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Sudden Stops and Output Drops*

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ABSTRACT

We show that in standard equilibrium models sudden stops of capital inflows lead to increases in output. Existing models that generate output drops from sudden stops do so by adding other subtle economic frictions which overwhelm the direct effect of sudden stops.

*The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

Sudden stops in the sense of abrupt declines of capital inflows have been a characteristic of recent financial crises in emerging markets. (See Calvo 1998.) These crises have also been characterized by large declines in output. Here we ask whether theory predicts that sudden stops should lead to output drops. We ask this question in a standard equilibrium model in which sudden stops are generated by an abrupt tightening of a country's collateral constraint on foreign borrowing. We find that theory predicts that sudden stops by themselves should actually lead to increases in output. To generate declines in output during crises, other frictions must be added to the model that are large enough to overwhelm the direct effect on output from the sudden stops.

We begin by setting up a standard model of a small open economy in which foreign borrowing is subject to a collateral constraint. We view flucutations in this collateral constraint as arising from fluctuations in a country's reputation. In the model the country's budget constraint implies that a abrupt decline in capital inflows gives rise to an abrupt increase in net exports. Following the approach in Chari, Kehoe and McGrattan (2004), we show that the equilibrium outcomes in the small open economy are equivalent to those of a closed economy prototype growth model of the kind widely used in the business cycle literature. In particular, we show that a rise in net exports in the small open economy corresponds to a rise in government consumption in the prototype model. It is well-known that an increase in government consumption gives rise to an increase in output in models like our prototype growth model. It follows that a sudden stop that gives rise to an increase in net exports in the small open economy also leads to an increase in output.

Christiano, Gust and Roldos (2004), Mendoza (2004), and Neumeyer and Perri (2004) build small open economy business cycle models in which sudden stops lead to output drops. In these papers output drops arise because of other frictions that overwhelm the direct effect on output from sudden stops. In all three papers firms must borrow in advance to pay for inputs to production. In Neumeyer and Perri firms must borrow to pay for a fraction of the wage bill while in Christiano, Gust, and Roldos and Mendoza firms must borrow to pay for foreign intermediate inputs. This payment-in-advance requirement, by itself, does not introduce a friction because firms can simply borrow at the market interest rate to make these payments. In Mendoza (2004) and Neumeyer and Perri (2004) the key friction is that firms are effectively required to put the funds in a non-interest bearing escrow account. In Neumeyer and Perri this requirement gives rise

to a wedge between the marginal product of labor and the marginal rate of substitution between leisure and consumption. In Mendoza this requirement gives rise to a shock to total factor productivity. In Christiano, Gust, and Roldos, the payment in advance requirement interacts with the collateral constraint to produce a shock to total factor productivity.

We have shown that in standard models sudden stops, by themselves, do not lead to output drops. Moreover, simply adding payment-in-advance requirements to such models does not lead to predict output drops. The key frictions that generate output drops in the existing literature on sudden stops are subtle ones for which, at least currently, there is little direct evidence. To assess the quantitative importance of the key frictions, future research should focus on providing direct evidence of their existence and magnitude.

1. Collateral Constraints

In this section we develop a model of collateral constraints facing a small open economy. We show that equilibrium allocations in this original model coincide with those in a prototype growth model with shocks to government consumption. Specifically, a decline in net exports resulting from a tightening of the collateral constraint corresponds to a rise in government consumption in the prototype growth model.

A. Original Economy

Consider the following model of a small open economy embedded in a world economy with a single homogenous good in each period. The economy experiences one of finitely many events s_t, which index the shocks. We denote by $s^t = (s_0, \ldots, s_t)$ the history of events up through and including period t. The probability, as of period 0, of any particular history s^t is $\pi(s^t)$. The initial realization s_0 is given.

The representative consumer in this economy has preferences

$$(1) \qquad \sum \beta^t \pi(s^t) U(c(s^t), l(s^t))$$

where $c(s^t)$ and $l(s^t)$ denote consumption and labor. The country's budget constraint is

$$(2) \qquad c(s^t) + b(s^t) + k(s^t) \leq F(k(s^{t-1}), l(s^t)) + (1-\delta)k(s^{t-1}) + \sum_{s^{t+1}|s^t} q(s^{t+1}) b(s^{t+1})$$

where $b(s^{t+1})$ denotes the amount of state contingent borrowing from the rest of the world by the country in period t, $q(s^{t+1})$ denotes the the corresponding state contingent price and $k(s^t)$ denotes the capital stock

chosen in period t for use in period $t+1$. The country faces a collateral constraint on borrowing

$$(3) \qquad b(s^{t+1}) \leq V\left(s_{t+1}\right)$$

where $V\left(s_{t+1}\right)$ depends on the shock in period $t+1$. We assume that $V\left(s_{t+1}\right)$ is uniformly bounded above to avoid Ponzi schemes. The constraint (3) implies that having more collateral allows the country to borrow more. We interpret shocks to these collateral constraints as arising from changes in the relationship between the country and international financial markets.

The government of this country maximizes the utility of the representative household (1) subject to the budget constraint (2) and the collateral constraint (3). The world gross interest rate is constant and equal to R across both states and time. Arbitrage requires that, in equilibrium,

$$(4) \qquad q(s^{t+1}) = \frac{\pi(s^{t+1}|s^t)}{R}.$$

An *equilibrium for the original economy* is a set of allocations $\left(c(s^t), k(s^t), l(s^t), b(s^{t+1})\right)$ and prices $(q(s^t))$ such that these allocations solve the government's problem and the prices satisfy the arbitrage condition.

In this economy we think of a *sudden stop* as an abrupt increase in net exports. Here, net exports are clearly $F(k(s^{t-1}), l(s^t)) - [k(s^t) - (1-\delta)k(s^{t-1})] - c(s^t)$ which from (2) equals

$$(5) \qquad b(s^t) - \sum_{s^{t+1}|s^t} q(s^{t+1}) b(s^{t+1}).$$

Thus, a sudden stop is equivalently defined as an abrupt decrease in new borrowing. From (3), a sudden fall in $V(s_{t+1})$ leads to a reduction in $b(s^{t+1})$, and, when the collateral constraint is binding, a sudden stop.

To understand how the collateral constraint affects the equilibrium, consider the first order conditions of the government's problem. Let $\beta^{t+1}\pi(s^{t+1})\mu(s^{t+1})$ be the multiplier on the collateral constraint, so that $\mu(s^{t+1})$ is positive when the collateral constraint is binding and zero when it is not. Then the first order conditions imply

$$(6) \qquad -\frac{U_l(s^t)}{U_c(s^t)} = F_l(s^t)$$

$$(7) \qquad U_c(s^t) = \sum_{s^{t+1}|s^t} \beta\pi(s^{t+1}|s^t) U_c(s^{t+1}) \left[F_k(s^{t+1}) + 1 - \delta\right]$$

and

$$(8) \qquad U_c(s^t) = \beta R[U_c(s^{t+1}) + \mu(s^{t+1})].$$

Notice that the collateral constraint does not distort either the first order condition,(6), governing labor supply or the intertemporal Euler equation, (7), governing capital accumulation. As can be seen from (8) the collateral constraint only affects the intertemporal marginal rate of substitution in consumption.

B. Associated Prototype Economy

Consider a prototype economy with an exogenous stochastic variable, government consumption $g(s^t)$, which we call the *government consumption wedge*. Consumers maximize (1) subject to the budget constraint

$$(9) \qquad c(s^t) + k(s^t) \leq w(s^t)l(s^t) + \left[r(s^t) + 1 - \delta\right] k(s^{t-1}) + T(s^t)$$

where $w(s^t)$, $r(s^t)$, and $T(s^t)$ are the wage rate, capital rental rate and lump sum transfers. In each state s^t, firms choose k and l to maximize $F(k,l) - r(s^t)k - w(s^t)l$. The government's budget constraint is

$$(10) \qquad g(s^t) + T(s^t) = 0.$$

The resource constraint for this economy is

$$(11) \qquad c(s^t) + g(s^t) + k(s^t) = F\left(k(s^{t-1}), l(s^t)\right) + (1-\delta)k(s^{t-1}).$$

An *equilibrium of the prototype economy* consists of allocations $(c(s^t), k(s^t), l(s^t), g(s^t), T(s^t))$ and prices $(w(s^t), r(s^t))$ such that these allocations are optimal for consumers and firms and the resource constraint is satisfied.

The following proposition shows that the government consumption wedge in the prototype economy consists of net exports in the original economy.

Proposition 1. Consider an equilibrium $\left(c(s^t), k(s^t), l(s^t), b(s^{t+1})\right)$, $(q(s^t))$ for the original economy. Let the government consumption wedge be

$$(12) \qquad g(s^t) = F\left(k(s^{t-1}), l(s^t)\right) - \left[k(s^t) - (1-\delta)k(s^{t-1})\right] - c(s^t),$$

let the wage and rental rates be $w(s^t) = F_l(s^t)$ and $r(s^t) = F_k(s^t)$, and let $T(s^t)$ be defined by (10). Then $(c(s^t), k(s^t), l(s^t), g(s^t), T(s^t))$ and prices $(w(s^t), r(s^t))$ are an equilibrium for the prototype economy.

Proof. The first order conditions for the prototype economy are (6) and (7). Under the construction of the government consumption wedge given in (12), the resource constraint (11) is equal to (2). The proposition then follows. *Q.E.D.*

Consider then a sudden stop in the original economy. In the prototype economy this sudden stop manifests itself as an abrupt increase in the government consumption wedge. As is well-known from the business cycle literature an increase in government consumption by itself leads to an increase in labor and an increase in output. (See, for example, Aiyagari, Christiano and Eichenbaum 1992.) Thus, in this economy a sudden stop does not generate an output drop. Instead, it generates an output rise.

Note that for simplicity we have abstracted from any government consumption in the original economy. If we let the original economy have government consumption, then the government consumption wedge in the prototype economy would be the sum of government consumption and net exports in the original economy.

Note also that given an equilibrium in the prototype economy we can construct the associated equilibrium in the original economy if $U_c(s^t)/\beta R - U_c(s^{t+1})$ is nonnegative for all s^{t+1} for some choice of $R > 1$. This R serves as the world interest rate in the original economy and at such an R the multiplier on the collateral constraint is nonnegative. In this constructed equilibrium the value of the initial debt in the original economy is set equal to the present discounted value of government consumption in the prototype economy. The debt in the original economy at state s^t is, of course, the present discounted value of net exports and hence in the prototype economy corresponds to the the present discounted value of future government consumption.

Proposition 1 is closely related to a proposition in Chari, Kehoe, McGrattan (2004). In that proposition we consider an original economy with no collateral constraints but with fluctuating world interest rates. We establish a similar equivalence proposition. In the original economy fluctuations in world interest rates lead to fluctuations in net exports. In the prototype economy these fluctuations in net exports show up as fluctuations in the government consumption wedge.

C. Extensions to Uncontingent Asset Markets

Consider now a version of the original economy in which we replace the state contingent bonds with an uncontingent bond. Here the budget constraint becomes

$$c(s^t) + b(s^{t-1}) + k(s^t) \leq F(k(s^{t-1}), l(s^t)) + (1-\delta)k(s^{t-1}) + q(s^t)b(s^t) \tag{13}$$

where $b(s^t)$ denotes the amount of state uncontingent borrowing from the rest of the world by the country in period t, $q(s^t)$ denotes the the corresponding price. The collateral constraint becomes

$$b(s^t) \leq V(s_{t+1}) \text{ for all } s_{t+1}. \quad (14)$$

Here $q(s^t) = 1/R$. The first order conditions associated with the problem are (6), (7), and

$$U_c(s^t) = \beta R \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) \left[U_c(s^{t+1}) + \mu(s^{t+1})\right]$$

With this setup the analog of Proposition 1 immediately applies. In particular, in the prototype economy fluctuations in the government consumption wedge play the same role as fluctuations in net exports in the original economy.

2. The Role of Other Frictions in Generating Output Drops

We have shown that in our simple model, sudden stops do not lead to output drops. In this section we consider adding other frictions to the simple model and ask if sudden stops, interacting with these frictions, lead to output drops. We first consider an economy with endogenous collateral constraints and show that a binding collateral constraint in this model corresponds to a subsidy to investment in the prototype growth model and therefore an increase in output. We then consider the role of advance payment constraints. We show that these constraints can lead to output drops from sudden stops only when coupled with yet other frictions.

A. Investment Wedges from Endogenous Collateral Constraints

Consider a version of the original economy with one change, namely the collateral constraint (3) is replaced by

$$b(s^{t+1}) \leq V\left(k(s^t), s_{t+1}\right). \quad (15)$$

Here the maximal amount that can be borrowed, $V\left(k(s^t), s_{t+1}\right)$, depends on the capital stock chosen in period t and the shock in period $t+1$. We assume that $V_k(k(s^t), s_{t+1}) > 0$ so that having more collateral allows the country to borrow more and that $V\left(k(s^t), s_{t+1}\right)$ is uniformly bounded above to avoid Ponzi schemes. This formulation is similar to that of Kiyotaki and Moore (1997) and is motivated by the idea that a portion of

the capital stock can effectively be seized by foreign lenders in the event of default and hence foreign lenders will not lend more than the value of the seizable portion.

An equilibrium in this *endogenous collateral constraint economy* is defined as before. The first order conditions for this economy are the same as in the original economy except that the intertemporal Euler equation (7) is replaced by

$$(16) \quad U_c(s^t) = \sum_{s^{t+1}|s^t} \beta\pi(s^{t+1}|s^t)U_c(s^{t+1})\left[(F_k(s^{t+1}) - \delta) + 1\right] + \sum_{s^{t+1}|s^t} \beta\pi(s^{t+1}|s^t)\mu(s^{t+1})V_k(s^{t+1})$$

The associated prototype economy has an *investment wedge* $1 - \tau_k(s^t)$ along with the government consumption wedge. In it, the consumers' budget constraint is now

$$(17) \quad c(s^t) + k(s^t) \leq w(s^t)l(s^t) + \left[\left(1 - \tau_k(s^t)\right)\left(r(s^t) - \delta\right) + 1\right]k(s^{t-1}) + T(s^t).$$

Note that the investment wedge resembles a tax on capital income. The government's budget constraint is now

$$(18) \quad g(s^t) + T(s^t) = \tau_k(s^t)\left(r(s^t) - \delta\right)k(s^{t-1}).$$

and the resource constraint is unaffected. An equilibrium of the prototype economy is defined as before.

The following proposition shows that the endogenous collateral constraint manifests itself as an investment wedge in the associated prototype economy.

Proposition 2. Consider an equilibrium for the endogenous collateral constraint economy. Let the investment wedge be given by

$$(19) \quad \tau_k(s^{t+1}) = -\frac{\mu(s^{t+1})V_k(s^{t+1})}{U_c(s^{t+1})\left(F_k(s^{t+1}) - \delta\right)},$$

and let $T(s^t)$ be given by (18) and let the rest of the variables be given as in Proposition 1. Then the resulting allocations, prices and taxes are an equilibrium for the associated prototype economy.

The proof of the proposition follows immediately from a comparison of the first order conditions, budget constraints and resource constraints of the two economies.

Note that if the collateral constrained economy is in the neighborhood of a steady state then $F_k(s^{t+1}) - \delta$ is positive so that in this neighborhood a binding collateral constraint ($\mu(s^{t+1}) > 0$) corresponds to a subsidy

to capital accumulation ($\tau_k(s^{t+1}) < 0$). The intuition for this result is that if the collateral constraint in the original economy is binding in period $t+1$ then capital accumulation in period t helps to relax the collateral constraint in period $t+1$ and hence provides an additional benefit beyond that from the marginal product of capital.

Consider a sudden stop in the endogenous collateral constraint economy generated by a tightening of the collateral constraint. Proposition 2 shows that such a sudden stop corresponds to an subsidy to capital accumulation along with an increase in government consumption. A well-known result in the business cycle literature is that a subsidy to capital accumulation stimulates investment and output. We have already argued that an increase in government consumption also stimulates output. Thus, in the endogenous collateral constraint economy, sudden stops do not lead to output drops.

B. Labor and Efficiency Wedges from Advance Payment Constraints

In the literature on sudden stops, Perri and Neumeyer (2004), Mendoza (2004), Christiano, Gust and Roldos (2004) introduce various forms of advance payment constraints and show that sudden stops interacting with these constraints can produce output drops. Perri and Neumeyer introduce advance payment constraints on wages and Mendoza and Christiano, Gust and Roldos introduce advance payment constraints on payments of intermediate goods.

Consider first the advance payment constraints on wages. In the business cycle literature, it has been widely argued that requiring firms to pay workers in advance of production introduces a wedge between the marginal product of labor and the marginal rate of substitution between leisure and consumption that we call a *labor wedge.* Here we argue that such a requirement, by itself, does not introduce a labor wedge, but does so when coupled with a requirement that firms escrow future wages in non-interest bearing accounts.

Consider the following deterministic version of our economy with an advance payment constraint. In it, in period $t-1$ firms must escrow with the government a fraction θ of its wage bill $w_t l_t$ due in period t. To do so the firm borrows $w_t l_t$ at $t-1$ from foreign lenders at the world (gross) interest rate R_t and escrows these funds with the government. (Note that the firms cash flows in period $t-1$ with respect to these transactions net out to zero.) In period t, the firm uses the escrowed funds to pay the workers and it must repay the

foreign lenders $R_t w_t l_t$. The firm's problem is to maximize profits at t given by

$$F(k_t, l_t) - R_t w_t l_t - r_t k_t.$$

The government rebates the interest on escrowed funds as a lump sum transfer to consumers. Consumers maximize utility $\sum \beta^t U(c_t, l_t)$ subject to the budget constraint

$$c_t + R_t b_t + k_t \leq w_t l_t + [r_t + (1 - \delta)]k_t + b_{t+1} + T_t.$$

The resulting first order conditions imply that

$$-\frac{U_{lt}}{U_{ct}} = \frac{F_{lt}}{R_t}.$$

This economy is equivalent to a prototype economy with tax rate on labor τ_{lt} equal to $1 - 1/R_t$. The advance payment constraint with an escrow provision induces what Chari, Kehoe and McGrattan (2004) call a labor wedge of $1 - \tau_{lt} = 1/R_t$.

To see that an advance payment constraint, by itself, does not introduce a labor wedge let the government pay interest on the escrow accounts at rate R_t. Here the firms would borrow $w_t l_t / R_t$ at $t - 1$ and their problem would be to maximize $F(k_t, l_t) - w_t l_t - r_t k_t$ and there would be no labor wedge.

It is easy to show that advance payment constraints on intermediate goods by themselves do not distort decisions. When coupled with other frictions, such as subjecting these payments to collateral constraints, these constraints manifest themselves as efficiency wedges that resemble shocks to total factor productivity.

3. A Quantitative Analysis of a Sudden Stop

Here we use the logic of Proposition 1 to examine the quantitative effects of a the sudden stop in Mexico in 1994-1995.

In Figure 1 we plot Mexican data on real net exports and the government consumption wedge (the sum real government consumption and real net exports). We normalize both series by the level of real GDP in 1994:4. The figure shows an abrupt and dramatic increase in the consumption wedge and that this increase was due almost entirely to the rise in net exports. That is, Mexico experienced a sudden stop.

We study the quantitative effects of this sudden stop in a prototype growth model. We use a version of the model in Chari, Kehoe, and McGrattan (2004), in which the stochastic processes for the government

consumption wedge, the labor wedge, the efficiency wedge and the investment wedge are estimated using the model and Mexican data from 1980:1 to 2003:4. (For details see Chari, Kehoe, and McGrattan 2005.)

To assess the effects of the sudden stop we feed in the realized values of the government consumption wedge over the the period 1994:4 to 1996:4 into our prototype growth model, holding the values of the other wedges fixed at their 1994:4 levels. Figure 2 shows that a sudden stop, by itself, leads to a small increase in output. Figure 2 also shows that Mexican real GDP fell sharply in 1995. Clearly, the sudden stop by itself cannot account for the output drop.

4. Conclusion

We show that in standard equilibrium models sudden stops of capital inflows lead to increases in output. The key frictions that generate output drops in the existing literature on sudden stops are subtle ones for which, at least currently, there is little direct evidence. To assess the quantitative importance of the key frictions, future research should focus on providing direct evidence of their existence and magnitude.

References

Aiyagari, S. Rao; Christiano, Lawrence J.; Eichenbaum, Martin. "The Output, Employment, and Interest Rate Effects of Government Consumption." *Journal of Monetary Economics,* October 1992, 30 (1), pp. 73-86.

Calvo Guillermo. "Capital Flows and Capital-Market Crises: The Simple Economics of Sudden Stops." *Journal of Applied Economics*, 1 (1), November 1998, pp. 35-54.

Chari, V.V.; Kehoe, Patrick, J.; and McGrattan, Ellen. "*Business Cycle Accounting.*" Federal Reseve Bank of Minneapolis, Staff Report, 2004.

Chari, V.V.; Kehoe, Patrick, J.; and McGrattan, Ellen. "*Sudden Stops and Output Drops*" Federal Reseve Bank of Minneapolis, Staff Report, 2005.

Christiano, Lawrence J.; Gust Christopher; and Roldos Jorge. "Monetary policy in a financial crisis." *Journal of Economic Theory,* pp. 64-103

Kiyotaki, Nobuhiro and Moore, John. "Credit Cycles." *Journal of Political Economy*, April 1997, v. 105, (2), pp. 211-48.

Mendoza, Enrique, G. " 'Sudden Stops' in an Equilibrium Business Cycle Model with Credit Contraints: A Fisherian Deflation of Tobin's q." Working paper, University of Maryland, 2004.

Neumeyer, Pablo, A and Perri, Fabrizio. " Business Cycles in Emerging Economies: The Role of Interest Rates." NBER working paper, march 2004.

Figure 1. The Mexican Sudden Stop in 1994:4

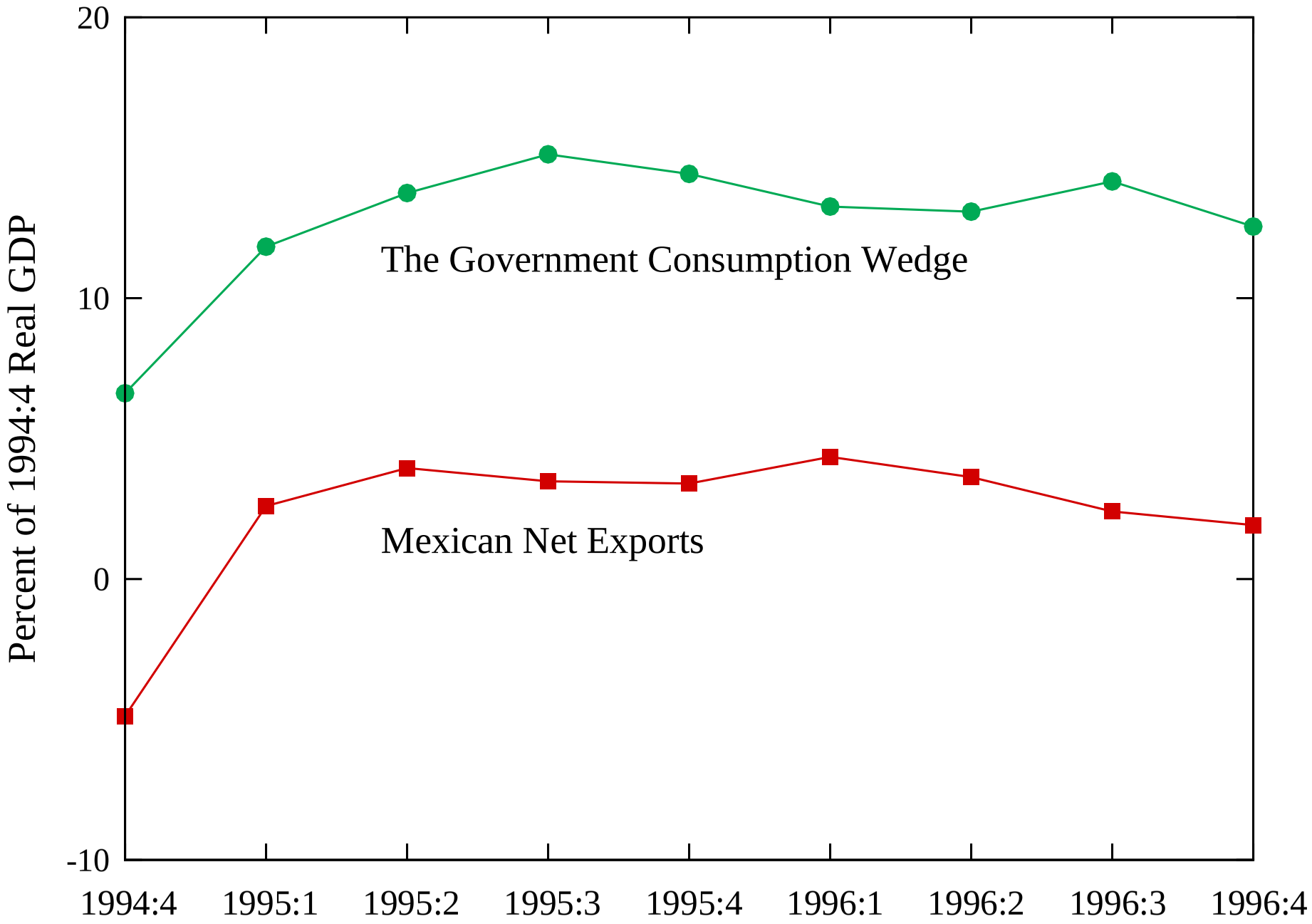

Figure 2. The Output Effect of a Pure Sudden Stop