μ and μ' are

$$
\mu = \left(\sum_k \alpha_k^i N_k(x)\right)^{-1}
$$

$$
\mu' = \left(\sum_k \alpha_k^j N_k(x')\right)^{-1}
$$

since $\hat{c}_p = \sum_k \alpha_k^i N_k(x)$. The private investments i_p and i'_p satisfy resource constraints:

$$
\hat{i}_p = x^{\theta}\left(z(i)\,(1-a(i))\,n\right)^{1-\theta} h^{\phi} - \sum_k \alpha_k^i N_k(x) - \hat{c}_g(i) - \hat{i}_g(i)
$$

$$
\hat{i}'_p = (x')^{\theta}\left(z(j)\,(1-a(j))\,n'\right)^{1-\theta}(h')^{\phi} - \sum_k \alpha_k^j N_k(x') - \hat{c}_g(j) - \hat{i}_g(j).
$$

The next period capital stock is given by:

$$x' = \left(\left(1-\delta\right)x + \left(\hat{i}_p + \hat{i}_g\left(i\right)\right)\right)/[(1+\gamma_n)(1+\gamma_z)].$$

If we have μ, x, and the exogenous variables, then we can solve the following two equations for the two unknowns, hours of work h and the employment level n:

$$V'\left(1-h\right) = \mu\left(1-\tau_l\left(i\right)\right)\phi\left(1-\theta\right)\left(x/n\right)^{\theta} z\left(i\right)^{1-\theta}\left(1-a\left(i\right)\right)^{-\theta} h^{\phi-1}$$

$$0 = V\left(1-h\right) + V'\left(1-h\right)h/\phi - p'\left(n\right).$$

Derivatives of the residual equation are given as follows:

$$\begin{aligned}
\frac{\partial R\left(x,i;\alpha\right)}{\partial\alpha_k^i} = & \left(\frac{\partial\mu}{\partial\hat{c}_p} - 2\zeta\min\left(\hat{i}_p,0\right)\frac{d\hat{i}_p}{d\hat{c}_p}\right)\frac{d\hat{c}_p}{\alpha_k^i} \\
& + \hat{\beta}\sum_j \pi_{i,j}\left\{\left(1-\tau_k\left(j\right)\right)\left[\theta\left(x'\right)^{\theta-1}\left(z\left(j\right)\left(1-a\left(j\right)\right)n'\right)^{1-\theta}\left(h'\right)^{\phi} - \delta\right] + 1\right\}\frac{d\mu'}{d\hat{c}_p'}\frac{d\hat{c}_p'}{d\alpha_k^i} \\
& - \hat{\beta}\sum_j \pi_{i,j}\,\mu'\left(1-\tau_k\left(j\right)\right)\theta \\
& \quad \left(\left(\theta-1\right)\left(x'\right)^{\theta-2}\left(z\left(j\right)\left(1-a\left(j\right)\right)n'\right)^{1-\theta}\left(h'\right)^{\phi}\frac{dx'}{d\alpha_k^i}\right. \\
& \quad + \left(1-\theta\right)\left(x'\right)^{\theta-1}\left(z\left(j\right)\left(1-a\left(j\right)\right)\right)^{1-\theta}\left(n'\right)^{-\theta}\left(h'\right)^{\phi}\frac{dn'}{d\alpha_k^i} \\
& \quad \left. + \phi\left(x'\right)^{\theta-1}\left(z\left(j\right)\left(1-a\left(j\right)\right)n'\right)^{1-\theta}\left(h'\right)^{\phi-1}\frac{dh'}{d\alpha_k^i}\right) \\
& + \hat{\beta}\left(1-\delta\right)\zeta\min\left(\hat{i}_p,0\right)\frac{d\hat{i}_p'}{d\alpha_k^i}
\end{aligned}$$

and for $j \neq i$

$$\begin{aligned}
\frac{\partial R\left(x,i;\alpha\right)}{\partial\alpha_k^j} = & \hat{\beta}\pi_{i,j}\left\{\left(1-\tau_k\left(j\right)\right)\left[\theta\left(x'\right)^{\theta-1}\left(z\left(j\right)\left(1-a\left(j\right)\right)n'\right)^{1-\theta}\left(h'\right)^{\phi} - \delta\right] + 1\right\} d\mu' d\hat{c}_p'\frac{d\hat{c}_p'}{d\alpha_k^j} \\
& - \hat{\beta}\pi_{i,j}\,\mu'\left(1-\tau_k\left(j\right)\right)\theta\left(1-\theta\right)\left(x'\right)^{\theta-1}\left(z\left(j\right)\left(1-a\left(j\right)\right)\right)^{1-\theta}\left(n'\right)^{-\theta}\left(h'\right)^{\phi}\frac{dn'}{d\alpha_k^j} \\
& - \hat{\beta}\pi_{i,j}\,\mu'\left(1-\tau_k\left(j\right)\right)\theta\phi\left(x'\right)^{\theta-1}\left(z\left(j\right)\left(1-a\left(j\right)\right)n'\right)^{1-\theta}\left(h'\right)^{\phi-1}\frac{dh'}{d\alpha_k^j} \\
& + \hat{\beta}\left(1-\delta\right)\zeta\min\left(\hat{i}_p,0\right)\frac{d\hat{i}_p'}{d\alpha_k^j}.
\end{aligned}$$

The partial derivative of μ is

$$\frac{d\mu}{d\hat{c}_p} = -\frac{1}{\hat{c}_p^2}.$$

For the partial derivatives of n, n', h, and h', totally differentiate the static consumer's first order conditions (3.1) and (3.2) to get:

$$\begin{aligned}
&-V''(1-h)\,dh = V'(1-h)\left(d\mu/\mu + \theta dx/x - \theta dn/n + (\phi-1)\,dh/h\right)\\
&-V'(1-h)\,dh + (1-\theta)\,V'(1-h)\,/\phi dh - (1-\theta)\,V''(1-h)\,h/\phi dh\\
&\qquad\qquad = p''(n)\,dn
\end{aligned}$$

where $\kappa = (1-\tau_l)\phi z^{1-\theta}(1-a)^{-\theta}$. For the current period, $dx = 0$ since x is a given state variable. For the next period, dx' is given by:

$$\begin{aligned}
dx' &= d\hat{i}_p/[(1+\gamma_n)(1+\gamma_z)]\\
&= \left(\left(x^{\theta}\left(z(1-a)\,n\right)^{1-\theta}h^{\phi}\right)\left((1-\theta)\,dn/n + \phi dh/h\right) - d\hat{c}_p\right)/[(1+\gamma_n)(1+\gamma_z)].
\end{aligned}$$

Finally we need

$$\frac{d\hat{c}_p'}{d\alpha_k^i} = \left(\sum_l \alpha_l^j \frac{\partial N_k(x')}{\partial x'}\right)\frac{dx'}{d\alpha_k^i}$$

$$\frac{d\hat{c}_p'}{d\alpha_k^j} = N_k(x')$$

$$\frac{dx'}{d\alpha_k^i} = \frac{1}{(1+\gamma_n)(1+\gamma_z)}\left(x^{\theta}\left(z(1-a)\,n\right)^{1-\theta}h^{\phi}\left[\frac{(1-\theta)}{n}\frac{dn}{d\alpha_k^i} + \frac{\phi}{h}\frac{dh}{d\alpha_k^i}\right] - \frac{d\hat{c}_p}{d\alpha_k^i}\right)$$

$$d\hat{i}_p' = (x')^{\theta}\left(z(j)\,(1-a(j))\,n'\right)^{1-\theta}(h')^{\phi}\,\left[\theta dx'/x' + (1-\theta)\,dn'/n' + \phi dh'/h'\right] - d\hat{c}_p'.$$

4. Results

In this section, we describe results that are not reported in the main text. The first set of results is for the stochastic version of the benchmark model. The second set of results is for the model with variable capacity utilization.

4.1. Benchmark stochastic model

For the stochastic model, we need transition matrices, denoted by Φ, for the Markov chain on exogenous variables. We assume that there are a maximum of nine possible states: a state with 1939 values of exogenous variables, a state with 1940 values of exogenous variables, and so on up to 1945, a postwar state with 1946 values of exogenous variables, and a postwar state that is used for counterfactual experiments.

We restricted attention to transition matrices that generated episodes of war and peace consistent with U.S. wars prior to World War II since these prior wars would have been the basis of households' expectations. In order to generate episodes of war and peace that are consistent with the U.S. experience, we imposed the following conditions on the matrices: (*i*) the average duration of war in the range of 2.6 to 4.8 years; (*ii*) the frequency of the outbreak of war in the range of 2.9 percent to 5.3 percent of the time; and (*iii*) the fraction of time spent in war in the range of 10.6 percent to 19.8 percent. The U.S. statistics were at the midpoint of these ranges and the lower end of the ranges is 70 percent of the U.S. observation, while the upper end of the ranges is 130 percent of the U.S. observation. We place no other restrictions on these probabilities so that we can obtain a wide range of expectations and thus check the sensitivity of our results to differences in expectations.

We first generated candidate matrices by drawing each element of Φ independently from the same distribution (and then normalizing columns so they sum to 1). This approach was inefficient because it yielded very few Markov chains that satisfy the above criteria. A much more efficient procedure is to generate probabilities that place relatively more mass on the first off-diagonals of Φ so that the probability of transiting to the next phase of the war is sufficiently high as to obtain more draws that satisfy the above duration and frequency criteria.

In particular, we preset a parameter $\alpha \in [0, .5]$ and then use the following algorithm to set transitions through the states of war, with $n = 1, \ldots, N$ and N large:

- Initialize elements of the candidate transition matrix, $\Phi_n(i, j) = 0$
- For $j = 1, \ldots J - 1$,
  - Draw $\zeta \in \text{uniform}[0, 1]$ and set $\Phi_n(j + 1, j) = \alpha + .5\zeta$

- Draw $J-1$ more uniform random variables on [0,1] for the remaining elements of column j, normalizing them so the probabilities sum to 1

- For $j = J$,
  - Draw $\zeta \in$ uniform$[0,1]$ and set $\Phi_n(J,j) = .9 + .1\zeta$
  - Draw J-1 more uniform random variables on [0,1] for the remaining elements of column 6, normalizing them so the probabilities sum to 1

- Check to see if candidate Φ_n is within 70 percent to 130 percent of U.S. averages for the above criteria. If so, keep. Continue to next n.

With $\alpha = 0.5$, the algorithm allows off-diagonals to range from 0 to 1, with probabilities over 0.5 more likely. To ensure that we span the probability space, we also repeat the procedure for values of α (discretely) chosen between 0 and 0.5. The probabilities that we construct span most of the relevant probability space in the transition matrix.

Following the generation of these transition matrices, we then compute the equilibrium for the model economy for each of the accepted transition matrices. Figures A1–A3 show the results. Each graph presents the U.S. data and regions of color representing the model predictions for all probability matrices satisfying the frequency and duration criteria described earlier. The colors indicate the relative mass of the realizations from the experiments, which are approximately normally distributed. Thus, the redder area toward the midpoint between the bounds indicates relatively more mass, and the bluer area closer to the bounds indicates less mass.

The results have several noteworthy features. One is that the changes in output, consumption, investment, labor, and the marginal products of labor and capital are similar between the benchmark stochastic model and the data; the actual data typically lie between the upper and lower bounds of the stochastic model, and in most cases there are small differences between the highest and lowest model prediction in any year. For example, there is at most a 10 percent difference between the upper and lower bounds for nonmilitary hours during the war. The fact that the marginal products of labor and capital in the model are similar to those in the data is particularly interesting given the statutory government regulations in the labor and capital markets. Our findings thus suggest that wage controls during the war did not have a significant effect on aggregate labor productivity. Similarly, the findings suggest that the interest rate controls on government bonds did not have a significant effect on aggregate capital productivity.

Another noteworthy feature of the results is that the general patterns of the variables in the stochastic model are similar to those in the perfect foresight model. The largest difference between the perfect foresight and the stochastic case is that consumption declines in the stochastic case, compared to the very flat pattern in the deterministic case, and that on average, nonmilitary hours in the stochastic case do not rise as much early in the war. Otherwise, the differences between the perfect foresight case and the stochastic case are quite small. This similarity is surprising, given that the probabilities for the Markov transition matrix typically differed substantially from those in the (degenerate) transition matrix for the perfect foresight case. An interesting implication of the similarity between the perfect foresight model and the stochastic model is that the results are not very sensitive to variations in expectations, provided that the expectations are consistent with historical war episodes.

4.2. Variable capacity utilization model

Figures A4–A7 summarize the main results for the model with variable capacity utilization and perfect foresight expectations. For comparison, we include the results for the benchmark model.

To generate these pictures, we use the same parameters as in the benchmark model (Table 1 of the paper) with the exception of ψ and ξ; here, we use $\psi = .62$, $\xi = -2$ which implies a lower labor elasticity relative to the benchmark case but the same average hours between 1946 and 1960. A lower labor elasticity is chosen in order to generate a quantitatively important role for varying the workweek. Having nontrivial costs $p(n)$ also helps in this dimension so we chose $\eta = 1.3$ and $\rho = 2$. Values of ψ and η imply that the levels for the fraction employed and the hours per worker are consistent with U.S. data. The parameters ξ and ρ govern elasticities of hours and employment. We varied these to see how they affected our results. For technology, we need to also choose ϕ. Here, we set it equal to 1 so that there are no diminishing returns to the workweek.

In the model with variable capacity utilization, we also have a different sequence for TFP than in the benchmark since the production technology is slightly different. However, the procedure for deriving the sequence is the same; we use data on output and capital and labor inputs to determine TFP residually. In the variable capacity utilization case, TFP rises by roughly 10 percent between 1941 and 1944.

The main finding from Figures A4–A6 is that the predictions for GNP, consumption, investment, total hours, nonmilitary hours, the return to capital, and nonmilitary labor productivity are very similar to the benchmark model.

In Figure A7, we see the decomposition of the labor input into hours per worker and the fraction employed. With $p(n) = 0$, the hours per worker is constant. As we increase ρ, and thus the costs to varying employment, we can get a larger response in the hours per worker. However, we were unable to generate as large of a response as is seen in the data (even if we raise ρ from 2 to 10 and adjust the utility parameters to get the levels of n and h consistent with the data). A comparison of the two plots in Figure A5 shows that the model is matching up well on the total labor input because we underpredict the change in hours per worker and overpredict the change in employment.

5. Sensitivity analysis

In this section we describe several computational experiments that we conduct to check the sensitivity of our results. We describe how the results change when we (*i*) start our simulations in 1941; (*ii*) use alternative tax rate series on labor; (*iii*) use alternative tax rates on capital and labor in 1946 and after; (*iv*) use alternative preferences with a much lower labor elasticity; and (*v*) vary the expectations about postwar fiscal variables and TFP.

5.1. Simulations starting in 1941

In Figures A8–A10, we display results for the benchmark model for two choices of initial date: 1939 and 1941. The main difference between these simulations is the initial condition for capital, which is set equal to the U.S. observed levels in the corresponding year, and two fewer years of anticipating the war for the case in which we start in 1941.

Overall, starting in 1941 versus 1939 does not have a big effect on the model predictions during the war. There is a slight shift in the paths of consumption and factor returns, but all other paths are close.

5.2. Alternative labor tax rates

In the benchmark simulations, we used labor tax rates of Joines (1981). In this section, we consider three alternative tax rate series. The first is the rate $\hat{\tau}_t$ described in the main text that implies

$$l_{pt}^{m}\left(\hat{\tau}_t\right) = l_{pt}^{d},$$

where l_{pt}^{m} is nonmilitary hours generated by the model and l_{pt}^{d} is U.S. nonmilitary hours divided by the population over 16. The second is the tax rate that implies that nonmilitary hours in war years are at their trend level. The third is the income tax rate series of Barro and Sahasakul (1986), which mixes taxes on labor and capital. This is the rate that Mulligan (1998) uses in his analysis.

In Figure A11, we display the Joines (1981) measure of labor tax rates used in the baseline experiment and the three alternatives. To match the path of U.S. nonmilitary hours, we need a tax rate series $\hat{\tau}_t$ that is about 15 percent and relatively constant over the period 1939–1946. Otherwise, if households perfectly anticipate higher labor tax rates in the future—as is true for the Joines measure—then they increase their hours early in the period to take advantage of the low tax rates. During the first few years, TFP is also relatively low, but it is a less important factor for labor supply than the tax on labor.

The alternative tax that implies no change in wartime nonmilitary hours rises from roughly 18 percent to 30 percent and falls back to 20 percent by the end of the war. This path is needed to choke off any increase in labor during the war, especially in the peak of the war when government spending is very high.

Finally, the alternative measure based on U.S. data is the Barro and Sahasakul (1986) tax rate, which increases much more rapidly between 1939 and 1945 than the Joines (1981) rate. If we use the Barro-Sahasakul rate in our numerical experiments, then we find a small change in GNP and its components but a significant change in nonmilitary hours and its return. This is shown in Figures A12–A14. For these results, we adjust ψ as in the benchmark parameterization in order to match the level of the U.S. per capita hours series for 1946–1960. Specifically, we set $\psi = 2.24$.

The main difference between these results and those of the benchmark (shown in the figures) is the initial labor supply response. With the Barro-Sahasakul rate, predicted hours are significantly above actual hours in 1939–1941. The reason is that the alternative tax rate series rises by more during the war. Joines' labor tax rate is roughly 12 percent in 1941 and rises to a peak of 19 percent in 1945. Barro and Sahasakul's tax rate is roughly 12 percent in 1941 and rises to a peak of 26 percent in 1945. Most of the rise in the Barro-Sahasakul tax rate is between 1941 and 1942 when the United States enters the war. Thus, most of the difference in the predicted labor response is between 1941 and 1942. High tax rates during the war induce households to increase hours before the war starts. The higher the anticipated rise, the higher the jump in hours.[3]

5.3. Alternative tax rates in 1946

For our benchmark results, we set the postwar tax rates on labor and capital equal to 18.8 percent and 61.7 percent, respectively. (See Figure 1 of the main text.) In this case, the wartime debt is paid off by 1975 in our benchmark numerical simulation. In this section, we discuss the sensitivity of our results to alternative rates (that yield sufficient revenue to retire the accumulated debt).

If we set the labor tax equal to 25 percent for the postwar period, then the debt would have been paid off by 1960. Our predictions for consumption during the war are lower by roughly 1 percentage point and investment in 1943 and 1944 is higher, roughly 7 percent, and closer to the actual investment. Also, the predicted nonmilitary hours series during the war is slightly higher than in the benchmark simulation since the postwar labor tax rate is higher. However, overall the quantitative and qualitative effects are small. If we lower the labor tax to delay paying the debt for 15 additional years (that is, by 1990), we have to set the tax rate slightly above 17 percent. With such a small change in the tax rate, the effects on the wartime series are negligible.

[3] If households are not expecting to enter the war, then the jump is not so large. For example, compare the perfect foresight case with a complete surprise. In the perfect foresight case, hours rise 0.55 percent between 1941 and 1942. In the perfect surprise case, they rise 4.4 percent. In the United States, hours rose 5.8 percent.

To pay off the debt by 1960 with higher tax rates on capital (and the labor tax rate at 18.79) would require tax rates above 90 percent. Such an increase has a large effect on the simulations, especially investment and the after-tax return to capital. A delay until 1990, on the other hand, requires a postwar rate around 54 percent. The quantitative effects for this setting are small.

5.4. Alternative labor elasticity

Above, we discussed tax rates on labor that would be required for the model to induce no increase in nonmilitary hours during the war. Here, we describe a similar exercise that involves preferences. We ask, what labor elasticity is needed to choke off increased hours during the war? The answer is we need the labor supply elasticity to drop by more than a factor of eight relative to the log utility case used in the benchmark model. The elasticity for the log utility case is 2.75 percent compared to 0.32 percent in this experiment.

This elasticity is much too low to account for business cycles. To see this, consider replacing the log utility function in the prototype business cycle models studied by McGrattan (1994) with

$$U(c,l) = \log(c) + \psi (1-l)^{\xi} / \xi.$$

We can set ψ and ξ so as to achieve the same steady-state hours worked and lower labor elasticities. In her benchmark case with technology shocks only and divisible labor, a labor elasticity of 0.5 generates a standard deviation of hours worked equal to 0.3; the standard deviation for U.S. hours is 1.52. (See McGrattan 1994, Table 1.) Similar results are found for her model with taxes. In that case, a labor elasticity of 0.5 generates a standard deviation of hours worked equal to 0.51—again, much lower than that in the data. For a labor elasticity of 0.32, the results are even more striking: the standard deviations of hours worked predicted by the model are in the range of 0.22 to 0.38—significantly below the data. This implies that implausibly low aggregate labor supply elasticities are required in our model to choke off the World War II economic expansion.

5.5. Alternative expectations about the postwar

As a final experiment, we rerun the benchmark model with a slightly different transition matrix over the exogenous states. As in the benchmark, we assume that households have perfect foresight in periods 1939–1944 about the state in the following year. For 1946, however, we assume that households almost surely expect something different from what happened (i.e., with a probability of 0.999). In 1946, they find out that their expectations were incorrect. Figure A15 shows the paths of investment and nonmilitary hours for the different experiments we tried.

If they expect (but do not get) a return to the Great Depression, households invest and work a lot between 1939 and 1945. Notice that the top lines of Figure A15 are significantly higher than the equilibrium paths of the benchmark case. If households expect that fiscal variables will be at levels seen in the 1930s, but TFP will be at its 1946 level, then the impact on investment and hours is negligible. If households assume that spending, the draft, and TFP will be at their 1946 levels, while taxes on factors of production are at the levels of the 1930s, we do find some differences with the benchmark case. However, we view the assumption that households expected to pay off their debts through future lump-sum taxes as less plausible than our benchmark assumption.

6. References

Barro, Robert J., and Chaipat Sahasakul. 1986. Average marginal tax rates from social security and the individual income tax. *Journal of Business* 59(4): 555–566.

Braun, R. Anton and Ellen R. McGrattan. 1993. The macroeconomics of war and peace. In Stanley Fischer and Olivier J. Blanchard, eds., *NBER Macroeconomics Annual 1993*, pp. 197–247. Cambridge, MA: MIT Press.

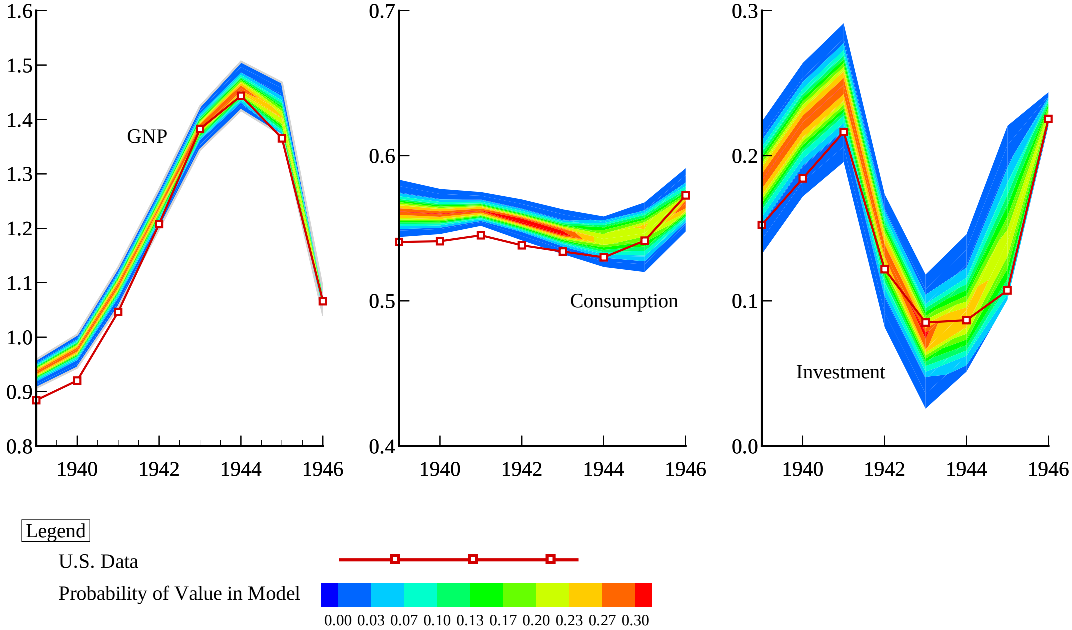
Joines, Douglas H. 1981. Estimates of effective marginal tax rates on factor incomes. *Journal of Business* 54(2): 191–226.

McGrattan, Ellen R. 1994. A progress report on business cycle models. *Federal Reserve Bank of Minneapolis Quarterly Review* 18(4): 2–16.

McGrattan, Ellen R. 1996. Solving the stochastic growth model with a finite element method. *Journal of Economic Dynamics and Control* 20(1–3): 19–42.

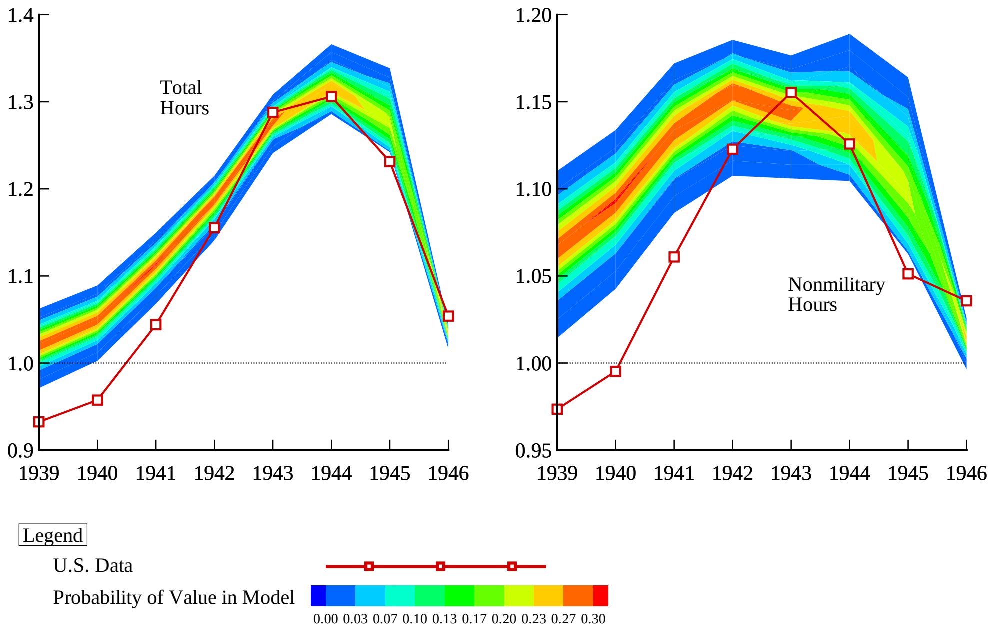
Mulligan, Casey B. 1998. Pecuniary incentives to work in the United States during World War II. *Journal of Political Economy* 106(5): 1033–1077.

Figure A1. Real Detrended GNP, Private Consumption, and Private Investment, 1939–1946
(Benchmark Stochastic Model, Starting 1939)



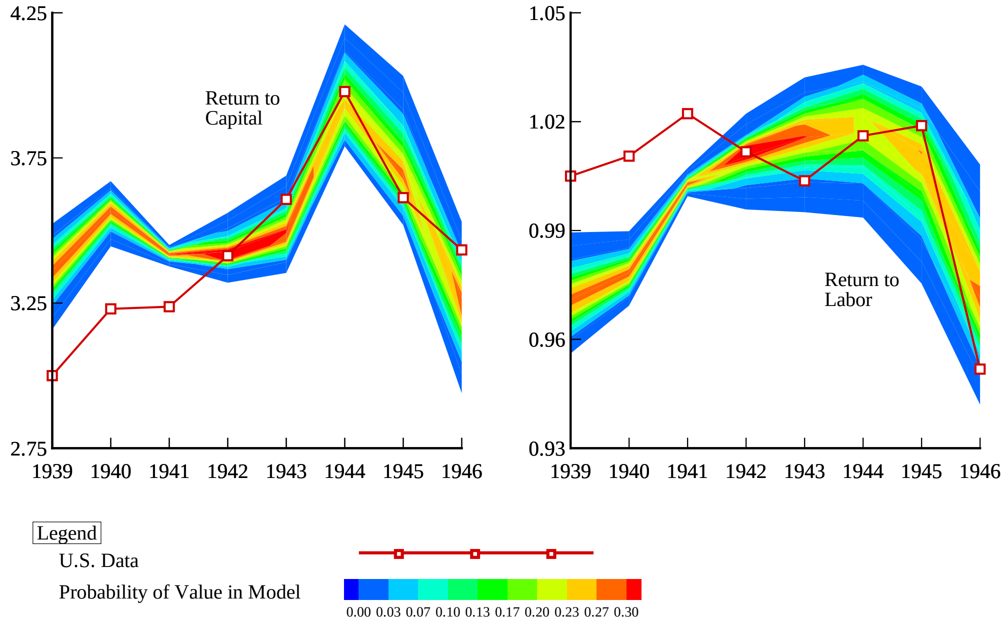
Note: Data series are divided by the 1946 real detrended level of GNP less military compensation.

Figure A2. Per Capita Total and Nonmilitary Hours of Work, 1939–1946
(Benchmark Stochastic Model, Starting 1939)



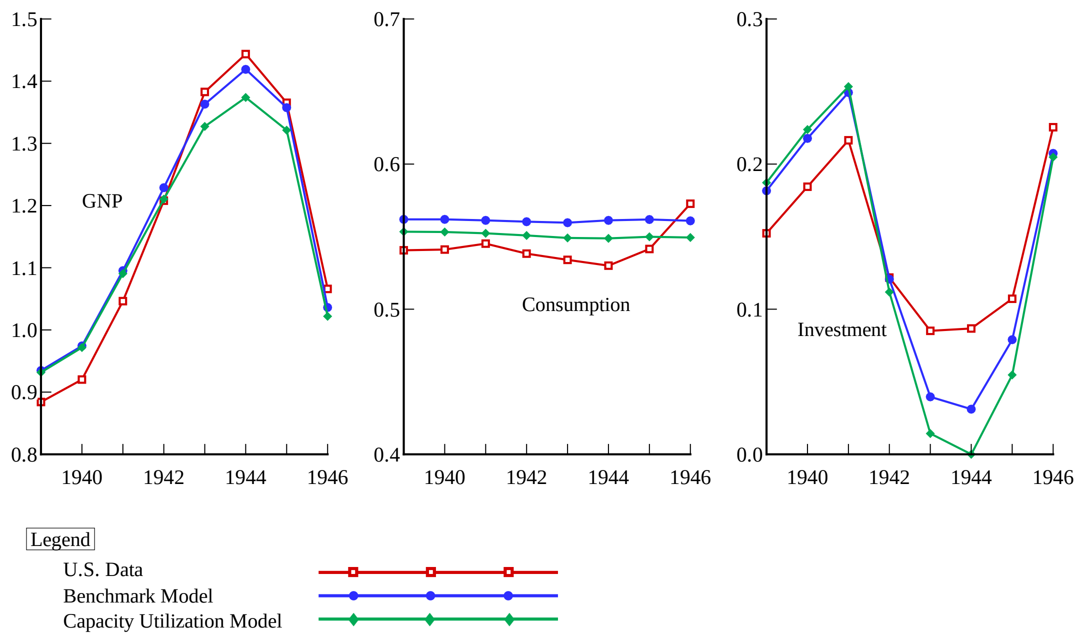
Note: Hours series are divided by the 1946–1960 U.S. averages.

Figure A3. After-tax Return to Capital and Nonmilitary Labor, 1939–1946
(Benchmark Stochastic Model, All Series Constructed Using Marginal Productivities)



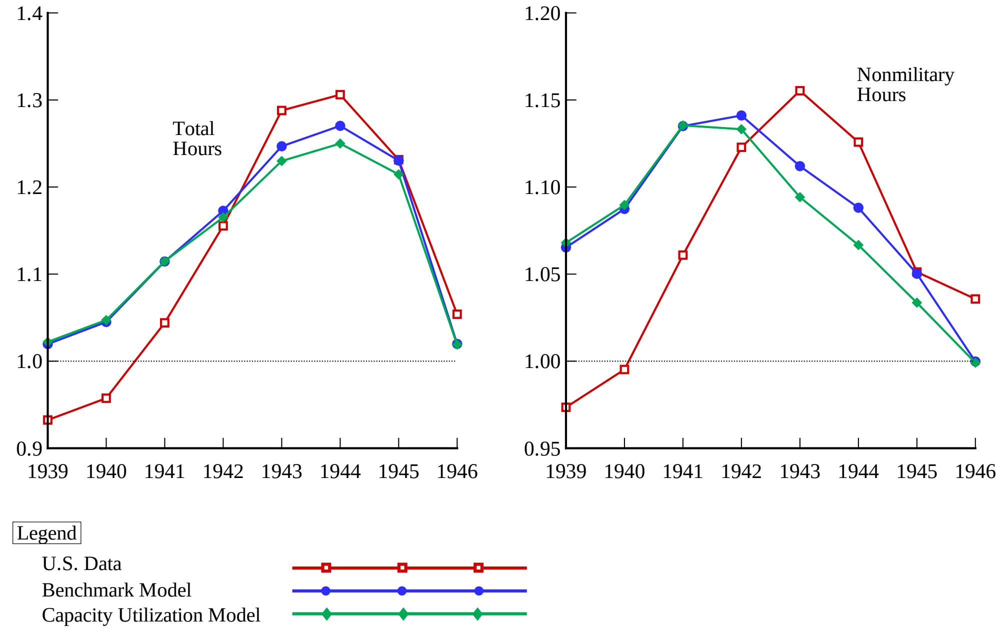
Note: Return to capital is equal to $100(1-\tau_k)(\theta Y/K-\delta)$.
Return to labor is after-tax nonmilitary labor productivity, with hours normalized by the 1946–1960 U.S. average.

Figure A4. Real Detrended GNP, Private Consumption, and Private Investment, 1939–1946
(Benchmark and Capacity Utilization Deterministic Models)



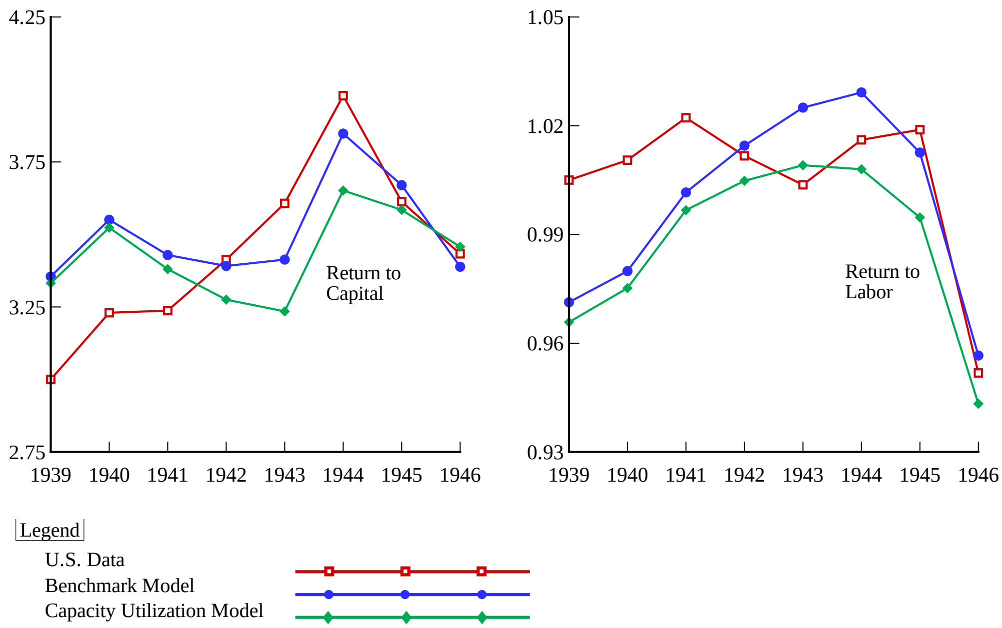
Note: Data series are divided by the 1946 real detrended level of GNP less military compensation.

Figure A5. Per Capita Total and Nonmilitary Hours of Work, 1939–1946
(Benchmark and Capacity Utilization Deterministic Models)



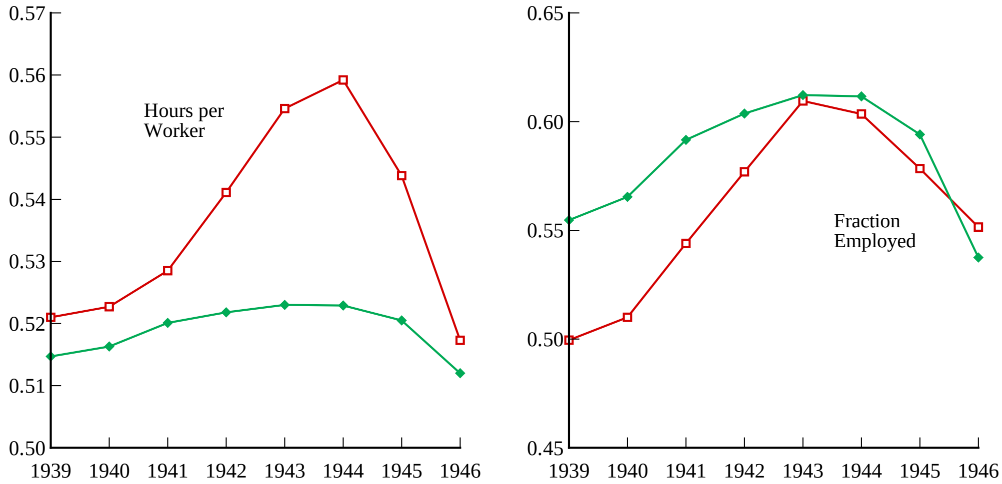
Note: Hours series are divided by the 1946–1960 U.S. averages.

Figure A6. After-tax Returns to Capital and Nonmilitary Labor, 1939–1946
(Benchmark and Capacity Utilization Deterministic Models,
All Series Constructed Using Marginal Productivities)



Note: Return to capital is equal to $100(1-\tau_k)(\theta Y/K-\delta)$.
Return to labor is after-tax nonmilitary labor productivity, with hours normalized by the 1946–1960 U.S. average.

Figure A7. Decomposition of Average Civilian Hours, 1939–1946
(Capacity Utilization Deterministic Model)



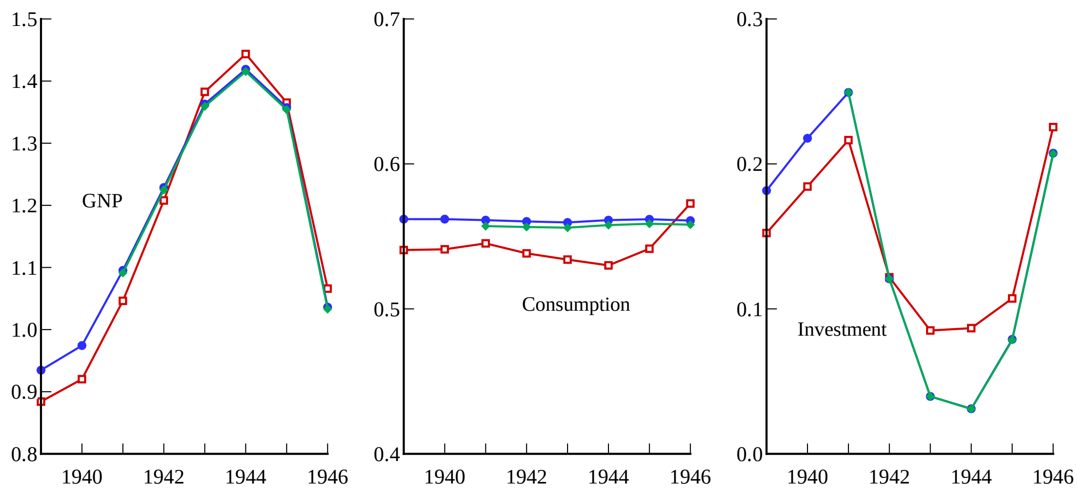
Legend

U.S. Data

Capacity Utilization Model

Note: The product of these two series is the civilian labor input.

Figure A8. Real Detrended GNP, Private Consumption, and Private Investment, 1939–1946
(Benchmark Deterministic Model with Alternative Starting Dates)



Legend

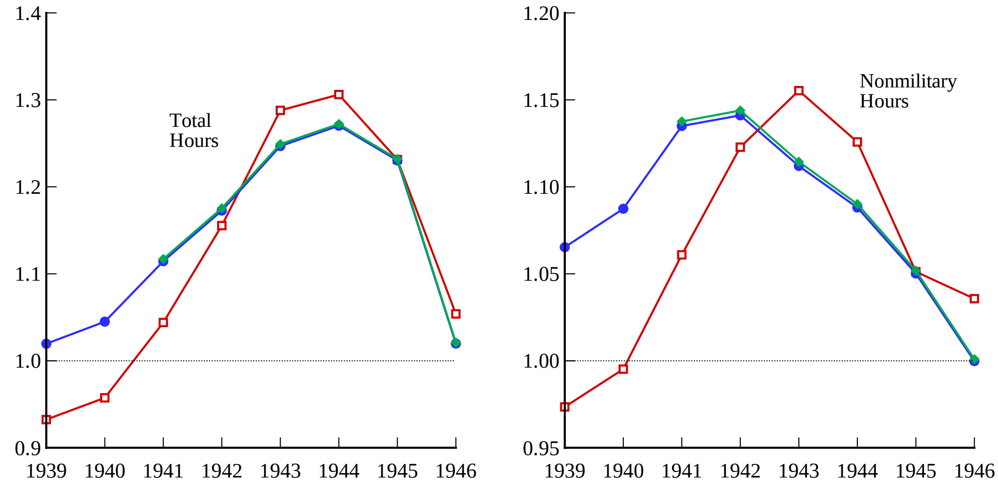
U.S. Data

Benchmark Model, Starting 1939

Benchmark Model, Starting 1941

Note: Data series are divided by the 1946 real detrended level of GNP less military compensation.

Figure A9. Per Capita Total and Nonmilitary Hours of Work, 1939–1946
(Benchmark Deterministic Model with Alternative Starting Dates)



Legend

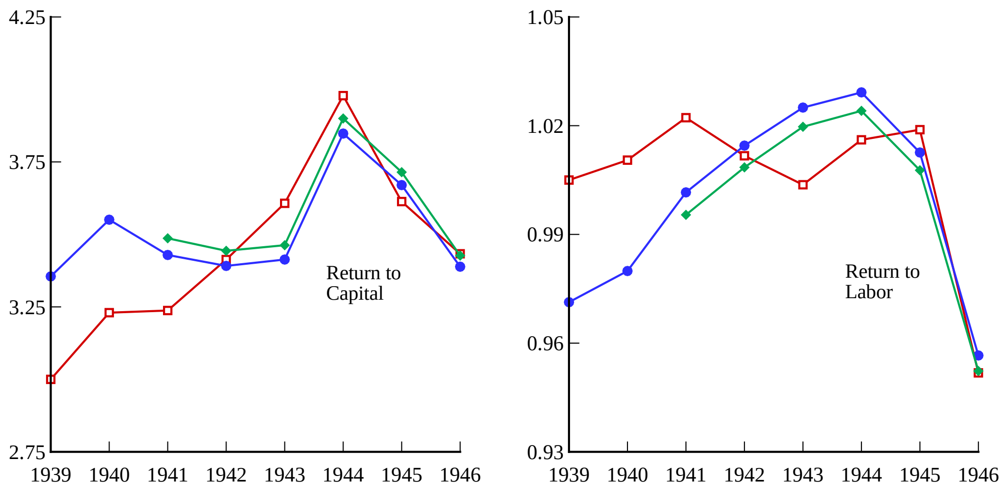
U.S. Data

Benchmark Model, Starting 1939

Benchmark Model, Starting 1941

Note: Hours series are divided by the 1946–1960 U.S. averages.

Figure A10. After-tax Returns to Capital and Nonmilitary Labor, 1939–1946
(Benchmark Deterministic Model with Alternative Starting Dates,
All Series Constructed Using Marginal Productivities)



Legend

U.S. Data

Benchmark Model, Starting 1939

Benchmark Model, Starting 1941

Note: Return to capital is equal to $100(1-\tau_k)(\theta Y/K-\delta)$.
Return to labor is after-tax nonmilitary labor productivity, with hours normalized by the 1946–1960 U.S. average.

Figure A11. Four Measures of the Tax Rate on Labor, 1939–1946

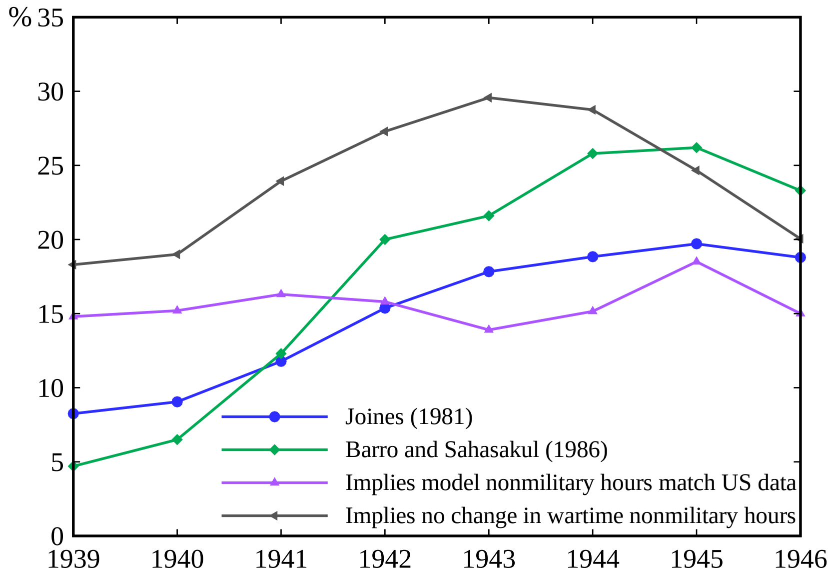
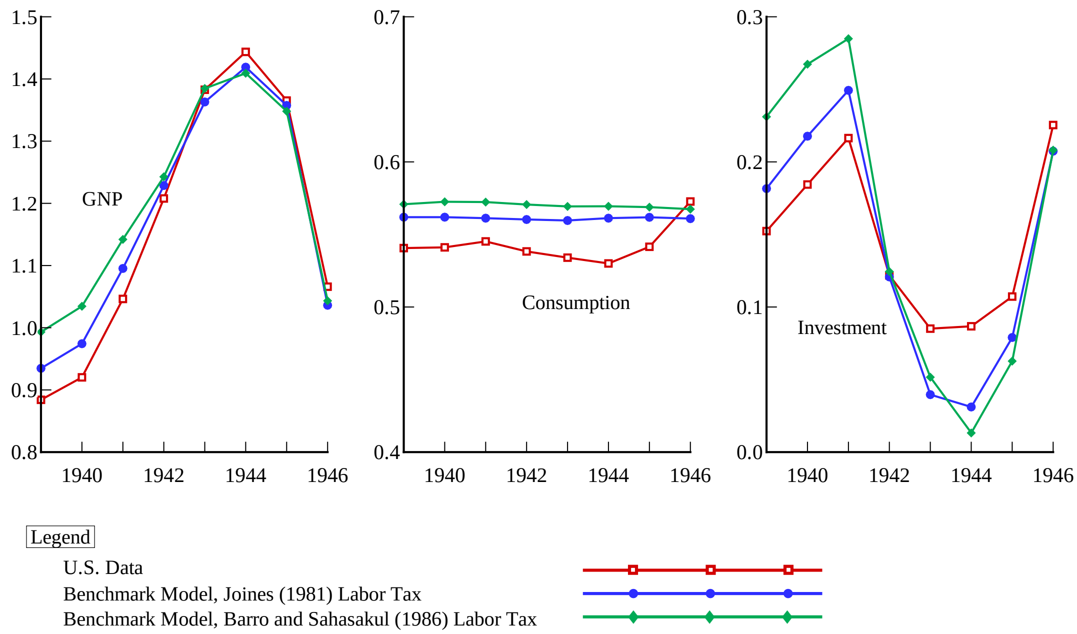
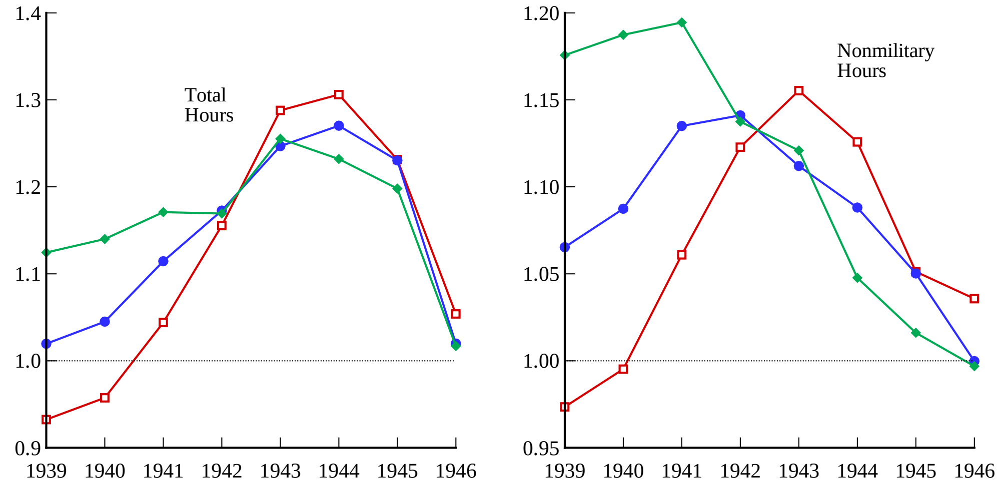


Figure A12. Real Detrended GNP, Private Consumption, and Private Investment, 1939–1946
(Benchmark Model with Alternative Labor Tax Rates)



Note: Data series are divided by the 1946 real detrended level of GNP less military compensation.

Figure A13. Per Capita Total and Nonmilitary Hours of Work, 1939–1946
(Benchmark Model with Alternative Labor Tax Rates)



Legend

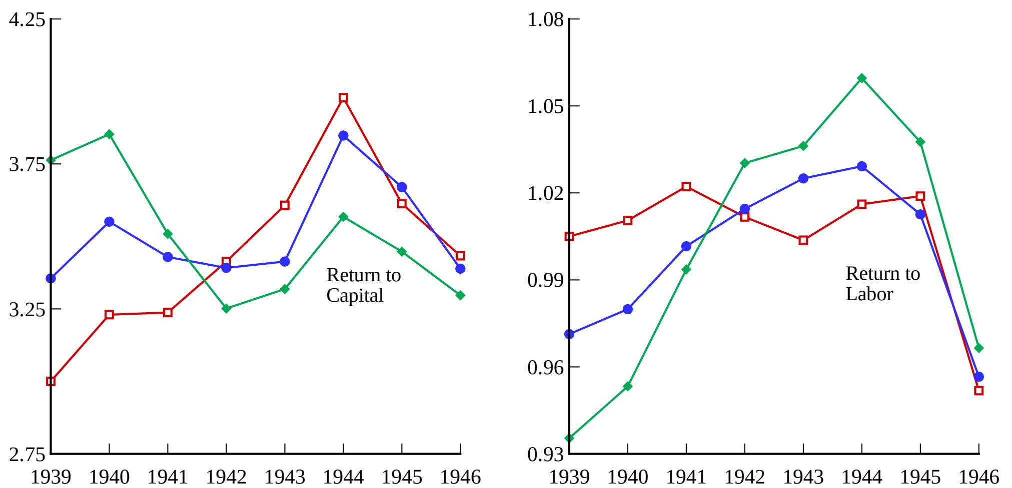
U.S. Data

Benchmark Model, Joines (1981) Labor Tax

Benchmark Model, Barro and Sahasakul (1986) Labor Tax

Note: Hours series are divided by the 1946–1960 U.S. averages.

Figure A14. After-tax Returns to Capital and Nonmilitary Labor, 1939–1946
(Benchmark Model with Alternative Labor Tax Rates,
All Series Constructed Using Marginal Productivities)



Legend

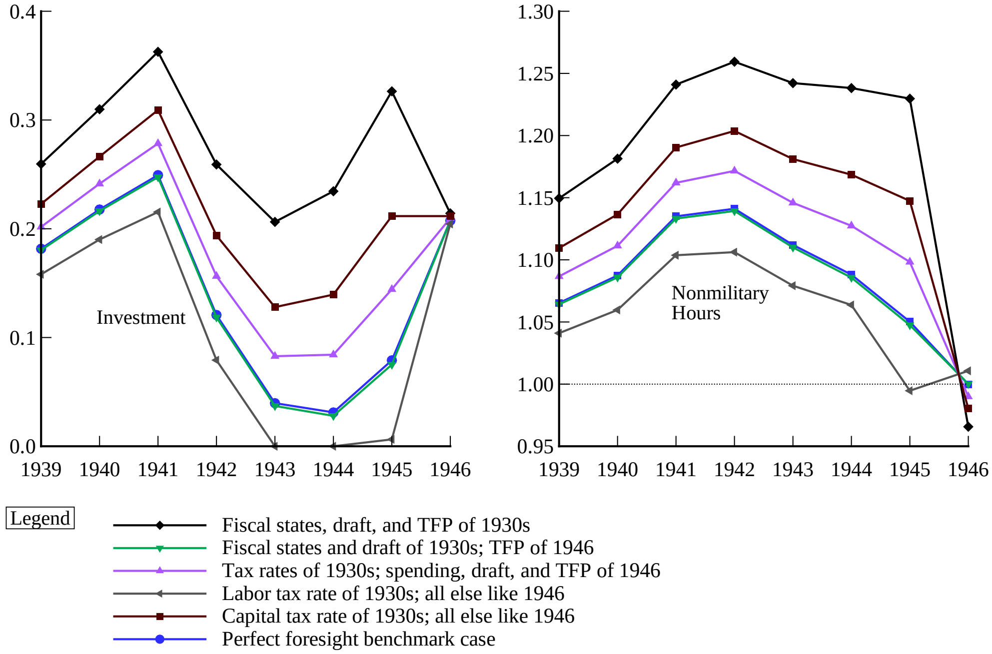
U.S. Data

Benchmark Model, Joines (1981) Labor Tax

Benchmark Model, Barro and Sahasakul (1986) Labor Tax

Note: Return to capital is equal to $100(1-\tau_k)(\theta Y/K-\delta)$.
Return to labor is after-tax nonmilitary labor productivity, with hours normalized by the 1946–1960 U.S. average.

Figure A15. Real Detrended Private Investment and Per Capita Nonmilitary Hours, 1939–1946
(Alternative Expectations for Postwar Economy)



Note: Investment series are divided by 1946 real detrended level of GNP less military compensation.
Hours series are divided by the 1946–1960 U.S. averages.