

Federal Reserve Bank of Minneapolis  
Research Department Staff Report 534

Revised December 2016

## **An Aggregate Model for Policy Analysis with Demographic Change\***

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### ABSTRACT

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Many countries are facing challenging fiscal financing issues as their populations age and the number of workers per retiree falls. Policymakers need transparent and robust analyses of alternative policies to deal with the demographic changes. In this paper, we propose a simple framework that can easily be matched to aggregate data from the national accounts. We demonstrate the usefulness of our framework by comparing quantitative results for our aggregate model with those of a related model that includes within-age-cohort heterogeneity through productivity differences. When we assess proposals to switch from the current tax and transfer system in the United States to a mandatory saving-for-retirement system with no payroll taxation, we find that the aggregate predictions for the two models are close.

Keywords: retirement, taxation, Social Security, Medicare

JEL classification: H55, I13, E13

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\* All supplemental materials are available at [users.econ.umn.edu/~erm](http://users.econ.umn.edu/~erm). The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

# 1. Introduction

Many countries are facing challenging fiscal financing issues as their populations age and the number of workers per retiree falls. In this paper, we propose a simple overlapping generations model with people differing only in age. The model can easily be matched to aggregate data from the national accounts and used to analyze alternative policies when there is demographic change. We demonstrate the usefulness of this aggregate model by comparing its quantitative predictions for U.S. data with those of a related model analyzed in our earlier work in which we allowed for within-age-cohort heterogeneity. (See McGrattan and Prescott (2016).) When we assess an often-discussed proposal to switch from the current tax and transfer system in the United States to a mandatory saving-for-retirement system with no payroll taxation, we find that the aggregate predictions for the two models are close.

The aggregate predictions we report are the welfare gains of switching policy regimes and the resultant changes in national account statistics. If the current system is continued, taxes must be increased because the number of retirees in the United States is projected to grow, and their retirement consumption must be somehow financed. If the system is reformed, payroll taxes and the associated transfers for Social Security and Medicare are to be phased out, and individuals have to save for their own retirement consumption.<sup>1</sup> Regardless of whether current policy is continued or reformed, we assume that spending on all other government transfer programs and purchases of goods and services remain at their current level as a share of gross national product (GNP).

As in McGrattan and Prescott (2016), we restrict attention to reforms that are by design welfare improving for all individuals. To ensure that no one is made worse off, we broaden the tax base and lower marginal tax rates, at least temporarily, during the transition to the new system. We report results for both a temporary and a permanent

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<sup>1</sup> Of course, in practice, saving would be mandatory; otherwise, individuals would want to opt out and apply for transfer programs targeted to the poor.

change in the workers' tax schedules. We verify in the aggregate model with only one productivity type that there is a welfare gain for all age cohorts, and we show that the gains are close in magnitude to the population-weighted average gains in McGrattan and Prescott's (2016) benchmark model that has more than one productivity type.

We then compare the models' aggregate predictions for statistics in the national accounts and flow of funds, along with factor inputs and prices. Like McGrattan and Prescott (2016), we find that reforming Social Security and Medicare would have a large impact on aggregate statistics. For example, McGrattan and Prescott (2016) predicted that GNP would be 4.5 percentage points below the current trend if current policy is continued and 11.4 percentage points above trend if policy is reformed and workers' tax schedules are changed only temporarily during the transition. For the aggregate model with one productivity type, we predict that GNP would be 6.3 percentage points below the current trend if current policy is continued and 10.4 percentage points above trend if policy is reformed. Taking differences, the predictions are 15.9 percentage points versus 16.7, respectively. If tax schedules are permanently changed, the differences in GNP predictions are 20.6 percentage points for McGrattan and Prescott (2016) and 19.5 percentage points for the aggregate model proposed here. Moreover, we find similarly close predictions for consumption, investment, factor inputs and incomes, the interest rate, tax revenues, and household net worth.

In Section 2, we present the model economy. In Section 3, we discuss the parameter estimates and policy experiments. Section 4 compares predictions of the models with and without heterogeneity in productivity levels. Section 5 concludes.

## **2. The Model Economy**

The framework we use is a relatively standard overlapping generations model. The only nonstandard feature that we introduce is the inclusion of multiple business sectors to

account for the fact that U.S. Schedule C corporations are subject to the corporate income tax, while pass-through businesses (for example, sole proprietorships, partnerships, and Schedule S corporations) are not. Allowing for differences in these businesses helps us match total incomes and tax revenues.

We consider two versions of the model: the *homogeneous within-cohort* version assumes people differ only in age, and the *heterogeneous within-cohort* version assumes people differ both in age and in their level of productivity. We are interested in comparing results for these two versions of the model to see the impact that within-cohort heterogeneity has on *aggregate* predictions.

## 2.1. The Population

We use  $h \in \{1, 2, \dots, H\}$  to index the year since entering the workforce, and we refer to this as *age*. We use  $j \in \{1, 2, \dots, J\}$  to index the productivity level of the household members. The measure of age  $h$  households with productivity level  $j$  at date  $t$  is denoted  $n_t^{h,j}$ , and these parameters define the population dynamics. The measure of people arriving as working-age households with productivity level  $j$  at date  $t$  is  $n_t^{1,j}$ , and we assume

$$n_{t+1}^{1,j} = (1 + \eta_t) n_t^{1,j}, \quad (2.1)$$

with  $\sum_j n_0^{1,j} = 1$ , where  $\eta_t$  is the growth rate of households entering the workforce. The probability of an age  $h < H$  household of any type at date  $t$  surviving to age  $h + 1$  is  $\sigma_t^h > 0$ .

## 2.2. The Households' Problem

In each period, households choose consumption  $c$  and labor input  $\ell$  to maximize utility, and they take as given their own level of assets  $a$  and the law of motion for the aggregate states,  $s' = F(s)$ . The states in  $s$  are the distribution of assets in the economy, the level of government debt, and the aggregate stocks of tangible and intangible capital. The value

function of a household of age  $h$  with productivity level  $j$  satisfies

$$v_h(a, s, j) = \max_{a', c, \ell \geq 0} \{u(c, \ell) + \beta \sigma_t^h v_{h+1}(a', s', j)\} \quad (2.2)$$

subject to

$$(1 + \tau_{ct})c + a' \sigma_t^h = (1 + i_t)a + y_t - T_t^h(y_t) \quad (2.3)$$

$$y_t = w_t \ell \epsilon^j \quad (2.4)$$

$$s' = F(s), \quad (2.5)$$

where a prime indicates the next period value of a variable,  $\tau_{ct}$  is the tax on consumption,  $i_t$  is the after-tax interest rate,  $w_t$  is the before-tax wage rate,  $\epsilon^j$  is the productivity of an individual of type  $j$ ,  $T_t^h(y_t)$  is the net tax function, and  $v_{H+1} = 0$ . Households with  $h > H_R$  are retired and have  $\ell = 0$ . The net tax schedule for retirees ( $h > H_R$ ) is  $T_t^j(y) = T_t^r(0)$  and is equal to the (negative) transfers to retirees since they have no labor income. The net tax schedule for workers ( $h \leq H_R$ ) is  $T_t^h(y) = T_t^w(y)$  and is equal to their total taxes on labor income less any transfers. Savings are in the form of an annuity that makes payments to members of a cohort in their retirement years conditional on them being alive. Effectively, the return on savings depends on the survival probability as well as the interest rate.<sup>2</sup>

In solving the dynamic program in (2.2), households take the aggregate state  $s$  and its evolution as given. Variables that define the aggregate state are time  $t$ , the distribution of household assets, the aggregate capital stocks used by the firms in production, and the government's fiscal policy variables. We turn next to a discussion of the firms' problem and government policy.

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<sup>2</sup> See McGrattan and Prescott (2016), who show that policy predictions are robust across many variations of this basic framework.

### 2.3. The Firms' Problem

There are two sectors indexed by  $i$ , and competitive firms in each of these sectors use inputs of capital and labor to produce output with the following technologies:

$$Y_{it} = K_{iTt}^{\theta_{iT}} K_{iIt}^{\theta_{iI}} (\Omega_t L_{it})^{1-\theta_{iT}-\theta_{iI}}, \quad (2.6)$$

where  $i = 1, 2$ . The inputs to production are tangible capital  $K_{iTt}$ , intangible capital  $K_{iIt}$ , and labor  $L_{it}$ , and outputs in both sectors grow with labor-augmenting technical change at the rate  $\gamma$ :

$$\Omega_{t+1} = (1 + \gamma) \Omega_t. \quad (2.7)$$

Firms in the first sector are subject to the corporate income tax and produce intermediate good  $Y_{1t}$ ; the empirical analogue of these firms are Schedule C corporations, sole proprietorships, and partnerships. Firms in the second sector are not subject to the corporate income tax and produce intermediate good  $Y_{2t}$ ; the empirical analogue of these firms are pass-through entities like Schedule S corporations. The aggregate production function of the composite final good is

$$Y_t = Y_{1t}^{\theta_1} Y_{2t}^{\theta_2}, \quad (2.8)$$

where  $\theta_1 + \theta_2 = 1$ . Capital stocks depreciate at a constant rate, so

$$K_{iT,t+1} = (1 - \delta_{iT}) K_{iTt} + X_{iTt} \quad (2.9)$$

$$K_{iI,t+1} = (1 - \delta_{iI}) K_{iIt} + X_{iIt} \quad (2.10)$$

for  $i = 1, 2$ , where  $X_{iTt}$  and  $X_{iIt}$  denote tangible and intangible investments in sector  $i$ , respectively. Depreciation rates are denoted as  $\delta$  and are indexed by sector and capital type. With competitive firms, factors of production—labor and both types of capital—in equilibrium are paid their marginal products, which are therefore the same in both sectors.

The accounting profits of Schedule C corporations are given by

$$\Pi_{1t} = p_{1t} Y_{1t} - w_t L_{1t} - X_{1It} - \delta_{1T} K_{1Tt}, \quad (2.11)$$

where  $p_{1t}$  is the price of the intermediate good relative to the final good. Accounting profits are equal to sales less compensation, intangible investment, and tangible depreciation. Notice that intangible investments are fully expensed, while tangible investments are capitalized. Distributions to the corporations' owners are given by

$$D_{1t} = (1 - \tau_{1t}^\pi) \Pi_{1t} - K_{1T,t+1} + K_{1Tt}, \quad (2.12)$$

where  $\tau_{1t}^\pi$  is the corporate income tax levied on Schedule C profits. These distributions are the after-tax profits after subtracting retained earnings,  $K_{1T,t+1} - K_{1Tt}$ .

Other businesses are pass-through entities, so their distributions are equal to their profits, which in this case is given by

$$D_{2t} = \Pi_{2t} = p_{2t}Y_{2t} - w_tL_{2t} - X_{2It} - \delta_{2T}K_{2Tt}, \quad (2.13)$$

where  $p_{2t}$  is the price of the intermediate good relative to the final good. These payouts are equal to sales less compensation, intangible investment, and tangible depreciation. Firms in both sectors maximize the present expected value of after-tax dividends which are paid to the owners of all capital, namely, the households.

The relevant equilibrium price sequences for the households are interest rates  $\{i_t\}$  and wage rates  $\{w_t\}$ . The term  $i_t a_t$  in (2.3) is the combined after-tax dividend income to the households, which is intermediated costlessly. If after-tax returns on all assets are equated, it must be the case that the after-tax interest rate  $i_t$  paid to households is equal to the returns of both types of capital in both sectors.

## 2.4. The Government's Fiscal Policy

The law of motion of government debt is given by

$$B_{t+1} = B_t + i_t B_t + G_t - \sum_{h,j} n_t^{h,j} T_t^h \left( w_t \ell_t^{h,j} e^j \right) - \tau_t^c C_t - \tau_{1t}^\pi \Pi_{1t} - \tau_{1t}^d D_{1t} - \tau_{2t}^d D_{2t}. \quad (2.14)$$

Thus, next period's debt  $B_{t+1}$  is this period's debt  $B_t$  plus interest on this period's debt  $i_t B_t$ , plus public consumption  $G_t$ , minus tax revenues net of transfers. As noted earlier, households pay taxes on labor and consumption and receive after-tax earnings on their capital income. The taxes levied on capital income are taxes on Schedule C corporate profits and distributions at rates  $\tau_{1t}^\pi$  and  $\tau_{1t}^d$ , respectively, and taxes on distributions of other business income at rate  $\tau_{2t}^d$ .

## 2.5. Market Clearing

The market for goods must clear in equilibrium, and this implies

$$Y_t = C_t + X_{Tt} + X_{It} + G_t, \quad (2.15)$$

where  $X_{Tt} = \sum_i X_{iTt}$  and  $X_{It} = \sum_i X_{iIt}$ . Aggregate labor supply is denoted by  $L_t$ , and assuming the labor market clears, it must be the case that

$$L_t = \sum_{h,j} n_t^{h,j} \ell_t^{h,j} \epsilon^j. \quad (2.16)$$

Finally, assuming that capital markets clear, it must be the case that the household policy functions  $\{a' = f_h(s, k)\}_h$  imply the aggregate law of motion  $s' = F(s)$ , where  $F$  is taken as given by the private agents.

Next, we parameterize the model and work with two versions: one that has within-cohort heterogeneity ( $J = 4$ ) as in McGrattan and Prescott (2016) and another with one productivity level  $J = 1$  and  $\epsilon^j = 1$  for all workers. The  $J = 4$  specification is the heterogeneous within-cohort version of the model, and the  $J = 1$  specification is the homogeneous within-cohort version.<sup>3</sup> We also refer to the latter as our aggregate model since there is a representative agent in each cohort.

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<sup>3</sup> McGrattan and Prescott (2016) also report results for the case with  $J = 7$ , but the main findings are unchanged.



### 3. Parameters and Policies

In McGrattan and Prescott (2016), we describe in great detail the U.S. data that are used to parameterize the heterogeneous within-cohort model.<sup>4</sup> Here, we summarize their parameter choices and the choices we make here for the nested homogeneous within-cohort model.

#### 3.1. Parameters

Table 1 reports parameters calibrated to generate a balanced growth path that is consistent with U.S. aggregate statistics averaged over the period 2000–2010. More specifically, the model predicts the same national account and fixed asset statistics as reported by the Bureau of Economic Analysis (BEA), regardless of the choice of  $J$ . (See Tables 1 and 2 in McGrattan and Prescott (2016) for full details.)

The first set of parameters listed in Table 1 are demographic parameters: growth in population and years of working life. We set the growth rate of the population equal to 1 percent and the work life to 45 years. In addition, we chose survival probabilities  $\sigma_t^h$  to match the life tables in Bell and Miller (2005). These choices for population growth, working life, and survival probabilities imply that the models' ratio of workers to retirees is 3.93, which is equal to the ratio of people over age 15 in the 2005 CPS March Supplement not receiving Social Security and Medicare benefits to those who are receiving these benefits.

The second set of parameters listed in Table 1 are preference parameters. The utility function is logarithmic, that is,  $u(c, \ell) = \log c + \alpha \log(1 - \ell)$ . We set  $\alpha$  equal to 1.185 to get the same predicted fraction of time to work, roughly 28 percent, for the model that we observe in the data. The discount factor  $\beta$  is set equal to 0.987, and this choice along

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<sup>4</sup> The main sources of the data are the Board of Governors (1945–2015), U.S. Congress (2012), U.S. Department of Commerce (2007), U.S. Department of Commerce (1929–2015), U.S. Department of Labor (1962–2015), and U.S. Department of Treasury (1962–2015).

with the choice of utility guarantees that 58.5 percent of income goes to labor, which is the U.S. share of compensation of employees plus 70 percent of proprietors' income. The remaining income is paid to capital owners.

The next parameters in Table 1 are technology parameters: the growth rate, capital shares, and depreciation rates. Growth in labor-augmenting technology is set equal to 2 percent, which is roughly trend growth in the United States. The share parameter  $\theta_1$  is the relative share of income to Schedule C corporations and is set equal to 50 percent to be consistent with IRS data on corporate receipts and deductions (because we do not have detailed national account data that split corporate income shares for Schedule C and Schedule S). The remaining shares and the depreciation rates are chosen to be consistent with investments and capital stocks reported by the BEA and the flow of funds. To attribute shares of investments and stocks to Schedule C and all other businesses, we use information on depreciable assets from the IRS. For the tangible stocks, this implies the following values:  $\theta_{1T} = 0.182$ ,  $\theta_{2T} = 0.502$ ,  $\delta_{1T} = 0.050$ , and  $\delta_{2T} = 0.015$ . In the case of intangible stocks, we cannot uniquely identify all capital shares and depreciation rates. We somewhat arbitrarily assume that two-thirds of the intangible capital is in Schedule C corporations and one-third in other businesses, and we set the depreciation rates on intangible capital equal to that of tangible capital in Schedule C corporations. We also set the depreciation rates on intangible capital equal to the tangible capital in Schedule C corporations.<sup>5</sup> These choices imply that  $\theta_{1I} = 0.190$ ,  $\theta_{2I} = 0.095$ ,  $\delta_{1I} = 0.050$ , and  $\delta_{2I} = 0.050$ .

The last set of parameters in Table 1 are policy parameters that remain fixed during the numerical experiments, namely, spending and debt shares and capital tax rates. The level of government consumption  $\phi_{Gt}$  in all  $t$  is set equal to 0.044 times GNP, which is the average share of U.S. military expenditures over the period 2000–2010.<sup>6</sup> The debt to

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<sup>5</sup> See McGrattan and Prescott (2016) for an extensive sensitivity analysis.

<sup>6</sup> The remainder is included with transfers, since it is substitutable with private consumption.

GNP share  $\phi_{Bt}$  in all  $t$  is set equal to 0.533, which is the average U.S. ratio for 2000–2010. Capital tax rates are assumed to be the same for all asset holders, since most household financial assets are held in accounts managed by fiduciaries. There are three rates: the tax on Schedule C profits,  $\tau_1^\pi$ , the tax on Schedule C distributions,  $\tau_1^d$ , and the tax on distributions of all other businesses,  $\tau_2^d$ . The statutory corporate income tax rate, which is assessed on Schedule C profits, is 40 percent with federal and state taxes combined. Not all firms pay this rate, so we use instead an estimate of the ratio of total revenue to total Schedule C profits. This ratio is 33 percent. The tax rate on Schedule C distributions is 14.4 percent, which is the average marginal rate computed by the TAXSIM model described in Feenberg and Coutts (1993) times the fraction of equity in taxed accounts. The tax rate on all other businesses is set equal to 38.2 percent, which is Barro and Redlick’s (2011) estimate of the income-weighted average marginal tax rate on wagelike income for federal, state, and FICA taxation during the period 2000–2010.

To parameterize the initial net tax schedules  $T_0^h(\cdot)$ , we use distributions of adjusted gross incomes (AGIs), wages, taxes, and transfers from the 2005 Current Population Survey (CPS), along with BEA totals (allowing us to scale up any categories in which the CPS total is less than the BEA total). (See McGrattan and Prescott (2016, Tables 4 and 5) for complete details on the data used in the estimation.) The tax data are available for tax filers who typically file on behalf of themselves and other family members. Thus, when organizing the data, we first assign individuals in the CPS to families, and we compute a family AGI. Families are defined to be a group of people living in the same household that are either related or unmarried partners and their relatives, and family AGI is the total AGI summed across AGIs for all tax filers in the family. Individuals in the family that are at least 15 years old are assigned an equal share of the family AGI, and this assignment determines their income brackets when we parameterize the net tax functions,  $T_0^h(\cdot)$ .

First, consider the net tax schedule for workers. We model it as a piecewise function

found by linearizing  $T^w(y)$  on each AGI income interval  $[y_i, \bar{y}_i]$ ,  $i = 1, \dots, I$ , as follows:

$$\begin{aligned}
T^w(y) &= T_i(y) - \Psi_i^w \\
&\simeq T_i'(\bar{y})y - \{[T_i'(\bar{y}) - T_i(\bar{y})/\bar{y}]\bar{y} + \Psi_i^w\} \\
&\equiv \beta_i y + \alpha_i,
\end{aligned} \tag{3.1}$$

where  $\bar{y}$  is the midpoint in  $[y_i, \bar{y}_i]$  and  $\Psi_i^w$  is a constant transfer to workers with labor income in this bracket. The  $\beta_i y$  term is the *marginal component* of the net tax schedule, since it depends on income  $y$ , and the intercept  $\alpha_i$  is the *nonmarginal component* of the net tax schedule, since it is the same for all income earners in the  $i$ th income bracket.

The marginal tax rates,  $T_i'(\bar{y})$  in (3.1), based on U.S. data are shown in panel A of Figure 1. Twelve rates are plotted and correspond to the twelve family AGI brackets in our sample. (The eleventh and twelfth rates are so close as to be indistinguishable.) These rates are income-weighted average marginal tax rates, adjusted for employer-sponsored pensions. For the adjustment, we assume that the pension benefits increase with an additional hour of work and, therefore, lower the effective marginal tax rates. We compute the change in the benefits relative to the change in per capita compensation and subtract the result from the rates. We then fit a smooth curve through the adjusted rates, and the result is plotted in Figure 1A.

Figure 1B shows the data underlying the nonmarginal components ( $\alpha_i$ ) of the net tax schedule in (3.1) for each of the twelve family AGI bins in our sample. Estimates are in per capita terms. The first two categories are government spending on nondefense spending and transfers other than Social Security and Medicare. The third category is employer contributions that we categorize as nonmarginal. For example, if the family receives a fringe benefit  $f$ , which is deducted from total wages, then the budget set includes a term equal to  $f$  times their marginal tax rate (and, therefore, we use the value of benefits multiplied by the relevant tax rates). The main benefit in this category is employer contributions for insurance. The fourth category is the residual category, namely,  $[T_i'(\bar{y}) - T_i(\bar{y})/\bar{y}]\bar{y}$  in

(3.1). This is constructed as the sum of the differences between the marginal and average tax rates from federal and state income filings and the employee’s part of FICA, multiplied by the amount of taxed labor income—wages and salaries plus 70 percent of proprietors’ income.

The estimates in Figure 1 are the underlying data for the tax schedule in Table 2. The tax rates in panel A of Figure 1 are the slopes, and the transfers in panel B are needed to estimate the intercepts. To smooth out the function, we regress the expenditures in panel B on the midpoints of the intervals  $[\underline{y}_i, \bar{y}_i]$  and use the linear approximation for the nonmarginal components of the net tax schedule (that is, the  $\alpha_i$  intercept terms). The outcome is shown under the heading “Current Policy” in Table 2. All families in the model face this same net tax schedule, regardless of their productivity type.

For retirees, we estimate transfers  $T_0^r(\cdot)$  using data for Social Security and Medicare and their share of all other government spending.<sup>7</sup> The total varies little across family AGI groups, and therefore we assume it is the same for all income groups in our model.<sup>8</sup> Over the period 2000–2010, the expenditures are \$32,526 in 2004 dollars per retiree and, in the aggregate, a little over 10 percent of GNP.

One more tax rate must be specified, namely, the tax rate on consumption  $\tau_{ct}$ . This tax rate is set residually to impose balance on the government budget. The rate is 6.5 percent in our baseline parameterization.

For the heterogeneous within-cohort model ( $J > 1$ ), we also need to parameterize the productivity levels. The baseline parameterization has  $J = 4$  types of families, which we call *low*, *medium*, *high*, and *top 1 percent*. The productivity level  $\epsilon^j$  for the low types is

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<sup>7</sup> In the United States, individuals can claim Social Security benefits while still in the labor force. When parameterizing the model, we split our sample into two: households receiving benefits and those that are not, regardless of their hours. In most cases, Social Security recipients are supplying little if any labor.

<sup>8</sup> Benefits increase with income, but higher incomes pay taxes on these benefits. See Steurle and Quakenbush (2013) for estimates of lifetime benefits of different groups. Also, we work with families and assume that benefits are attributed to all members over the age of 15.

chosen so that the share of their labor income in the model is 8 percent and matches the share of labor income for U.S. families in AGI brackets covering \$0 to \$15,000 in 2004 dollars. Thirty-eight percent of the population over 15 years old is included with this group. We set the values of  $\epsilon^j$  for the medium, high, and top 1 percent types in a similar way, matching labor income shares in the model to that of U.S. families in AGI brackets \$15,000 to \$40,000, \$40,000 to \$200,000, and over \$200,000, respectively. These groups have population shares equal to 40, 21, and 1 percent, respectively, and labor income shares of 38, 47, and 7 percent, respectively. (See McGrattan and Prescott (2016).) To generate these values in our model, we need to set  $\epsilon^j$  equal to 0.33, 0.98, 2.05, and 6.25 for the four productivity types.

When we simulate the model using the parameters in Tables 1 and 2, our national account and fixed asset statistics match up exactly with the U.S. aggregates averaged over the period 2000–2010. In the case that  $J > 1$ , we also find that the distribution of labor income for the baseline model is the same as the U.S. 2005 CPS sample. In both, the Gini index is 0.49.

### 3.2. Policies

To evaluate the usefulness of the homogeneous within-cohort model, we conduct an often-discussed policy reform, comparing the continuation of the current U.S. policy—taxing workers to finance retiree consumption—with a switch to a new policy in which individuals save for their own retirement. We compute aggregate statistics during and after the transition, as well as the welfare consequences for all current and future families. The policy experiments are conducted in the two versions of the model discussed earlier: the first assumes that productivity levels differ across four family types ( $J = 4$ ), and the second assumes productivity levels are the same for all families ( $J = 1$ ). In all simulations considered, a demographic transition takes place, with the number of workers per retiree falling from 3.93 to 2.40. To generate the decline in the ratio, we assume that the population

growth rate falls linearly from 1 percent to 0 percent over the first 45 years of the transition and the working life is shortened by 2 years.

### 3.2.1. *Continue U.S. Policy*

The continuation policy we consider assumes that transfers for Social Security and Medicare rise at the same rate as the retiree population. According to annual reports summarized in U.S. Social Security Administration (2013), this is a conservative estimate for the growth rate of these transfers. The rise in retiree transfers necessitates increased taxation. Here, we hold the ratio of debt to GNP and defense spending to GNP fixed and raise the consumption tax rate to make up the necessary financing. The net tax schedules for workers and retirees also remain unchanged, but revenues change in response to the demographic transition because economic decisions change. The initial state is summarized by the level of government debt and the distribution of household asset holdings consistent with U.S. current policy.

### 3.2.2. *Policy Reform*

We compare a continuation of the current U.S. policy to a saving-for-retirement regime, with FICA taxes and transfers to retirees phased out. Following McGrattan and Prescott (2016), we also change the tax schedules for workers during the transition period in order to produce a Pareto-improving transition. Two changes in these schedules are made. First, we suspend deductibility of certain employer benefits. Second, we partially flatten the net tax schedule of workers.

Along the transition, we assume that net taxes and transfers are computed with a linear combination of the initial tax schedules,  $T_0^h(\cdot)$ , and the final tax schedules,  $T_\infty^h(\cdot)$ . The rate of change of retiree transfers is equal to the rate of change of the fraction retired. More specifically, let  $r_t$  be the fraction of the population that is retired in year  $t$ , and let  $\mu_t$  be the ratio of new retirees in period  $t$  relative to new retirees on the final balanced

growth path, that is,  $\mu_t = (r_t - r_1)/(r_\infty - r_1)$ , which starts at 0 and rises to 1 over time. We assume that transfers for Medicare and Social Security paid to retirees fall at the same rate as  $-\mu_t$ .

The rate of change of workers' net tax functions is assumed to be faster. If tax rates are lowered at the same rate that old-age transfers fall, the current retirees are indifferent between a continuation of current policy and a shift to the new system because their benefits are not affected. But workers are worse off; they face higher tax rates on their labor income when young but receive lower transfers by the time they reach retirement age. If payroll taxes are lowered more quickly than Social Security and Medicare transfers, then current workers can immediately take advantage of lower taxes on their labor income. Specifically, we let  $\xi_t = \tanh(1.5 - .1t)$ , which is a smoothly declining function with range  $[-1,1]$ , and we assume that the workers' net tax schedule at time  $t$  is given by  $T_t^w(y) = 1/2(T_0^w(y) + T_\infty^w(y)) + 1/2(T_0^w(y) - T_\infty^w(y))\xi_t$ , which falls at a rate that is a little more than twice as fast as the phaseout of the transfers.

When the reform is complete, the payroll tax rates are zero. This means that the slopes  $\{T'_{it}(\bar{y})\}$  in future years at all income intervals,  $[\underline{y}_i, \bar{y}_i]$ , are lower. It also means that the residual in (3.1) is lower. In other words, we have a new piecewise linear net tax schedule for the economy on the final balanced growth path, with new values for  $\{\alpha_i, \beta_i\}$  on AGI intervals  $i = 1, \dots, I$ . We report this new schedule in the columns under the heading "No FICA" in Table 2. When the FICA taxes are eliminated, so too are the retiree transfers associated with Social Security and Medicare. The final retiree transfers in this case are equal to  $T_\infty^r = -13,344$ , which is an estimate of per capita expenditures that, when aggregated, provides the roughly 19 percent of adjusted GNP of resources needed to maintain current spending levels for public goods and transfers (other than Medicare and Social Security).

When we phase out the deductibility of employer contributions, we effectively lower a



component of  $\alpha_i$  in (3.1), namely, the nonmarginal employer benefits shown in Figure 1B. The eventual net tax schedule for workers is reported in Table 2 under the column heading “Suspend Deductibility.” This broadening of the labor income tax base provides a source of revenue for financing the transition in addition to consumption taxes. In addition, when we change the net tax schedule for workers by lowering marginal rates on labor income, we can accomplish our goal of constructing a Pareto-improving policy reform.

If the marginal rates are lowered permanently, then the new tax schedule is that shown in the last two columns of Table 2 under the heading “Lower Marginal Rates.” If it is temporary, we assume a reversal in policy and set the final net tax schedule to be the same as the “No FICA” case in Table 2. For this case, the deductibility of employer benefits is reintroduced and the (non-FICA) marginal tax rates are restored to their earlier levels (found by subtracting the FICA taxes from the total tax rates). The reversion occurs at the midpoint of the transition, roughly 50 years after the changes begin.

The time series we use for phasing out FICA taxes and transfers generate a Pareto-improving transition by design, but not uniquely. We experimented with variations on the time path of workers’ net tax schedule and found others that generated Pareto improvements. The one we report is particularly simple and easy to interpret because the nonmarginal components ( $\alpha_i$ ) are roughly constant across AGI brackets, at \$13,344 per person in 2004 dollars, since the residual category  $T'_i(\bar{y})\bar{y} - T_i(\bar{y})$  is reduced when we lower marginal rates, holding fixed average rates. If we aggregate the spending per person, we find an amount equal to all transfers (other than Social Security and Medicare) and nondefense spending recorded in the national accounts. In other words, we continue funding general public service, public order and safety, transportation and other economic affairs, housing and community services, health, recreation and culture, education, income security, unemployment insurance, veterans’ benefits, workers’ compensation, public assistance, employment and training, and all other transfers to persons unrelated to Social Security or Medicare.

## 4. Results

In this section, we report model predictions for welfare and aggregate statistics, comparing the economy if we continue with current policy or switch to a saving-for-retirement system without payroll taxation or transfers to the elderly.

### 4.1. Welfare

Figure 2 reproduces the main findings in McGrattan and Prescott (2016) in the case that  $J = 4$ . In panel A of Figure 2, we show the welfare gains of gradually eliminating FICA taxes and old-age transfers, while temporarily changing the workers' net tax functions to suspend deductibility of employer benefits and lower marginal rates. The welfare measure that we use is remaining lifetime consumption equivalents of cohorts by age at the time of the policy change and lifetime consumption equivalents of future cohorts. The figure shows the main result: this reform is Pareto-improving for all ages and productivity levels. The gains are slight but positive for cohorts alive at the time of the policy change and over 16 percent for all but the least productive in the future. Figure 2B shows the gains in the case that workers' tax functions are changed permanently. The gains for those alive at the time of the policy change are nearly the same by design, while gains for future cohorts are increasing in level of productivity. The gains for the least productive are roughly 10 percent gains, while the gains for the most productive are more than twice as large.

Figure 3 compares the weighted averages from Figure 2 with the results of the homogeneous within-cohort model that has only one productivity type ( $J = 1$ ), with weights equal to population shares. As before, there are two cases: one with the workers' net tax functions changed temporarily and another with the net tax functions changed permanently. Here, we verify that the aggregate model shows a Pareto improvement for all age cohorts. The figures also show that the averages of McGrattan and Prescott's (2016) heterogeneous-agent model are indistinguishable from the homogeneous-agent model gains

for the cohorts alive at the time of the policy change. For future cohorts, there are some differences when the tax functions are changed only temporarily (shown in panel A). For example, on the new balanced growth path, the welfare gains are predicted to be 14 percent, which overstates the weighted average of 11 percent for the model with  $J = 4$ . However, in the case of a permanent change, both models predict a 12 percent gain for cohorts born after the transition is complete.

## 4.2. Aggregate Data

In Table 3, we report changes in aggregate statistics between the initial and final balanced growth path. (Time series for the full transition are reported in a separate appendix.) Columns (1) and (4) show the results if we continue U.S. policy, and these results are the same regardless of whether we change the workers' net tax functions temporarily or permanently when reforming policy. Columns (2) and (5) show the results if we switch to a saving-for-retirement system, with workers' net tax functions changed temporarily (panel A) or permanently (panel B). We report predictions for GNP, consumption, investments, factor inputs, factor prices, tax revenues, and household net worth. The results for the two versions of the model are remarkably close and confirm the main finding in McGrattan and Prescott (2016), who found large differences in economic activity between staying with current policy and switching to the new saving-for-retirement policy.

Consider first a continuation of current U.S. policy. If we compare GNP on the new balanced growth path with GNP on the original balanced growth path, we find a decline of  $-4.5$  percent for the McGrattan and Prescott (2016) benchmark model with heterogeneous agents (and  $J = 4$ ). In the homogeneous-agent model, which assumes all individuals have the same productivity level (that is,  $J = 1$ ), the prediction is slightly lower at  $-6.3$  percent. In fact, for all aggregate statistics, we find the predictions are biased downward by roughly 1 to 2 percentage points, with the exception of the interest rate, which is the same for both models.

But the aggregate bias is small in comparison to the overall economic impact of financing the retirement of the U.S. aging population. For example, both models predict that a continuation of policy leads to sizable declines in tangible and intangible investments: over 20 percent for tangibles and over 15 percent for intangibles. Both predict large declines in factor incomes, factor inputs, and total tax revenues. Both predict a rise in the wage rate. In fact, the direction of change is the same for all variables.

Next, consider results for the reform with net tax functions of the workers changed only temporarily, which are in columns (2) and (5) of Table 3, panel A. The McGrattan and Prescott (2016) benchmark model with heterogeneous agents (column 2) predicts that GNP would rise 11.4 percent above the old balanced growth path following the reform. The model with homogeneous agents (column 5) predicts a 10.4 percent rise. If we compare GNP following a continuation of policy and GNP following a reform, we also find similar results for the comparison: in the case of heterogeneous agents, the difference in GNPs between the two future regimes is 15.9 percentage points, and in the case of homogeneous agents, the difference is 16.7 percentage points. In other words, we find huge differences for both models. Furthermore, if we rank the policies across *all* of the aggregate variables, we find sizable differences in favor of reform, and the differences are roughly the same magnitudes for the heterogeneous and homogeneous-agent models. Especially noteworthy is the difference in household net worth, which is about 27 percent for both models.

The main findings are unchanged for the policy experiments with workers' tax functions changed permanently. The results in this case are shown in panel B of Table 3. The difference in GNPs between the two future regimes is 20.6 percentage points in the heterogeneous-agents case and 19.5 percentage points in the homogeneous-agents case, and the difference in household net worth between the two future regimes is 27.9 percentage points in the heterogeneous-agents case and 25.7 percentage points in the homogeneous-agents case. Furthermore, a comparison of the differences in all other variables shows that gains to reform are sizable and that the model predictions are remarkably close.

## 5. Conclusions

In this paper, we proposed a simple overlapping generations framework for analyzing the aggregate impact of fiscal policies in economies undergoing demographic change. The proposed framework does not introduce any within-cohort heterogeneity, but generates aggregate predictions in line with our earlier work that does (McGrattan and Prescott, 2016). Our hope is that its simplicity can be exploited by policymakers who need timely analysis.

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TABLE 1  
PARAMETERS OF THE ECONOMY CALIBRATED TO U.S. AGGREGATE DATA

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|   |       |
|---|-------|
| DEMOGRAPHIC PARAMETERS                                |       |
| Growth rate of population ( $\eta$ )                  | 1%    |
| Work life in years                                    | 45    |
| PREFERENCE PARAMETERS                                 |       |
| Disutility of leisure ( $\alpha$ )                    | 1.185 |
| Discount factor ( $\beta$ )                           | 0.987 |
| TECHNOLOGY PARAMETERS                                 |       |
| Growth rate of technology ( $\gamma$ )                | 2%    |
| Income share, Schedule C corporations ( $\theta_1$ )  | 0.500 |
| Capital shares  |       |
| Tangible capital, Schedule C ( $\theta_{1T}$ )        | 0.182 |
| Intangible capital, Schedule C ( $\theta_{1I}$ )      | 0.190 |
| Tangible capital, other business ( $\theta_{2T}$ )    | 0.502 |
| Intangible capital, other business ( $\theta_{2I}$ )  | 0.095 |
| Depreciation rates                                    |       |
| Tangible capital, Schedule C ( $\delta_{1T}$ )        | 0.050 |
| Intangible capital, Schedule C ( $\delta_{1I}$ )      | 0.050 |
| Tangible capital, other business ( $\delta_{2T}$ )    | 0.015 |
| Intangible capital, other business ( $\delta_{2I}$ )  | 0.050 |
| SPENDING AND DEBT SHARES                              |       |
| Defense spending ( $\phi_G$ )                         | 0.044 |
| Government debt ( $\phi_B$ )                          | 0.533 |
| CAPITAL TAX RATES                                     |       |
| Profits, Schedule C corporations ( $\tau_1^\pi$ )     | 0.330 |
| Distributions, Schedule C corporations ( $\tau_1^d$ ) | 0.144 |
| Distributions, other business ( $\tau_2^d$ )          | 0.382 |

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TABLE 2  
CURRENT AND FUTURE LABOR INCOME NET TAX SCHEDULES,  $T_{\infty}^w(y) = \alpha_i + \beta_i y$

| Earnings<br>Over: | Additionally:  |           |            |           |                          |           |                         |           |
|-------------------|----------------|-----------|------------|-----------|--------------------------|-----------|-------------------------|-----------|
|                   | Current Policy |           | No FICA    |           | Suspend<br>Deductibility |           | Lower<br>Marginal Rates |           |
|                   | $\alpha_i$     | $\beta_i$ | $\alpha_i$ | $\beta_i$ | $\alpha_i$               | $\beta_i$ | $\alpha_i$              | $\beta_i$ |
| 0                 | -11,762        | 0.059     | -11,401    | -0.066    | -11,376                  | -0.066    | -13,344                 | 0.000     |
| 5,132             | -12,819        | 0.246     | -12,619    | 0.056     | -12,268                  | 0.056     | -13,344                 | 0.044     |
| 11,664            | -13,518        | 0.264     | -13,427    | 0.123     | -12,752                  | 0.123     | -13,344                 | 0.088     |
| 17,418            | -14,365        | 0.293     | -14,403    | 0.160     | -13,302                  | 0.160     | -13,344                 | 0.132     |
| 23,718            | -15,211        | 0.316     | -15,379    | 0.175     | -13,836                  | 0.175     | -13,344                 | 0.176     |
| 29,692            | -15,971        | 0.332     | -16,255    | 0.187     | -14,353                  | 0.187     | -13,344                 | 0.220     |
| 36,351            | -17,000        | 0.349     | -17,447    | 0.192     | -15,245                  | 0.192     | -13,344                 | 0.230     |
| 45,274            | -18,503        | 0.367     | -19,182    | 0.242     | -16,505                  | 0.242     | -13,344                 | 0.240     |
| 58,274            | -20,359        | 0.382     | -21,325    | 0.275     | -17,189                  | 0.275     | -13,344                 | 0.250     |
| 74,560            | -22,880        | 0.396     | -24,236    | 0.301     | -19,382                  | 0.301     | -13,344                 | 0.260     |
| 106,007           | -28,810        | 0.409     | -31,083    | 0.332     | -25,571                  | 0.332     | -13,344                 | 0.270     |
| 191,264           | -45,792        | 0.409     | -50,690    | 0.372     | -43,894                  | 0.372     | -13,344                 | 0.290     |

*Note:* Earnings and net tax function intercepts are reported in 2004 dollars.



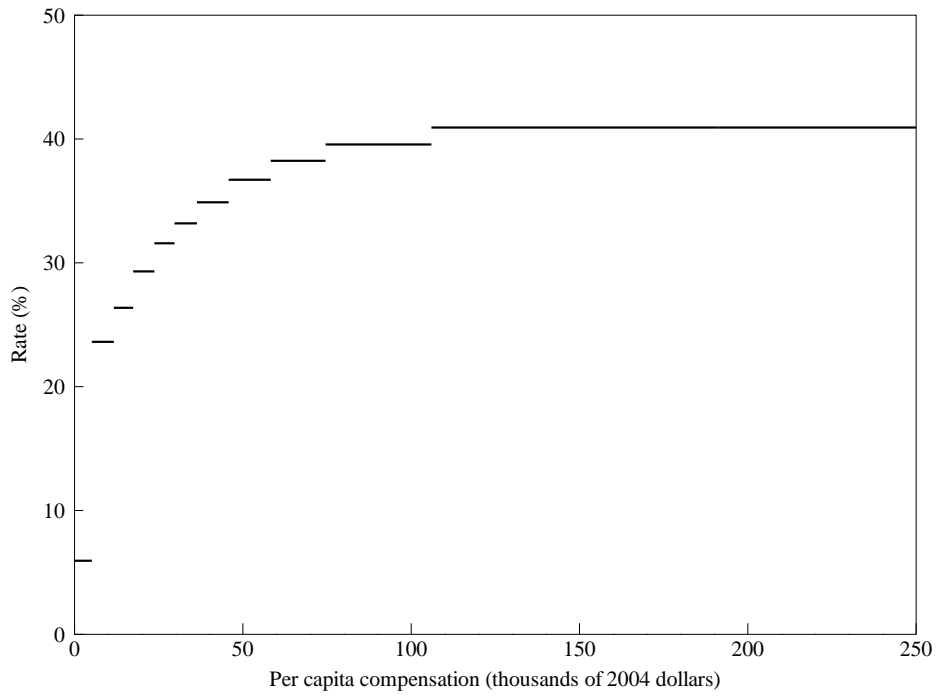
TABLE 3  
CHANGES IN BALANCED GROWTH AGGREGATE STATISTICS

|  | Heterogeneous within Cohort |               |                       | Homogeneous within Cohort |               |                       |
|--|-----------------------------|---------------|-----------------------|---------------------------|---------------|-----------------------|
|  | Continue<br>(1)             | Reform<br>(2) | Difference<br>(2)–(1) | Continue<br>(4)           | Reform<br>(5) | Difference<br>(5)–(4) |
| <b>A. WORKERS' NET TAX FUNCTIONS CHANGED TEMPORARILY DURING REFORM</b> |                             |               |                       |                           |               |                       |
| GNP  | –4.5                        | 11.4          | 15.9                  | –6.3                      | 10.4          | 16.7                  |
| Consumption  | –0.2                        | 13.5          | 13.7                  | –2.1                      | 13.7          | 15.7                  |
| Tangible investment  | –20.6                       | 3.0           | 23.6                  | –21.9                     | 1.1           | 23.0                  |
| Intangible investment  | –15.6                       | 5.8           | 21.4                  | –16.6                     | 4.5           | 21.0                  |
| Labor income   | –5.8                        | 10.8          | 16.6                  | –7.4                      | 9.6           | 17.1                  |
| Capital income   | –24.2                       | –15.5         | 8.7                   | –26.0                     | –16.1         | 10.0                  |
| Tangible capital   | –2.4                        | 27.3          | 29.6                  | –3.9                      | 24.9          | 28.8                  |
| Intangible capital   | –4.3                        | 20.0          | 24.3                  | –5.4                      | 18.5          | 23.9                  |
| Labor input  | –9.8                        | –1.8          | 8.0                   | –10.5                     | –1.3          | 9.2                   |
| Wage rate  | 4.4                         | 12.8          | 8.4                   | 3.4                       | 11.1          | 7.7                   |
| Interest rate, level (%)   | 4.3                         | 3.8           | –0.5                  | 4.3                       | 3.8           | –0.5                  |
| Tax revenues   | 7.3                         | –10.2         | –17.5                 | 6.4                       | –9.8          | –16.1                 |
| Household net worth  | –3.0                        | 24.4          | 27.4                  | –4.5                      | 22.3          | 26.8                  |
| <b>B. WORKERS' NET TAX FUNCTIONS CHANGED PERMANENTLY DURING REFORM</b> |                             |               |                       |                           |               |                       |
| GNP  | –4.5                        | 16.1          | 20.6                  | –6.3                      | 13.2          | 19.5                  |
| Consumption  | –0.2                        | 19.5          | 19.7                  | –2.1                      | 17.6          | 19.7                  |
| Tangible investment  | –20.6                       | 3.0           | 23.6                  | –21.9                     | –0.2          | 21.7                  |
| Intangible investment  | –15.6                       | 7.2           | 22.9                  | –16.6                     | 0.2           | 21.2                  |
| Labor income   | –5.8                        | 15.1          | 20.9                  | –7.4                      | 12.0          | 19.5                  |
| Capital income   | –24.2                       | –10.3         | 13.9                  | –26.0                     | –12.4         | 13.6                  |
| Tangible capital   | –2.4                        | 27.0          | 29.3                  | –3.9                      | 23.1          | 27.0                  |
| Intangible capital   | –4.3                        | 21.6          | 25.9                  | –5.4                      | 18.6          | 24.0                  |
| Labor input  | –9.8                        | 5.2           | 15.0                  | –10.5                     | 4.1           | 14.7                  |
| Wage rate  | 4.4                         | 9.4           | 5.0                   | 3.4                       | 7.6           | 4.2                   |
| Interest rate, level (%)   | 4.3                         | 4.0           | –0.3                  | 4.3                       | 4.0           | –0.3                  |
| Tax revenues   | 7.3                         | –29.1         | –36.4                 | 6.4                       | –29.8         | –36.2                 |
| Household net worth  | –3.0                        | 24.9          | 27.9                  | –4.5                      | 21.2          | 25.7                  |

*Note:* Statistics are percentage changes between the final and initial balanced growth paths, with the exception of the interest rate, which is the level on the final growth path.

FIGURE 1  
 MARGINAL AND NONMARGINAL COMPONENTS OF THE NET TAX FUNCTION

A. Marginal tax rates on labor income



B. Nonmarginal components that affect worker budget sets

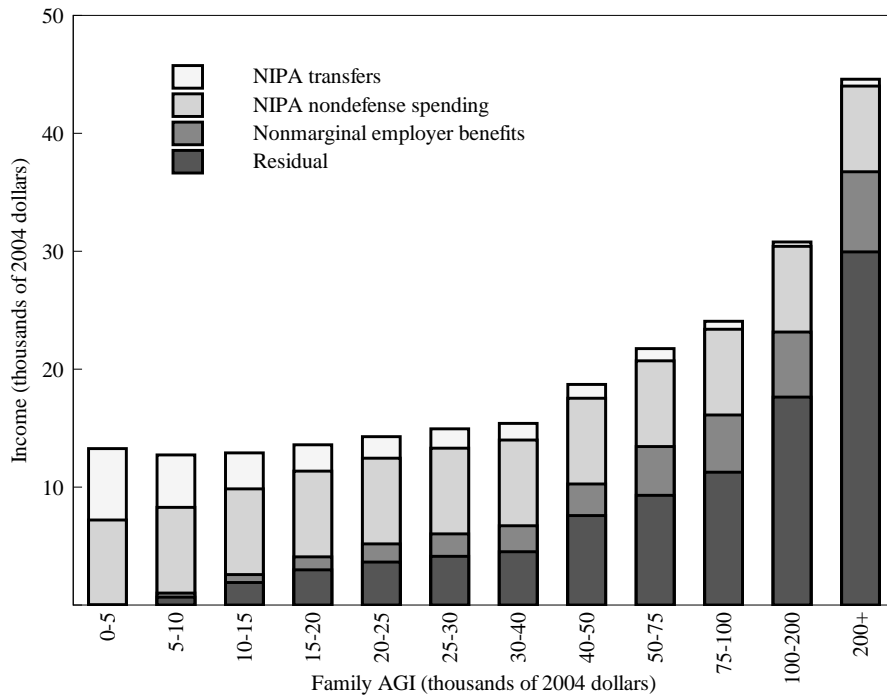
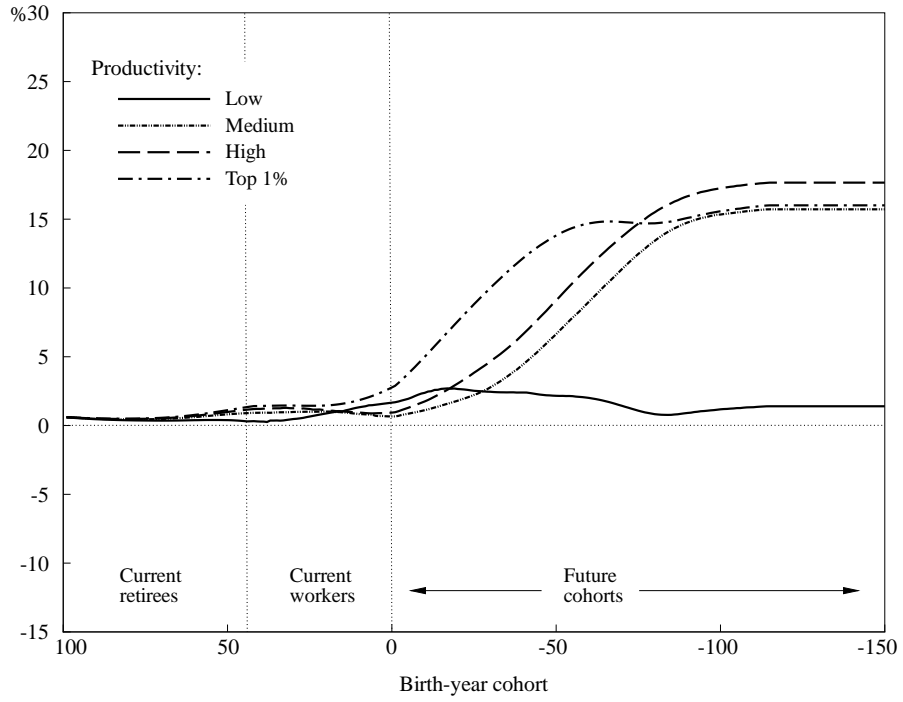


FIGURE 2  
 PERCENTAGE WELFARE GAINS BY AGE COHORT AND PRODUCTIVITY TYPE  
 HETEROGENEOUS-AGENTS MODEL

A.  $T^w(y)$  Changed Temporarily



B.  $T^w(y)$  Changed Permanently

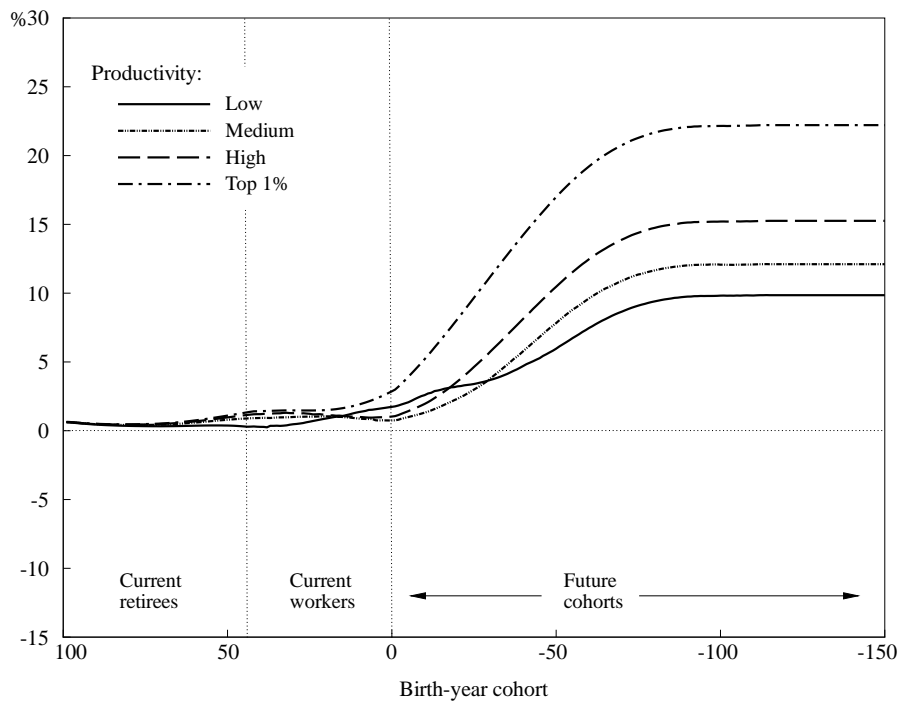
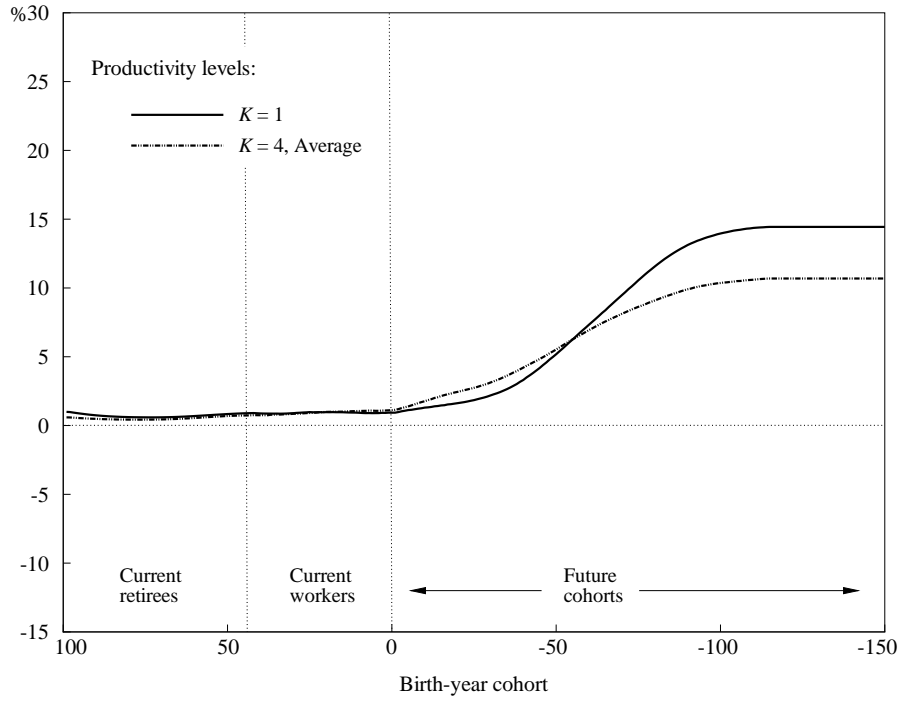


FIGURE 3  
 PERCENTAGE WELFARE GAINS BY AGE COHORT  
 HOMOGENEOUS-AGENTS MODEL AND HETEROGENEOUS-AGENTS MODEL AVERAGE

A.  $T^w(y)$  Changed Temporarily



B.  $T^w(y)$  Changed Permanently

