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ABSTRACT

We develop a theory of *sweat equity*—the value of business owners' time and expenses to build customer bases, client lists, and other intangible assets. We discipline the theory using data from U.S. national accounts, business censuses, and brokered sales to estimate a value for sweat equity in the private business sector equal to 1.2 times U.S. GDP, which is about the same magnitude as the value of fixed assets in use in these businesses. For a typical owner, 26 percent of the sweat equity is transferable through inheritance or sale. The equity values are positively correlated with business incomes and standard measures of markups based on accounting data, but not with owners' financial assets or standard measures of business total factor productivity. We use our theory to show that abstracting from sweat activity leads to a significant understatement of the impacts of lowering business income tax rates on private business activity for both the extensive and intensive margins. Despite finding larger responses, our model's implied tax elasticities of establishments and owner hours are in line with empirical estimates in the public finance literature. Allowing for financial constraints and superstar firms does not overturn our main findings.

Keywords: Intangibles, business valuation

JEL classification: E13, E22, H25

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1. Introduction

In the United States, private businesses now account for over 60 percent of yearly business net income.[1] Using changes in business income after owner retirements and premature deaths, Smith et al. (2019) argue that much of the income is a return on the owner's time. If the payments were just compensation to nontransferable human capital, at the time a business sells, we would observe that only financial and fixed assets are transferred. However, evidence from brokered sales of businesses suggest otherwise. A significant part of the sale price is compensation for building up sweat equity in their business in the form of valuable client lists, customer bases, and other intangible assets. In this paper, we develop a theory in which investments in such assets are a central feature, and we use it, along with U.S. national accounts and business census microdata, to measure net incomes and sweat equity in private business. Once measured, we quantify its role for tax policy reforms.

When businesses sell, the Internal Revenue Service (IRS) requires buyers and sellers to allocate the purchase price across different asset categories. This information is needed to determine the purchaser's basis in each acquired asset and the seller's gain or loss on its transfer. Included in the purchase price allocation are Section 197 intangible assets such as customer- and information-based intangibles, trademarks, trade names, franchises, contracts, patents, copyrights, formulae, processes, designs, patterns, licenses, permits, and goodwill. The IRS also requires terms of noncompete and management consulting contracts provided by the seller. Using data from brokered sales compiled by Pratt's Stats, we construct ratios of intangible asset to total asset values—what we call the *intangible intensity*. We find ratios that are large—an average of 58 percent and a median of 64 percent; the remaining value is attributed to cash, trade receivables, inventories, fixed assets, and real estate. We also find that most sales list a business noncompete or management consulting agreement.

To measure these intangible assets for ongoing concerns and analyze their importance for public policy issues, we develop a dynamic general equilibrium life-cycle model with privately and publicly held businesses. We explicitly model the time use of private business owners. Besides

[1] Bhandari et al. (2020) document a 60.6 percent share of taxable net income and a 65.5 percent share of post-audit net income for private U.S. businesses over the period 2004–2014.

leisure, they put time into two activities: production of goods and services and accumulation of sweat capital, which is an input to production, along with tangible capital, owner hours, and employee hours. The income generated from sweat capital can be thought of as dividends whose present value is the sweat equity we are interested in measuring. During each period over their life cycle, individuals choose to run their own business or work for another business—either privately or publicly held—with this occupational choice driven by stochastic productivities in each activity, financial and sweat assets, and tax policy. We assume nonsweat capital can be rented and external labor hired, and therefore the main start-up costs are owner time and expenses to accumulate sweat capital, an asset that cannot be pledged.

To parameterize the model, we use aggregated data from the U.S. national income and product accounts (NIPA), panel data on labor and business incomes from household surveys and the IRS, and cross-sectional data on business owners and valuations from the U.S. Census and Pratt's Stats. Many parameters in our model—such as those related to private business production and sweat accumulation—have not been estimated in other studies of entrepreneurial choice. Using the structure of the model, we identify these new parameters by utilizing information on entrepreneurial time inputs and occupational switching patterns by age estimated from the U.S. Census Survey of Business Owners (SBO); direct estimates of the market value of intangibles used in private businesses from Pratt's Stats; and studies of useful lives of intangible assets by the General Accounting Office (GAO 1991). We also ensure that the model-generated predictions for aggregate product and income shares by legal form of organization and business age profiles are consistent with their empirical counterparts.

Our calibrated model yields estimates of income flows and valuations for private businesses. We find the private business owners' labor income is 11 percent of GDP. From the NIPA, we estimate a share of 9 percent of GDP for pass-through business owners—sole proprietors, partners, and shareholders in S corporations—who only pay individual income taxes on their share of business net income. Imputations are needed in the case of private C corporation owners, for whose income we estimate a share of roughly 2 percent of GDP. The present value of this labor income is partly a wage payment for time in production and partly the value of sweat equity. Conceptually, our measure of sweat equity is the shadow value for a hypothetical mutual fund that passively invests in all potential private business owners, reaping the net returns after paying owners for their labor in producing private goods and services. Using our calibrated model, we estimate an aggregate sweat equity value of 1.2 times GDP, which is equal to the value of tangible fixed assets in use in these businesses.

Our sweat equity measure includes both transferable wealth in the form of sweat capital and nontransferable wealth in the form of an owner-specific endowment of productivity to run a business. Values reported in brokered sales or business surveys would include only transferable assets. To estimate this value for ongoing concerns, we survey owners in our model at a point in time and ask at what price they would be willing to sell the transferable sweat capital. For current owners, we find that the sale value is 26 percent of the sweat equity value. This estimate implies a price for intangible assets of 1.3 times annual business income for the median owner, which is consistent with Pratt's Stats sales data. We also find that the model predicts much greater dispersion in sale prices if sold today than in sweat equity, since the latter is the value of all future cash flows.

Although we cannot directly observe the sweat capital—or the implied sweat equity—of ongoing businesses, the latent capital stock is positively correlated with some observable variables that could serve as useful proxies. For example, we find that the sweat capital is positively correlated with business incomes, since production cannot occur without clients or customers. Sweat capital is also positively correlated with standard measures of markups—sales relative to variable costs—if expenses are incurred when building the client list. This is true even though there are no actual markups in the baseline model. We find a negative correlation with standard measures of total factor productivity (TFP) that count only the tangible capital stocks, and we find no relation with financial assets, even when we allow for working capital constraints.

We next investigate the quantitative role of including owner time in production and sweat capital accumulation for the study of business taxation. Specifically, we compare the predictions of lowering taxes on incomes of privately and publicly held businesses in our baseline model with those of a nested model in which owner time is fixed and there is no sweat capital. This nested model is Lucas's (1978) span-of-control model, which has become the standard framework in the literature on entrepreneurial choice. As compared with no-sweat Lucas-type models, we find much larger effects of lowering taxes on private businesses in our baseline model on both the intensive and extensive margin. The no-sweat model has a negligible intensive margin effect because a tax on business income ultimately falls on the return to a *fixed* managerial input and hence is not distortive. Introducing sweat, we find a large effect on the intensive margin as owners work longer hours and hire less outside labor. We also find a large effect on the extensive margin as owners' sweat investment increases future returns of other factor inputs. Across business owners, we find that most of the changes are attributed to businesses that have high productivities and large sweat capital stocks. Although true productivities are exogenous and true markups are constant, standard measures of total factor productivities and markups are significantly higher after the tax change. If we additionally lower tax rates on corporate profits, then we find larger responses for the effects on the private business sector across the models with and without sweat activity but similar predictions for the effects on the C-corporate sector and in the aggregate.

Despite finding large differences in theoretical predictions when we add sweat capital, our estimates of short-run tax elasticities of business counts and owner hours in response to changes in business tax rates are in line with comparable empirical estimates. We estimate that a 1 percentage point decrease in the private business tax rate leads to a 0.42 percentage increase in the number of private businesses in the first year, which is in line with Giroud and Rauh's (2019) estimate of 0.43 for the establishment elasticity of pass-through businesses. Our estimate of the short-run tax elasticity of owner hours is equal to 0.29, which is comparable to the 0.33 point estimate of the intensive-margin labor-supply elasticity reported in Chetty et al.'s (2011) meta-analysis. Thus, while our model matches macroeconomic data, we find an implied labor elasticity for owners that is lower than labor elasticities typically used in the macro business cycle literature.

Finally, we show that the economic effects of taxing income from self employment are quite different that those of taxing income from paid employment. Lowering tax rates on owner time results in more entry and firms that are smaller in scale. In contrast, lowering tax rates on employee time leads to fewer owners since owners can make more working for someone else. However, the owners that find paid employment more attractive are not typically the most productive in business. Even with lower taxes on paid employment, the very highly productive owners would still find it optimal to run a business. With fewer owners and continued demand for the goods they produce, relative prices rise, and more outside labor and capital are used to meet that demand. In effect, the tax change results in fewer firms that are larger in scale.

2. Related Literature

Our paper is related to studies of small businesses and entrepreneurship. There are now many quantitative theories of entrepreneurship. Most model entrepreneurs as owners of physical capital subject to uninsurable idiosyncratic risk and financing constraints. See, for example, Angeletos and Calvet (2006), Boar and Midrigan (2019), Buera (2009), Cagetti and De Nardi (2006), Dyrda and Pugsley (2017), Li (2002), Meh (2005), Peter (2019), Quadrini (1999, 2000), and Wellschmied and Yurdagul (forthcoming). These studies focus on the role of financial frictions in accounting for dispersion in survey-based measures of wealth and income.[2] We include working capital constraints disciplined by estimates of available funds to value added but find they have a negligible effect on the results of our tax analyses. We also include superstar firms—whose owners earn 10 times the median labor income—and show that the model can generate large wealth ginis without assuming extreme productivity differences in the distribution. (See, for example, Castaneda, Diaz-Gimenez, and Rios-Rull, 2003.)

Another related line of research models entrepreneurial choices as driven by the nonpecuniary benefits of owning a business. See, for example, Hamilton (2000), Moskowitz and Vissing-Jorgensen (2002), Hall and Woodward (2010), and Hurst and Pugsley (2011, 2017). This literature is informed by survey responses of small-business owners and evidence that these owners have lower accumulated earnings over time than they would have had if they had worked for someone else and made fewer risky investments. With regards to this finding, we find that differences in the effective marginal tax rates of business owners and wage earners can account for almost all differences in pre-tax earnings. Thus, altering preferences to include a role for nonpecuniary benefits does not alter our main quantitative findings.

None of the studies on entrepreneurial choice explicitly model the accumulation of the business

[2] The literature on factor misallocation uses similar theories of entrepreneurs to quantify cross-country differences in aggregate productivity. See, for example, Buera and Shin (2013), Midrigan and Xu (2014), and Restuccia and Rogerson (2008). See also Hopenhayn's (2014) survey for a complete list of references.

owner's sweat in building the business and therefore cannot be used to estimate aggregate or cross-sectional valuations of this key business asset or its role for tax policy reform.[3]

3. Theory

In this section, we develop a theory to measure sweat equity in private businesses and to serve as a tool for evaluating tax policy counterfactuals. We start with an overview of the environment and then turn to a full description of the dynamic programs solved by different agents in the economy. A nomenclature for all model variables is provided in Appendix A.

3.1. Overview

The economy is populated with individuals who are endowed with skills that govern their productivities in running businesses and paid employment. They stochastically age over the life cycle and make occupational choices: to work as employees or to own and operate a private business.

We assume that there are two business sectors: publicly held C corporations and privately held pass-through businesses that sell goods and services.[4] Businesses in the two sectors differ in their technologies, exposure to idiosyncratic risk, and tax treatment. Moreover, the goods produced in the two sectors are imperfectly substitutable.

In the case of private firms, owners bear idiosyncratic risk and put time into producing goods and services and building sweat capital—the business customer base, client list, and other non-pledgeable intangible assets. Private firms also use fixed assets and time of paid employees. Publicly held C corporations are assumed to have fully diversified ownership. They use fixed assets as well as the time of paid employees as inputs to a constant-returns production technology.

Business incomes in the two sectors face different tax treatment. C corporations pay corporate income tax on profits, and the shareholders pay individual income tax on any distributions, while

[3] In other literatures, researchers model investments in intangible capital—including brand and customer capital—to study trade patterns, asset pricing, firm dynamics, and business cycles, but they do not model the time-use decisions of private business owners. See, for example, Arkolakis (2010), Atkeson and Kehoe (2005), Belo, Lin, and Vitorino (2014), Drozd and Nosal (2012), Gourio and Rudanko (2014), and McGrattan and Prescott (2010a, 2010b).

[4] In the United States, there are privately held C corporations, for which we have very limited data from the IRS. When calibrating the model, we separate businesses into C corporations and pass-through businesses but later use the limited data that we have to impute incomes and valuations for privately held C corporations.

pass-through entities distribute all profits to their owners, who pay individual income taxes on the proceeds.

There is a competitive intermediation sector with risk-neutral financial intermediaries, which accept deposits and use the funds to purchase equities of publicly held firms, government bonds, and fixed assets that they rent to private firms.

Finally, there is a nonbusiness sector that includes production by the government, households, and nonprofit institutions that primarily serve households. Government purchases are financed by taxes on consumption, individual incomes, and corporate profits.

We next turn to a formal description of this environment.

3.2. Occupational Choice

At a point in time, the state vector s for any individual—whether an entrepreneur or an employee—includes financial assets a, sweat capital κ, the productivity of running a business z, the productivity of working for someone else ϵ, and age j. The occupational choice of an individual is made to maximize the overall value

$$V(s) = \max\{V_p(s), V_w(s)\},$$

where $V_p(s)$ is the value of self employment and $V_w(s)$ is the value of paid employment. To keep the life cycle problems tractable, we allow for stochastic aging between young ages, $j = y$, and old ages, $j = o$, as in Blanchard (1985). We also assume that individuals spend some fraction of their life in paid employment and another in self employment; the spells do not overlap. In reality, some individuals do both activities simultaneously: they work for someone else and run a business. However, data on time use show that average hours on the primary job are much higher than those on the secondary job.

Individuals who run a private business make decisions related to both their household and their business. They choose consumption of C-corporate goods and services, c_c, consumption of private firm goods and services, c_p, leisure ℓ, hours in production h_p, hours building sweat capital h_κ, employee hours n_p, fixed assets k_p, and financial assets next period a'. Given a continuation value V, the dynamic programming problem for owners is

$$V_p(s) = \max_{\substack{c_c, c_p, h_p, h_\kappa, \\ n_p, k_p, a'}} \{U_p(c(c_c, c_p), \ell) + \beta \sum_{z', \epsilon', j'} \pi(z', \epsilon', j' | z, \epsilon, j) V(s')\}, \tag{3.1}$$

subject to

$$\begin{aligned} a' = [&(1+r)a + py_p - (r+\delta_k)k_p - wn_p - e - (1+\tau_c)(c_c + pc_p) \\ &- T_j^b(py_p - (r+\delta_k)k_p - wn_p - e)]/(1+\gamma) \end{aligned} \tag{3.2}$$

$$\kappa' = [(1-\delta_\kappa)\kappa + f_\kappa(h_\kappa, e)]/(1+\gamma) \tag{3.3}$$

$$y_p = z\zeta_j f_p(\kappa, k_p, h(h_p, n_p)) \tag{3.4}$$

$$\ell = 1 - h_\kappa - h_p \tag{3.5}$$

$$a' \geq \chi p y_p, \tag{3.6}$$

where $V_p(s)$ is the discounted present value of utility for an owner with state $s = \{a, \kappa, z, \epsilon, j\}$ and π is the Markov transition kernel for shocks (z, ϵ, j). The utility function U_p is defined over a consumption composite $c(c_c, c_p)$ of goods and services produced by C corporations and private businesses and leisure ℓ. Owners allocate nonleisure hours between growing their businesses, h_κ, and producing goods and services, h_p. These hours enter the production functions for sweat capital (3.3) and goods and services (3.4), respectively. We allow for productivity differences between young and old with $\zeta_y = 1$ and $\zeta_o \leq 1$.

The change in asset holdings, $a' - a$, is equal to after-tax incomes less consumption expenditures, $c_c + pc_p$, where p is the relative price of goods and services sold by private businesses. Financial assets earn an after-tax interest rate of r and thus provide ra in financial income. Pre-tax business net income from production of output y_p is equal to sales py_p less rental payments on fixed assets rk_p, depreciation $\delta_k k_p$, employee wages wn_p, and intermediate expenses e. The constraint (3.6) on assets for the business owners depends on the term $\chi p y_p$, which can be interpreted as a working capital constraint for business owners. The tax schedule for this income is $T_j^b(\cdot)$, which includes government transfers that may be age-dependent.[5] Taxes are also paid on consumption at rate τ_c. Owners take as given the vector of prices (p, r, w) for goods, assets, and

[5] To avoid additional notation, we also include nonbusiness incomes that are taken to be exogenous with government transfers.

employee hours when solving the maximization problem (3.1). The term $(1+\gamma)$ in (3.2) appears because all nonstationary variables are detrended by the economy-wide growth rate $(1+\gamma)^t$.

We should note that our modeling of capital financing in private businesses is not conventional. We assume that owners of private firms rent the tangible capital used in their businesses—a choice motivated by data on business tax filings showing large deductions for interest payments, rents, and leasing. In the presence of rental markets, the financial constraints operate through minimum liquidity requirements for working capital. Most of the existing literature, in contrast, models financial constraints as a requirement that capital be largely self-financed.

We assume that owners cannot produce without sweat capital—that is, $f_p(0, k_p, h(h_p, n_p)) = 0$. We have in mind that the business needs a customer base before producing goods and services. This is an asset accumulated and maintained with owner time and expensing, as in (3.3), and is potentially transferable through inheritance or sales, while productivity z is specific to the owner. Part of the value transferred may be the knowledge and reputation of the business owner, as evidenced by business transactions in the Pratt Stats database indicating transitional training of the buyer and maintenance of customer relationships. The alienability of sweat capital is a key distinction of our model, relative to entrepreneurial choice models in the tradition of Lucas (1978) and human capital models in the tradition of Ben-Porath (1967).

The problem of working for someone else is relatively standard. (See, for example, Aiyagari 1994, Huggett 1996, and Imrohoroglu et al. 1995) In this case, the individuals choose consumption of C-corporate goods and services, c_c, consumption of private firm goods and services, c_p, leisure ℓ, and financial assets next period a'. Given the continuation value V, the problem solved by workers is given by

$$V_w(s) = \max_{c_c, c_p, \ell, a'} \{U_w(c(c_c, c_p), \ell) + \beta \sum_{z', \epsilon', j'} \pi(z', \epsilon', j' | z, \epsilon, j) V(s')\}, \tag{3.7}$$

subject to

$$a' = [(1+r)a + w\epsilon\zeta_j n - (1+\tau_c)(c_c + pc_p) - T_j^w(w\epsilon n)]/(1+\gamma) \tag{3.8}$$

$$\kappa' = (1-\lambda)\kappa, \tag{3.9}$$

$$\ell = 1 - n \tag{3.10}$$

$$a' \geq 0, \tag{3.11}$$

where $V_w(s)$ is the discounted present value of utility for a worker with state s. The utility function U_w is defined over a consumption composite $c(c_c, c_p)$ and leisure ℓ. The change in asset holdings carried to the next period, $a' - a$, is equal to financial income ra plus wage earnings $w\epsilon n$ less consumption expenditures $c_c + pc_p$ and tax payments. Workers earn wage rate w per effective hour ϵn, regardless of whether they work for a privately or publicly held firm. The net tax schedule for wages is given by $T_j^w(\cdot)$, and consumption expenditures are taxed at rate τ_c. As in the case of the owners' problem, the tax schedule and productivity shocks are age dependent.

The only feature of the problem in (3.7) that is not standard is the inclusion of sweat capital κ in the state vector. Workers who have previously run or inherited a business may have accumulated sweat capital. The value of this capital deteriorates at rate λ while not in use.[6]

3.3. C Corporations

There is a competitive C-corporate sector with firms choosing hours n_c and investment in fixed assets x_c to solve the following dynamic program:

$$v_c(k_c) = \max_{n_c, x_c} \{(1 - \tau_d) d_c + \left(\frac{1+\gamma}{1+r}\right) v_c(k_c')\}, \tag{3.12}$$

subject to

$$k_c' = [(1 - \delta_k) k_c + x_c] / (1 + \gamma)$$
$$y_c = AF(k_c, n_c)$$
$$d_c = y_c - wn_c - x_c - \tau_p (y_c - wn_c - \delta_k k_c),$$

where k_c is fixed assets, d_c is dividends that are taxed at rate τ_d after paying corporate income taxes at rate τ_p, and y_c is output from a constant returns to scale technology F, with TFP given by A.[7] Employees working for C corporations earn the same hourly wage, w, as employees in private businesses. Here, we assume that the capital used in the C-corporate sector is equity financed and therefore $\delta_k k_c$ is the only component of the cost of capital that is tax deductible.[8]

[6] In effect, we are assuming that sweat capital is not a productive input and cannot be accumulated while in paid employment. With panel data tracking individuals across paid and self employment spells, one could relax this assumption.

[7] We assume that x_c includes intangible investments made by C corporations and later exploit the fact that the BEA collects data on many of these investments.

[8] For a more comprehensive treatment of debt financing by corporations see Barro and Furman (2018).

Our formulations of the problems of privately and publicly held businesses in (3.1) and (3.12) ignore changes in legal forms of organization that might occur—for example, if C corporations want to avoid double taxation by choosing pass-through status or if pass-throughs want to pursue better financing opportunities by choosing C-corporate status. There are several reasons for this abstraction. First, in practice, privately held C corporations largely avoid double taxation, which is why we combine privately held C corporations with pass-throughs and only model privately held businesses.[9] Second, only a few firms switch between being privately held and being publicly held. In Bhandari and McGrattan (2020), we show this using evidence from the Kauffman Firm Survey over the period 2004–2011 and the sample of employer firms in the Longitudinal Business Database analyzed by Dyrda and Pugsley (2017) over the period 1980–2011.[10]

3.4. Market Clearing and Equilibrium

The model includes a competitive financial sector with risk-neutral intermediaries that accept deposits and use the funds to invest in C-corporate equities, government bonds, and fixed assets. Details of the problem solved by intermediaries and the asset market clearing condition, along with the government budget constraint are provided in Appendix B. For our model economy, we analyze a stationary recursive competitive equilibrium. The equilibrium concept is standard and fully detailed in the appendix.

4. Model Parameters

In this section, we set parameters of preferences, technologies, stochastic processes, and government policies to match key moments for U.S. aggregate data and microsamples of businesses. We start with functional form choices and a summary of data sources used to estimate the model parameters.[11]

[9] See Hamill (2005) and Quantria Strategies (2009). These studies show that effective taxes for small C corporations are comparable to those for pass-throughs.

[10] There are, of course, nonzero public listings (IPOs), and our estimates of aggregate sweat equity will miss the compensation to owners' sweat that is realized in an IPO. However, IPO activity is small in the aggregate, accounting for only about 0.2 percent of GDP. Also, pass-through firms such as S corporations are restricted from issuing shares with differential voting rights. Therefore, nearly all IPO activity, including the pre-IPO venture financing or angel investing, occurs via non-pass-through firms. Adding a listing decision is an interesting extension to our framework but not a focus of this study.

[11] See Bhandari and McGrattan (2020) for full details on data sources and replication codes.

4.1. Functional Forms

The following functional forms are used for specifying preferences, technologies, and dynamic transitions:

$$
\begin{aligned}
U_w(c,\ell) &= \left(c^{1-\psi}\ell^{\psi}\right)^{1-\sigma}/(1-\sigma) \\
U_p(c,\ell) &= U_w(c,\ell)+\xi \\
c(c_c,c_p) &= c_c^{\eta}c_p^{1-\eta} \\
F(k_c,n_c) &= k_c^{\theta}n_c^{1-\theta} \\
f_{\kappa}(h_{\kappa},e) &= \left(h_{\kappa}^{\vartheta}e^{1-\vartheta}\right)^{\varphi} \\
f_p(\kappa,k_p,h) &= \kappa^{\phi}k_p^{\alpha}h^{\nu} \\
h(h_p,n_p) &= \left(\omega h_p^{\rho}+(1-\omega)n_p^{\rho}\right)^{\frac{1}{\rho}} \\
\pi(z',\epsilon',j'|z,\epsilon,j) &= \begin{cases} \tilde{\pi}(z',\epsilon'|z,\epsilon)\,\pi_y & \text{if } j=y, j'=y \\ \tilde{\pi}(z',\epsilon'|z,\epsilon)\,(1-\pi_y) & \text{if } j=y, j'=o \\ \tilde{\pi}^e(z',\epsilon')\,(1-\pi_o)\,\iota & \text{if } j=o, j'=y \\ \tilde{\pi}(z',\epsilon'|z,\epsilon)\,\pi_o & \text{if } j=o, j'=o, \end{cases}
\end{aligned}
$$

where $\phi+\alpha+\nu=1$. The parameter ξ captures nonpecuniary benefits from running a business. The function $\tilde{\pi}(z',\epsilon'|z,\epsilon)$ is the transition matrix for productivity shocks with ergodic probabilities denoted by $\tilde{\pi}^e(z',\epsilon')$. The probabilities of remaining in the young and old states are π_y and π_o, respectively. Upon death, the next generation receives all assets a and a share $1-\lambda_d$ of the sweat capital but draws new productivity levels from the ergodic distribution. The parameter $\iota \in [0,1]$ is a measure of altruism, with the two extremes capturing no (0) or full (1) altruism of parents.

In addition to the parameters of the functions specified here, we need to set depreciation rates for fixed assets δ_k and sweat capital δ_κ, the discount rate β, the growth rate γ, the rates of deterioration of sweat capital λ and λ_d, the severity of the financing constraint χ, the productivity processes, and government budgets. Without loss of generality, we set the level of TFP in C-corporate production, A, so that y_c is normalized to 1 in equilibrium.

4.2. Data Sources

The main sources of aggregate data for our study are the national accounts and fixed asset tables compiled by the BEA, along with aggregated tax filings for S and C corporations from the

IRS. In Appendix C, we show how these data can be used to construct model national accounts and fixed asset tables. In doing so, we introduce a new category of income that we call *sweat income*—which adds together BEA proprietors' income (from sole proprietorships and partnerships) and the part of corporate profits reported to the IRS that is compensation to owners of S corporations less any payments to capital or to employees. The sweat income share will turn out to be a critical moment to match. BEA and IRS data are also needed to align the government budget constraints in the model and data.

The main sources of microdata on businesses are the SBO for characteristics of businesses and their owners and Pratt's Stats for data on assets transferred in brokered sales. For the SBO, we use the public-use microsample for the year 2007 which consists of 2.3 million firms, and for Pratt's Stats, we use a sample of 6,858 sales of private businesses over the period 1994–2017. Critical information from the SBO includes owner age, hours, financing, and the year of acquiring a share of the business. Critical information from Pratt's Stats is the value of intangible assets listed as part of the purchase price allocation recorded on IRS Form 8594, which establishes the asset bases for the buyer and capital gains or losses for the seller. We use this information to construct an estimate of the intangible intensity of the businesses in the database; that is, $\text{ii}(s)$ for a business s is given by

$$\text{ii}\,(s) = \frac{v_{\kappa}\,(s)}{v_{\kappa}\,(s) + k_p\,(s)}, \tag{4.1}$$

where $v_{\kappa}(s)$ is the value of transferable intangible assets. We can also use this concept for continuing businesses in our model by interpreting $v_{\kappa}(s)$ as the amount of cash needed to leave a business owner indifferent between continuing in business with sweat capital κ and selling it; that is, $v_{\kappa}(s)$ satisfies

$$V_p\,(s) = V_w\,(a + v_{\kappa}\,(s)\,, 0, z, \epsilon, j)\,. \tag{4.2}$$

The Pratt's Stats database also has information on business age, which we use, along with the intangible asset valuation, to discipline patterns of sweat capital accumulation in the model.

Another important data source for studying sweat capital accumulation is the GAO (1991) study of intangible asset amortization. At the time of the GAO study, the tax law was such that businesses separately determined the specific useful life for each intangible asset, except in the case of goodwill, which could not be amortized for tax purposes. Because taxpayers have an incentive to depreciate their assets, the IRS frequently challenged taxpayers. The GAO analyzed 2,166 such cases, spanning the years 1979 to 1987, and covered businesses in nine different industries with 175 types of intangible assets. We use this analysis to study the useful lives of intangible assets.

Microdata on individuals are used to estimate tax schedules and productivity processes. Most private businesses pass incomes to owners, who file individual tax returns. Detailed data needed to compute the effective tax schedules for both owners and employees are published in the IRS Statistics of Income (SOI). Panel data from the IRS, the Survey of Income and Program Participation (SIPP), and the Panel Study of Income Dynamics (PSID) are used for the productivity processes governing self and paid employment.

Next, we provide details on specific parameters summarized in Table 1 for preferences, technologies, life cycle, financing, productivity processes, and tax rates. Given the large number of parameters and moments to be matched, we align model and data statistics with the help of a Nelder-Mead optimization algorithm.

4.3. Preferences

Certain information about private businesses helps discipline key preference parameters. Most notably, when we construct national accounts for the model, as in Appendix C, we find an estimate for sweat income equal to 9 percent of GDP. Sweat income depends crucially on demand for private good consumption and thus on the parameter η, which governs consumption shares of publicly and privately produced goods and is set to 0.449 in our baseline.

Using data from the SIPP, Hamilton (2000) finds that the typical entrepreneur earns significantly less running a business than working in paid employment—with a median earnings differential of 35 percent for those in business for 10 years. The parameter ξ governs the nonpecuniary benefit of running a business and is typically chosen to deliver this earnings differential. In our case, the differences in effective taxes on wage and pass-through income come close to guaranteeing a 35 percent higher *pre-tax* income to paid employment relative to self employment. Therefore, we set $\xi = 0$ in our baseline model.

Data on hours of work help discipline the weight ψ on leisure, which we set equal to 0.58. In our case, total business hours are the sum of hours for employees $n_c + \int n_p(s)\mu(s)\,ds$ and private business owners $\int(h_p(s) + h_\kappa(s))\mu(s)\,ds$, where $\mu(s)$ is the measure of individuals with state s. In 2007, the noninstitutional U.S. population ages 16 to 64 was 197 million, and the average annual hours per person were 1,465 according to the Bureau of Labor Statistics (BLS).[12] If each person has 5,200 hours of annual discretionary time, then 28.2 percent of aggregate available time is allocated to work. Government, nonprofit, and household employees—which we include with nonbusiness activity—contribute roughly 5.8 percent, and thus business hours are assumed to be 22.4 percent of aggregate available time.

For the two remaining parameters, we use standard estimates. The discount factor is set equal to $\beta = 0.98$, consistent with a 4 percent annual interest rate, and the inverse of the intertemporal elasticity is set equal to $\sigma = 1.5$.

4.4. Technologies

Next, consider parameters of technologies reported in panel A of Table 1, starting with those related to sweat capital production and followed by those that are more standard.

Relative to the no-sweat model in Lucas (1978), there are many new technology parameters: the shares of sweat capital and owner hours in private business production (ϕ, ω); the elasticity of substitution between owner and employee time (ρ); the curvature and share parameters in sweat capital accumulation (φ, ϑ); and the rates of sweat capital depreciation in active and nonactive use (δ_κ, λ). As we noted earlier, the microdata from SBO and Pratt's Stats are most informative for inference here because these technology parameters are quantitatively important in predicting the intangible asset valuations and age profiles of private businesses. We also use NIPA data on compensation shares by industry and by legal form of organization.

Although there is not a one-to-one mapping between each parameter and each statistic, there are intuitive forces at work that lead us to choose certain empirical moments to match. Two important moments turn out to be the average intangible intensity of 58 percent from Pratt's Stats and the business entry rate of 11.5 percent based on the SBO business age profile. We find

[12] For compatibility with the SBO microsample on business, we use data for 2007 where possible. We also check that nothing changes if we average over more years. See Bhandari and McGrattan (2020) for further details on the available data and the construction of moments described in this section.

that the choice of share parameter ϕ is critical for sweat capital valuations and the intangible intensity of the business, as expected, but it is also quantitatively important for predicting the shape of the age profile of businesses; the higher ϕ is, the longer the duration of benefits from investing in one's business is. Similarly, the deterioration rate λ is critical for the age profile: the higher the rate, the more costly it is to exit and reenter the business sector. This cost naturally lowers the fraction of young businesses in the cross section. For the baseline optimization, we find $\phi = 0.15$ and $\lambda = 0.6$.

One potential issue estimating intangible intensity with broker data is that we may encounter selection bias, conditioning only on businesses that eventually sold. There may be downward bias due to distressed selling, say because the owners had health issues. To investigate this, we compute average intensities by reason for sale and find that they are not too different. There may be upward bias if successful businesses are more likely to be sold and thus overrepresented in Pratt's Stats. To investigate this, we construct independent estimates of intangible intensities for ongoing S corporations using SOI data and proxies for valuations using Compustat data. For most industries, we find average intensities that are higher than those based on the Pratt's Stats data. (See Section 2.1.2 of Bhandari and McGrattan (2020).) Finally, in Section 7, we deal with selection within the model extending the framework to include brokered sales. After recalibrating, we find a similar estimate for ϕ as in the baseline model when matching the Pratt's Stats average intangible intensity.

The choice of curvature on sweat capital investment, φ, affects the model's prediction of firm value by age, much as an investment adjustment cost would. Higher values of φ imply faster growth in sweat capital value. Because of this relation, we regress the logarithm of intangible asset value on business age and age squared with data from Pratt's Stats and use the regression coefficients as targets for our model.[13] For the baseline model, we find a value for φ very close to 1, which implies a Cobb-Douglas form for f_κ. To estimate ϑ, we would ideally want cost shares of labor and materials in the production of sweat capital. Since cost shares are not available at such a disaggregated level, we use cost share information from the NIPA input-output tables for sectors that are the most sweat-capital intensive. Specifically, we compute a weighted average of labor to

[13] We also control for year and industry. The regression results are reported in Bhandari and McGrattan (2020).

intermediate factor shares with sales weights by sector for pass-through businesses in the Pratt's Stats database. This yields an estimate of $\vartheta = 0.408$.

Our estimate of the sweat capital depreciation rate is $\delta_\kappa = 5.8$ percent and is based on the GAO (1991) study. The study found that taxpayers claimed an average useful life of eight years for intangible assets with a determinable life, which is consistent with a 15.9 percent rate of depreciation.[14] At the time of the study, the IRS treated goodwill as a nonamortizable asset, although the Financial Accounting Standards Board (FASB) assumed a useful life of 40 years, which is consistent with a 3.4 percent rate of depreciation. Thus, we set $\delta_\kappa = 1-(1-.159)^{.18}(1-.034)^{.82}$ using the Pratt's Stats estimate that 82 percent of intangible assets are allocated to goodwill.[15]

Another new feature in our baseline model relative to Lucas (1978), is owner hours in production, which necessitates estimating the share ω and elasticity ρ of owner hours relative to employee time. Since employees can work in either the private or C-corporate sector, we require that our model match the fact that 33 percent of aggregate employee compensation recorded in NIPA is paid by pass-through businesses. The parameter ρ governs the elasticity of substitution between owner and employee hours in private business; the more substitutable they are, the greater the opportunity for an owner to scale up the business if productivity is high is. Therefore, to inform this parameter, we match the percentage change in payroll relative to the percentage change in owner hours in the SBO data. For the baseline calibration, we find $\omega = .425$ and $\rho = 0.5$.

Data from the BEA fixed asset tables are most relevant for capital income shares, α and θ, and the depreciation rate δ_k. With our parameter choices, the model delivers capital-output ratios for C corporations and pass-through businesses that are roughly 2 times GDP and 1 times GDP, respectively, as is the case for the United States.[16] The investment rates for C corporations and private businesses are also consistent. For the baseline optimization, we find $\alpha = 0.3$ and $\theta = 0.5$.

Given values for α and ϕ, the share of labor (owner plus employee time) is $\nu = 0.55$, and the

[14] In the GAO sample, the shortest lives were contract-based assets (for example, non-compete contracts) averaging 6.3 years and the longest were statutory-based assets (for example, patents and copyrights) averaging 10.6 years. The largest category was customer-based assets, which averaged 8.8 years.

[15] In Bhandari and McGrattan (2020), we show that our main quantitative results are hardly changed under alternative baselines with a lower curvature parameter φ and higher and lower shares on owner hours ϑ or assuming an eight-year useful life for all intangible assets.

[16] These estimates are based on an expanded notion of intellectual property product (IPP) investments, which are estimated at 12 percent of GDP, rather than those currently counted in the NIPA, which are estimated at 4 percent of GDP. See Bhandari and McGrattan (2020) for details.

predicted hours of work for business owners, $\int(h_p(s)+h_\kappa(s))\mu(s)\,ds$, are roughly 23 percent of all hours worked in business. This prediction is close to the 25 percent estimate based on microdata from the SBO. Finally, for aggregate growth in technology, γ, we use the U.S. trend rate of 2 percent.

4.5. Life Cycle

Parameters governing life-cycle patterns are chosen to match overall U.S. population statistics and age profiles for young and old business owners. These include age-dependent parameters of the transition probability matrix $\pi(z', \epsilon', j'|z, \epsilon, j)$ in (3.1) and (3.7), the old-age productivity shock ζ_o, old-age transfers, and parameters related to altruism and inheritance.

In our baseline model, we set the stochastic aging parameters equal to $\pi_y = 0.978$ and $\pi_o = 0.933$ to ensure that one-fourth of the model population is over 65, with the average duration of working years at 45. The parameter ι is set to 1, implying that parents are fully altruistic. We use SBO business age profiles for young and old owners to inform our parameterization of the old-age productivity shock ζ_o and the deterioration rate at death λ_d. For the baseline optimization, we find a significant fall off in productivity with $\zeta_o = 0.5$ and significant deterioration at death with $\lambda_d = 0.9$.

The estimates for ζ_o and λ_d are also consistent with evidence from tax data found by Smith et al. (2019). They identify pass-through businesses with owners that retire or die prematurely and compare their profit streams to businesses that did not experience a retirement or death. They find that profits of the two track closely prior to the event—either the retirement or death—but not afterward. In the case of owner retirements, business profits fall off by 60 to 72 percent depending on the sample analyzed. In the case of owner deaths, profits fall off by 73 to 92 percent.

Later, we extend the model to allow for the transfer of sweat capital through brokered sales as well as through inheritance. (See Section 7.)

4.6. Financing

We have one parameter related to business financing, which is the tightness of the working capital constraint in (3.6). Based on data from the SBO and surveys of the National Federation of

Independent Business (NFIB) members, we set $\chi = 0$. The SBO data reveal that only 11 percent of all owners reporting the source of start-up financing used external funds from banks, government, or venture capitalists. The NFIB collects information on problems faced by business owners, including issues with financing. When asked about their single most important problem, an average of only 3 percent reported financing and interest rates to be their main problem over the period 1994–2019. More often, the owners cite taxes, poor sales, government regulations, competition from big business, labor quality, and availability of insurance as their primary issues. When asked if their business was able to satisfy its borrowing needs, an average of only 5.6 percent of respondents answered "no" over the period 1994–2019.[17] Later, in the sensitivity analysis (Section 7), we set χ equal to the maximum observed ratio of available funds to value added using different samples from NIPA, SOI, and Compustat.

4.7. Productivity Processes

The productivity shocks are modeled as uncorrelated Markov chains with the states and transition matrices for z_t and ϵ_t shown in Table 1, panel B.

The transition matrix for z_t is estimated from information in Debacker, Panousi, and Ramnath (2013), who use a panel of businesses in the IRS Statistics of Income (SOI) unpublished subsample to construct transitions for business incomes. We use the same estimates for our productivity transition matrix and find that the implied transition matrix for business income is not significantly different than that reported by Debacker, Panousi, and Ramnath (2013). For the z grid, we face the challenge that the upper income bracket is top-coded to protect privacy. Since we know the income distribution is skewed, we use a squared log-normal autoregressive process with the variance chosen to generate the 90th percentile business income relative to the median wage income as in Debacker, Panousi, and Ramnath (2013). We view our choice of z grid as conservative. Later, in our sensitivity analysis (Section 7), we introduce a small number (1 percent) of superstar owners whose incomes are about 10 times larger than the median wage earner's and show that the differences between model predictions with and without sweat activity are even greater when the skewness of the productivity process is increased.

[17] The SBO and NFIB findings are consistent with those of Hurst and Lusardi (2004), who found no relationship between wealth and business entry, except for those at the very top of the wealth distribution.

The transition matrix for ϵ_t is consistent with the estimated wage processes of Low, Meghir, and Pistaferri (2010) for U.S. households in SIPP and PSID. To construct this matrix, we take a panel of simulated wages from their estimated quarterly model, annualize the simulated data, and then run a regression of log wages on one lag, individual fixed effects, and a set of controls (namely, age, age squared, education, and their interactions). We use the estimate of the coefficient on the log of lagged wages (0.7) and the estimate of the standard deviation of regression residuals (0.16) as parameters in an autoregressive process for ϵ_t. We then apply the discretization method of Tauchen (1986) to estimate the Markov chain shown in Table 1, panel B.

4.8. Taxes and Transfers

The third set of parameters are related to taxes and transfers. Here we summarize the estimates and refer interested readers to Section 2.8 in Bhandari and McGrattan (2020) for more details.

In panel C of Table 1, we report the marginal tax rates for workers and businesses. As we noted earlier, we include nonbusiness incomes with transfers, and therefore in the case of workers with income y, this means

$$T_j^w(y) = \tilde{T}_j^w(y) + \bar{y}_{nb} - \bar{x}_{nb},$$

where $\tilde{T}_j^w(y)$ is a piecewise linear tax schedule, $\bar{y}_{nb}$ is nonbusiness income, and $\bar{x}_{nb}$ is nonbusiness investment. The intercept of $\tilde{T}_j^w(y)$ for the young is set so that we match non-retirement government transfers for the population of workers. To accommodate old-age transfers, we also assume that the difference between $T_o^w(y)$ and $T_y^w(y)$ is payments for social security and Medicare. For the marginal rates ($T_j^{w\prime} = \tilde{T}_j^{w\prime}$) reported in the top part of Table 1 panel C, we compute the tax paid on an additional dollar of wages and salaries for each adjusted gross income (AGI) bracket in the SOI.

For the business owner tax schedule, T^b, we follow the same procedure as T^w and additionally adjust for noncompliance. To do that, we use the BEA estimate of total misreporting of unincorporated businesses (NIPA Table 7.14), GAO (2009, 2014) estimates of misreporting by S corporations, and Johns and Slemrod's (2010) estimates on misreporting for sole proprietorships. Overall, we find that marginal taxes on business income are about two-thirds of that on wage income (reported in the middle of Table 1 panel C), with most of the difference due to tax noncompliance.[18]

The remaining tax rates are reported at the end of Table 1 panel C. The tax rate on profits is a weighted sum of the marginal rates on domestic (federal plus state) and foreign earnings, which we find to be 36 percent.[19] We estimate a weighted marginal tax rate on dividends τ_d of 13.3 percent and the tax rate on consumption τ_c of 6.5 percent. Finally, in the model, we use NIPA data to set nonbusiness income, investment, and government spending shares to match U.S. empirical analogues.

4.9. Model Fit

With the baseline parameters in Table 1, we compute an equilibrium of the model and check that the implied national accounts and business age profiles are in fact aligned with U.S. data. Table 2 reports the model and data accounts using income and product categories of Appendix C.[20] Figure 1 shows the age profile for businesses in the model and data. The data are taken from the public-use microsample of the 2007 SBO.[21] We find that roughly 11.5 percent of owners started running their business in the year of the survey. For those who started more than four years before, we have only bracketed information and thus report the averages in the interval.

Given our model is now parameterized to match key statistics in U.S. data, we turn next to our main results and policy experiments.

5. Business Valuations

We use the model to estimate the sweat equity in U.S. private business, which is the present

[18] In Bhandari and McGrattan (2020), we check robustness of our main results with respect to the degree of tax noncompliance.

[19] This estimate is in line with Barro and Furman (2018), but higher than rates reported in Cooper et al. (2016) and Smetters (2018). As we show in Bhandari and McGrattan (2020), the lower estimates are based on taxes paid, which can vary year by year due to deferrals and timing of foreign distributions.

[20] For details on the construction of the national accounts by legal form of organization, see Section 2.3 of Bhandari and McGrattan (2020).

[21] SBO estimates are shown in Figure 1 for all owners, but we find similar results after conditioning on sector and hours in business.

discounted value of a hypothetical mutual fund that holds shares in all private businesses and receives the cash flow from investing in sweat capital.

5.1. Estimates of Private Business Values

We use $v_b(s)$ to denote the value of sweat capital for an individual with state $s = (a, \kappa, z, \epsilon, j)$ and compute it as follows:

$$v_b(s) = d_b(s) + \sum_{z', \epsilon', j'} \pi(z', \epsilon', j' | z, \epsilon, j) \, M(s, s') \, v_b(s'), \tag{5.1}$$

where $d_b(s) = \phi p y_p(s) - e(s)$ is the sweat dividend and $M(s, s')$ is the discount factor computed using the equilibrium under the baseline calibration. Note that the dividend does not include payments to owner hours in production, h_p, but does include payments to sweat capital κ accruing to all future generations. When we aggregate private businesses, and assume a mutual fund that holds shares in all private businesses, then the appropriate discount factor is $(1 + \gamma)/(1 + r)$. Conceptually, this mutual fund value is comparable to stock market share values and thus serves as a useful benchmark when comparing valuations of privately and publicly held businesses.

We can compute $v_b(s)$ for all individuals, including those currently working as employees, since employees could run a businesses in the future. When we aggregate, we find it to be large: the total sweat equity value for the United States is estimated to be 1 times GDP for pass-through businesses—roughly equal to the fixed assets in use in the businesses. The magnitude is easy to justify if we consider that 9 percent of national income is sweat income paid to owner time in production and building the business. Capitalizing this income with the mutual fund discount factor and multiplying the result by the fraction of time owners put in building sweat capital yields estimates that are on the order of 1 times GDP.

If private C corporations use the same production technologies as pass-through businesses, we can impute a value of sweat equity for all private businesses. Since the sweat equity value is the present value of pre-tax cash flow to a hypothetical mutual fund, we simply take the result for pass-through businesses (1 times GDP) and multiply by the ratio of post-audit incomes of all private businesses relative to pass-through businesses. Bhandari et al. (forthcoming) estimate this ratio using IRS data from corporate filings of Schedule M-3 and BEA imputations of misreported corporate incomes.[22] In 2007, the estimated income for privately held C corporations was 1.8 percent of GDP, which, if added to pass-through income, yields a net income of 10.8 percent and an estimate of the sweat equity value for all private businesses of 1.2 times GDP.

To provide some context for this estimate, we can compare the market value of shares in private businesses held by the hypothetical mutual fund investors with the market value of publicly held companies and other estimates of intangible asset values in the literature. To do this, we first adjust for taxes that would have to be paid on the mutual fund income—at a rate equal to roughly 24 percent. This implies an after-tax valuation of 0.92 times GDP for 2007 for the flow of sweat dividends and a share of sweat equity in total assets used in the private sector of 41 percent. In the U.S. flow of funds, the Federal Reserve estimates a stock market capitalization for publicly held firms of 1.25 times GDP in that year. Hall (2000) decomposed the stock market capitalization during the technology boom of the 1990s and estimated an intangible asset share of roughly 54 percent by the end of the decade. McGrattan and Prescott (2010a) analyzed intangible capital use in U.S. businesses (including their foreign subsidiaries) and estimated an intangible share of roughly 45 percent in 2007, although their analysis abstracts from investment in sweat capital. If they had included sweat capital, their share estimate would be closer to 60 percent.[23]

The incomes being valued in (5.1) are payments to both nontransferable productivity z, which is specific to an owner, and transferable sweat capital κ, which is eventually bequeathed or sold. Thus, the mutual fund shares would be worth more than the cash value v_κ in (4.2), given the latter is the price offered current owners for sweat capital only. If we condition on current business owners, we find that the average value for $v_b(s)$ is 1.22 times per capita GDP, whereas the average value for $v_\kappa(s)$ is 0.32 times per capita GDP. Thus, for owners, the transferable value is 26 percent of the total.

Thus far we have reported our main results from the perspective of a hypothetical mutual fund that can diversify across private firms. While this is useful for comparing to estimates from the literature that typically assumes complete markets, we can also use our model to quantify the effects

[22] The Schedule M-3 links IRS data with the Securities and Exchange Commission 10-K filings, allowing researchers to infer the split of income earned by privately and publicly held C corporations.

[23] Corrado, Hulten, and Sichel (2009) measure capital spending in the period 2000–2003 and report an intangible investment share of 58 percent.

of undiversifiable income risk. For instance, Heaton and Lucas (2000) argue that "background income risk"—an undiversifiable component of total income—would lower the marginal investors subjective valuation for stocks as long as such risk is correlated with equity returns. To generate an estimate of such a discount within our model, we compute the present value of sweat dividends using an alternative pricing kernel constructed from an individual owner's equilibrium consumption process, $M(s, s') = \beta U_c(c', \ell')/U_c(c, \ell)$. Since part of the consumption is financed by income from the business, there is a positive correlation between the idiosyncratic return on the business and the owner's consumption. In our baseline model, this leads to a valuation discount of 11 percent.

5.2. Cross-Sectional Private Business Statistics

In Table 3, we report cross-sectional information on business valuations, intangible intensities, and returns from the calibrated model.

In the first two columns, we report statistics for our two valuation concepts: sweat equity $v_b(s)$ and sale value $v_\kappa(s)$. The most noteworthy feature of the results is the difference in dispersion between the two distributions. Sweat equity values range from 64 percent of per capita GDP at the 10th percentile of the distribution to close to four times that at the 95th. Sale values of transferable sweat capital range from 1.3 percent of per capita GDP at the 10th percentile of the distribution to 94 times that at the 95th. Sweat equity measures all future cash flows of owners, which are not that different across owners facing the same productivity shocks even if they started their businesses in different years. In contrast, the current value of sweat capital is the cash owners would receive for the accumulated capital if the business were sold today. Accumulating higher and higher sweat capital stocks is very costly, given that it takes many years and some luck to build the business. Owners just starting out may have very little, while owners in the business for five to 10 years may have a significant stock.

In Figure 2, we show the pattern of sweat capital after sorting owners by the years since the acquisition of their business. The figure shows that significant sweat capital building occurs in the first five years for a typical business, followed by decumulation. In most cases, the decumulation of capital occurs either because productivity is low and the owner scales back or because productivity is high and the owner substitutes his or her time for outside labor and capital. In either case, the differences can lead to large dispersion in the sale values of businesses.

If we compare the ratio of sale values to business incomes in the model with that in the Pratt's Stats database, we find comparable values for the typical business. For example, the median firm in the model has a sale value of 1.3 times annual business income. This is consistent with the Pratt's Stats database, where we find a median ratio of 1.2 for sole proprietors, 1.5 for partnerships, and 1.6 for S corporations.

As we see in Table 3, the average intangible intensity is 57 percent by choice of the income share ϕ, but the distribution covers a wide range. Experienced business owners with high productivity can scale up their business by hiring outside labor and capital and have relatively low intensities, whereas new businesses that are just starting out have relatively high intensities. Similarly, we find a wide range of business holding returns constructed as follows:

$$r_b\left(s,s'\right)=\frac{\left(1+\gamma\right)v_b\left(s'\right)+d_b\left(s\right)}{v_b\left(s\right)}-1.$$

The mean gross return is 7.5 percent, with the 10th to 95th percentile covering a range of -11 to 52 percent. Much of the average return and its dispersion is due to capital gains as the dividend yield d_b/v_b averages 1.6 percent and covers a range of 0 to 17 percent. Because of the dispersion in returns, the commonly used procedure of estimating wealth as the ratio of income divided by a common rate of return—sometimes called capitalizing income—will lead to wrong answers.[24]

5.3. Correlates of Private Business Values

While the sweat equity and sale values are meaningful summary statistics in the model, we do not have reliable empirical counterparts of these statistics for ongoing concerns.[25] Here, we report statistics for variables that are empirically measurable and potentially correlated with sweat capital and the corresponding business valuations. In doing so, we illustrate how standard measures of firm markups and TFP can be misleading.

[24] This point has also been made in the critique of Smith, Zidar, and Zwick (2019).

[25] Surveys of business owners do ask for self-reports of business valuations. Bhandari et al. (forthcoming) show that the estimates of incomes and income-to-value ratios from these surveys are significantly overstated relative to counterparts in IRS, Pratt's Stats, and Center for Research in Security Prices (CRSP) data.

In Table 4, we report averages of these potential proxies. The first two columns show results for business incomes and the fixed asset input, which are highly positively correlated with the overall productivity level $z\kappa^{\phi}$ and thus serve as good proxies for κ. The higher their productivity, the more incentivized owners are to build up κ. If productivity remains persistently high, owners continue to build up sweat capital and scale up production by hiring more outside labor and renting more physical capital. In the third column, we show results for measured markups defined as the ratio of net income to cost of goods sold:

$$\mathrm{m}\left(s\right)=\left(\frac{py_{p}\left(s\right)-\left(r+\delta_{k}\right)k_{p}\left(s\right)-wn_{p}\left(s\right)-e\left(s\right)}{\left(r+\delta_{k}\right)k_{p}\left(s\right)+wn_{p}\left(s\right)+e\left(s\right)}\right). \tag{5.2}$$

Measured markups also turn out to be a good proxy for sweat capital. In our model, $m(s)$ is increasing with the size of the business. The expensing that occurs when businesses are young drives net incomes down and variable costs up. The opposite is true once sweat capital has been accumulated, implying a very high correlation between business incomes and measured markups despite the fact that true markups in the model are equal to zero. As shown in Table 4, we find a wide range of estimates after sorting businesses on the amount of their sweat capital, with a 35 percent markup in the top quintile. Abstracting from intangible assets, one might wrongly conclude that large firms in our model were earning significant monopoly rents.[26]

Statistics based on standard calculations of TFP might also mislead researchers faced with data from this model.[27] In Table 4, we report results for the following measure of total factor productivity computed for a business with state s:

$$\mathrm{tfp}\left(s\right)=\log y_{p}\left(s\right)-0.33\log k_{p}\left(s\right)-0.67\log n_{p}\left(s\right). \tag{5.3}$$

We find TFP is low for businesses with high sweat capital stocks, because those businesses are productive and can scale up their hiring of outside factors much more than they can scale up their own time. If we could observe the "true" productivities—that is, z—we would have predicted that high sweat capital businesses are the high productivity firms, the very opposite of what we would conclude by using typical measures of TFP.

[26] See, for example, Atkeson and Burstein (2008); Edmond, Midrigan, and Xu (2018); and Boar and Midrigan (2019).

[27] Crouzet and Eberly (2018) explore the relationships between TFPs, markups, and intangible investment shares for publicly held firms in the retail sector. No comparable data exist for private firms.

In the last column of Table 4, we report financial assets for the businesses, which show no relation to sweat capital and are thus a poor proxy. Note that here we include a but not v_κ, as is sometimes done in studies of business wealth. There is no pattern because firms have access to rental markets and can therefore scale up easily if they have high productivity or high sweat capital. However, even if working capital constraints were included, one would need a high value for χ to see much of a pattern because owners still have access to rental markets for fixed assets, but the key investment when building the business is their own time.

In summary, we find large sweat equity values for private businesses—with a significant fraction attributed to transferable sweat capital—and significant dispersion in business intangible intensities and returns. High-value businesses are larger in scale and appear to have higher measured markups.

Next, we consider the tax experiments and show that sweat capital and owner time in production play an important role for our quantitative results.

6. Tax Policy Experiments

To quantify the role of owner time in the business when evaluating tax changes, we make two types of comparisons. First, we contrast the effect of lowering tax rates on private pass-through businesses and C corporations in our baseline model with the effect of lowering them in a nested model that has owner time fixed.[28] Second, we contrast the impact of lowering tax rates on labor income earned by owners and by employees.

For comparability across experiments, we lower average marginal tax rates (τ_{AMTR}) by the same amount in all experiments, namely,

$$\Delta \log\left(1 - \tau_{\text{AMTR}}\right) = 0.156. \tag{6.1}$$

This choice is motivated by the size of the tax change for corporations in the 2017 U.S. tax reform. The average marginal rate for private business and wages is computed by taking a weighted average of each filer's marginal rate, with weights given by the taxable net income, as in Barro and Redlick

[28] The policy experiments we conduct are not intended to be a careful study of a particular reform in U.S. history but are instead a proof of concept intended to highlight the economic forces at work in our model with sweat activity. For related work that focuses specifically on the recent Tax Cuts and Jobs Act of 2017, see Barro and Furman (2018).

(2011). In the case of private business, lowering all marginal rates by 50.6 percent implies a 15.6 percentage point decline in the AMTR. For wage earners, we lower marginal rates by 25.5 percent to achieve the same result. In both cases, we adjust intercepts in the piecewise linear tax schedules to ensure the schedule remains continuous. For C corporations, there is only one rate, so the AMTR is simply the income tax rate τ_p, which is lowered from 36 percent to 26 percent. In each case, we compute the stationary equilibrium associated with the new taxes. Debt levels adjust so that the government budget constraint is satisfied.

6.1. Lower Taxes on Business

We start by comparing results in the baseline model with those of the nested model that abstracts from owner time in production and building sweat capital. The latter is the standard framework used in the study of entrepreneurial choice, which, as mentioned earlier, is a version of the Lucas (1978) span-of-control model. There are three main takeaways from this exercise. First, lowering private business tax rates has a much larger impact in the baseline model than in the Lucas (1978) model—both on the intensive and extensive margins—with most of the change due to highly productive owners and not necessarily those with the highest financial assets. Second, the implied short-run elasticities of the baseline model are in line with findings in the empirical public finance literature. Third, lowering business tax rates in the models with and without sweat activity has similar effects for most C-corporate and aggregate variables, with the exception of hours.

6.1.1. Role of sweat capital

Consider a lowering of just the private business tax rates. In the first two columns of Table 5, the main results for this experiment are shown for the baseline model with sweat and the nested Lucas (1978) model that has a production function given by

$$y_p = z k_p^{\alpha} n_p^{\nu}. \tag{6.2}$$

In this nested case, we set $\phi = \omega = 0$, and we reparameterize the consumption share η and labor share ν in order to match two statistics: the share of employee hours allocated to private business and the share of pass-through income in total income. The new values are $\eta = 0.51$ and $\nu = 0.38$.

Comparing results for private business activity across the two models, we find that the changes are much smaller in the Lucas (1978) model without sweat activity than in our baseline model. To understand the differences, consider first the decisions of very productive owners in the Lucas (1978) model. If we were to ignore any general equilibrium effects and assume Ricardian equivalence holds, we would expect no response to a lowering of tax rates from such owners. This is because they are not marginal in terms of their entry or exit decisions irrespective of the tax rates, and conditional on operating they expense all the variable factors leaving the only taxed factor to be the *fixed* managerial input.[29] Allowing for general equilibrium and non-Ricardian effects, we do predict some effects of lowering taxes, but they are quantitatively small.

On the other hand, in the baseline model, we predict a quantitatively large response to the lower income tax rates on the intensive margin. As we show in Table 5, owners increase time in production by 15.5 percent and time in building their business by 8.4 percent, which results in a 6 percent increase in their sweat capital and a 5 percent increase in their sweat equity. Although there is some substitution between owner and employee time, overall hours rise by 4.2 percent. As a result, private business output is higher by 2.2 percent as compared with the model without sweat activity, in which it barely changes.

In both models, lowering tax rates on business income leads to more entry. In the Lucas (1978) model, the main driver for the size of the extensive margin response is the mass of agents at the exogenous productivity entry threshold. In contrast, when we model sweat activity, the extensive margin changes are more substantial. Post-entry owners work harder, resulting in a larger endogenous productivity $z\kappa^{\phi}$, which amplifies the incentives to enter. In the baseline model, the fraction of owners increases by 6.6 percent and in the no-sweat Lucas (1978) model, the fraction of owners increases by only 2.5 percent.

When we compare the impacts on measured TFP and markups for this tax experiment for the models with and without sweat activity, we again find stark differences. For example, if we compare aggregate TFP in the private sector as it is typically measured—the logarithm of $\int y_p(s)\mu(s)\,ds$ divided by $(\int k_p(s)\mu(s)\,ds)^{.33}(\int n_p(s)\mu(s)\,ds)^{.67}$—we find an increase of 5.6 percent in the baseline

[29] To see this formally, consider a simpler case where the tax rate on business is linear and firms maximize: $(1-\tau_b)(py_p - wn_p - (r+\delta_k)k_p)$. In this case, the first-order conditions with respect to labor n_p and fixed assets k_p are independent of τ_b.

model but only 0.6 percent in the no-sweat model. This difference arises because measured TFP picks up changes in the unmeasured factors of production. If we compare the aggregate markup in the private sector as it is typically measured—total business net income divided by total variable costs for capital, labor, and expenses—we find a large increase of 5.6 percent in the baseline and no change in the no-sweat model. In fact, true markups are zero in both regardless of tax policy.

With the exception of total hours, the C-corporate and aggregate effects of lowering private business tax rates are similar in the two models, given the only modification we made was to private business production in (6.2). For example, differences in GDP and consumption are within 0.2 percentage points. For aggregate hours, we find an increase of 1.6 percent in the model with sweat, compared with a decline of 1 percent in the model without sweat.

In the right panel of Table 5, we show the results for lowering marginal rates on private business net incomes as before and additionally lowering the corporate profits tax rate τ_p. Again, we find significant increases in owner hours and sweat capital in the baseline model and a larger increase in private business output, compared with the no-sweat model. Despite differences in predictions for private output, private sales rise significantly in both models because households optimally equate spending shares on private and C-corporate goods, with the relative price adjusting to clear the goods market. Lower profits taxes also have a significant effect on fixed asset investments, with changes that are roughly the same in the two models. As before, the main differences are predictions for hours, TFP, and measured markups.

6.1.2. Exploring the mechanism

To better highlight the mechanisms at work in the baseline model with sweat activity, we next explore the distributional impacts of lowering just the tax on private business, T^b. We group businesses into bins using three different sorts: first by their productivity z, second by sweat capital stocks κ, and third by financial asset balances a. We then attribute the total change in a particular variable of interest to the different bins, with contributions summing to the total change. The point of the decomposition is to show that heterogeneity in business characteristics matters: high z businesses are not the same as high κ businesses or high a businesses.

In Table 6, we report the results of the decomposition for changes in sweat capital, κ; owner hours, $h_p + h_\kappa$; and private business hours, $h_p + h_\kappa + n_p$. Since z takes on five values, we sort businesses using the discrete values. Since κ and a are continuous, we assign owners to quintiles. Sorting by productivity, we find that the highest productivity (z) businesses contribute the most to the total changes in the private business production factors. For example, the high-z firms contribute 52 percent to total changes in sweat capital, 89 percent to total changes in owner hours, and 83 percent to total changes in hours in private business. This result is due partly to selection: individuals with low z do not choose to run businesses, so there are fewer owners in the lowest categories.

A different picture emerges if we sort businesses by sweat capital or financial assets. Despite an increase in the number of owners, the post-reform distributions of sweat and financial assets shift downward. In the case of sweat capital, most of the changes are attributed to businesses in the fourth quintile—those that are still growing and have high levels of capital. In the case of financial assets, most of the changes in sweat capital and owner hours are attributed to the businesses in the lowest quintile. Clearly, these businesses are not the same as the low-κ businesses.

6.1.3. Short-run elasticities

Results in Tables 5 and 6 are predictions of total long-run changes but are not easily compared with tax elasticities estimated in the empirical public finance literature. Next, we compute the implied short-run elasticities of business activity in response to a lowering of the private business tax rate T^b. We find our estimates for short-run changes in owner hours in line with empirical estimates of changes in employee hours, and we find our estimates for changes in business start-ups in line with empirical estimates of changes in new establishments.

To estimate these changes and the implied elasticities, we compare similar agents across different levels of taxes. To do this, we use the optimal policies in the two economies—the baseline and the economy with lower T^b—and simulate paths for individuals with the same initial conditions and the same sequence of exogenous shocks. We can compute the percentage differences in the paths of a particular outcome per owner and recover the distribution of changes over time.

For labor-supply elasticities of owners, we first condition on individuals running businesses in date 0 and compute the percentage increase in hours per owner to a 1 percentage point decrease in the rates $1 - \tau_{AMTR}$ applied to private business income. Calculating the responses for the median owner, we find an elasticity of 0.29, which is relatively constant across time. This estimate is in the range of intensive-margin labor elasticities for employees found in the literature. For example, in their meta-analysis, Chetty et al. (2011) report an estimate of 0.33 on the intensive margin. Both estimates are low when compared with the macro estimates of 2 to 3 typically found in the business cycle literature.

There are two reasons for the low implied labor-supply elasticities in our model. First, a lower tax rate on business income increases hours, but it also lowers the marginal rate of substitution between leisure and consumption—which is effectively the shadow wage of owners. Thus, unlike in the standard business cycle model, our model features a non-traded input in production—namely, owner time, which faces a higher tax incidence.

Second, part of the return to owner time is the increased option value of paid employment—in effect, the larger is κ, the higher the reservation wage for the owners and their offspring. Since this option value is not directly affected by business taxes, owner time—particularly h_κ—is less sensitive to tax changes than a standard model predicts.

If we compute an elasticity for total hours of owners, taking into account both intensive margin and extensive margin changes, we again find estimates of elasticities that are in the range found by Chetty et al. (2011) for employees and low relative to the business cycle literature. Our estimate for the elasticity of total hours of a typical owner is 0.6 in the first year and rises to 0.77 by the third year. Chetty et al. (2011) report an elasticity for total hours of 0.59.

Given our focus is business taxation, a more direct comparison can be made with respect to the impact of business activity such as the opening of new businesses or establishments. In this case, we compare the percentage change business counts during the transition, again assuming that initial conditions and sequences of shocks are held fixed. We estimate that a 1 percentage point decrease in the private business tax rate leads to a 0.42 percent increase in the number of businesses in the first year following the change. By the third year, the estimate is 0.53.

These estimates are in line with Giroud and Rauh's (2019) findings on how businesses respond to state-level changes in taxes. Their identification exploits a feature in the U.S. tax code, whereby firms that are similar in all respects except the legal form of organization face differences in the incidence of a change in the corporate or personal income tax. In the case of pass-through businesses, Giroud and Rauh (2019) estimate that a 1 percentage point decrease in the tax rates results in a 0.43 percent increase in the number of establishments within the first year.[30]

6.2. Lower Taxes on Labor

Private business activity is also affected by the taxation of employees, since the tax schedule T^w is relevant for owners' opportunity cost of time. In this section, we contrast the results in the baseline model of lowering the tax schedule on employee time, T^w, with the results above of lowering the tax schedule on owner time, T^b. We show that changing these schedules has different implications for self-employment rates, private business production, and the organization of firms.

As before, we assume a 15.6 percentage point decline in $\log(1 - \tau_{\text{ATMR}})$ using the average marginal tax rate on T^b and separately on T^w. In Table 7, we report results for key statistics that highlight the differences between these tax policy changes. In the first row, we report the self-employment rate. As we noted earlier, there is a 6.6 percent increase in this rate if T^b is lowered. Contrast this with lower taxes on employees, which result in an 18.1 percent decline in the self-employment rate. Although the changes in the average marginal tax rates are the same across the experiments, there are more individuals who can take advantage of lower taxes on paid employment. Lower taxes on self employment do little for a large fraction of owners who make losses or earning little as they build their businesses.

Evidence from these experiments reveals another important difference: lower business taxes result in more businesses that are smaller in scale (for example, mom and pops), while lower employee taxes result in fewer businesses that are larger in scale (for example, businesses with many employees). As we showed earlier, with lower tax rates on private business incomes, owners do more production themselves and hire fewer employees, whose time following the reform is relatively more costly. With lower tax rates on employees, individuals with high (but not too high) productivity levels in business, z—who would have run businesses if employment taxes had not

[30] Giroud and Rauh (2019) find smaller elasticities of 0.26 percent for the larger pass-through businesses that can reallocate establishments across states in response to tax changes. This reallocation margin is not operable in our model.

been lowered—choose to work for someone else. In Table 7, we report that total employee hours are higher by 14 percent because both publicly and privately held businesses hire more employees. For private businesses, the surge in employment is concentrated at the top, among the high-z, high-κ businesses. These businesses take the opportunity of a decline in tax rates on employees to scale up. Their owners put less time into production but more time into building sweat capital. Overall, there is a decline of 5.3 percent in sweat capital building following a lowering of T^w, but this result is due to an extensive margin response, since there are fewer low- and medium-κ businesses following the tax reform.

In summary, the tax experiments reveal that including owner time in business and sweat capital changes the economics of tax reform both qualitatively and quantitatively. We next turn to our sensitivity analysis.

7. Sensitivity

In this section, we show that adding features to the model that are common in the literature on entrepreneurial choice does not overturn any of our main findings. The features we include are financial constraints on working capital, superstar owners with high productivity levels, and brokered sales of businesses. Differences between the baseline model and extensions with financial constraints and brokered sales turn out to be quantitatively small. Differences between the baseline model and the extension with superstar owners are quantitatively larger for many predictions but overall strengthen the finding that abstracting from sweat activity leads to understatements of tax reforms' effect on measures of business activity.

7.1. Financial Frictions

In the baseline model, we set the parameter χ governing the severity of borrowing constraints in private businesses to zero. Here, we use evidence reported in Chari (2014) on available funds—which is a measure of funds available for gross investment or financial activities—divided by business value added. We conservatively set χ to the maximum of the estimates he reports.

The most relevant estimate is based on U.S. flow of funds data for nonfinancial corporations over the period 1952–2012. At the start of the sample, the ratio of available funds to value added was a little over 15 percent. After 2000 it rose steadily to 25 percent, the estimate we use. In economic terms, this estimate translates into a requirement that roughly a quarter of a year of sales be available as working capital. We should note that this maximal value is in line with the other evidence in Chari (2014) from large firms in the IRS and Compustat data that have maximal ratios of available funds to sales of about 12 to 14 percent. Given aggregate business receipts are roughly twice value added, we set $\chi = 0.25$.

In Table 8, we compare the baseline results for $\chi = 0$ with this alternative and find the differences to be negligible. We also experimented with much larger estimates found in the literature and still found negligible effects. High values for χ change the asset distribution but not our main findings because over time individuals save precautionarily to avoid hitting the constraints.

7.2. Superstar Owners

Next, we change the baseline model by adding an additional productivity state z at the top. Since Debacker, Panousi, and Ramnath (2013) could not publish any information about owners at the top of the income distribution, we add a new point on the productivity grid—that is, z_6—and use published IRS taxable incomes to set it. If we set $z_6 = 1.25 z_5$, the model generates business income at the 95th percentile that is 10 times the median labor income, consistent with the published IRS data. Because no panel data are published by the IRS, our choice of transition probabilities into and out of this state is necessarily somewhat arbitrary. Here, we assume that the only transition in and out of state 6 is via state 5. The average duration in state 6 is taken to be a typical working life of 45 years, which implies that the probability of remaining in state 6 must be equal to $1 - 1/45$. We assume that the probability of transitioning from state 5 to state 6 is equal to the probability of transitioning from state 6 to 5 and choose it in such a way so as to ensure that only 1 percent of owners are in the high productivity state in the stationary distribution. To generate model data consistent with U.S. data, we recalibrate the C-corporate fixed asset share θ to 52.2 percent, the private business owner hour share ω to 47.8 percent, and the deterioration rate λ to 80 percent.

We should note this version of the model passes a standard litmus test for researchers studying income and wealth dispersion—namely, that wealth is more dispersed than income. (See Castaneda, Diaz-Gimenez, and Rios-Rull, 2003.) Furthermore, Quadrini (2000) reports evidence that wealth-to-income ratios are significantly higher for business owners than for workers. Our model predicts both greater dispersion in wealth than in income and higher wealth-to-income ratios for business owners relative to workers. If we compare total wealth to total income for all business owners, we find it is twice as large as for workers.[31]

In Table 8, we report our main statistics for this case with superstar owners. With higher variability in the productivity process, we find a higher average sweat equity value of 1.64 times per capita GDP for mutual fund investors, a higher business sale value of 0.5 times per capita GDP, and a higher average gross return of 10.6 percent. In terms of tax experiments, we find larger average increases in self employment, sweat capital, and owner hours—reinforcing our claim that abstracting from sweat activity leads to an understatement of the impact of tax reform. Interestingly, looking across the distribution of owners, we find that the highest-z superstar owners account for a very small portion of the total change in hours. Most of the change, both on the intensive and extensive margin, comes from owners that have high z (state 5), but not the highest (state 6).

7.3. Brokered Sales

The final extension that we consider is to explicitly introduce brokered sales into the baseline model. In this section, we describe the problem of a broker who buys and sells sweat capital and the data used to discipline additional parameters. Because few U.S. businesses sell in any given year, we find that the extended model predictions are close to those of our baseline model without brokered sales.

With sales, individuals choose whether to buy or sell κ simultaneously with occupation. That is, given their state s, all individuals can decide whether to sell their sweat capital κ and become a worker, keep the κ and become a worker, run a business with the κ they currently have, or run a business after buying additional κ from the broker. If individuals are indifferent between selling

[31] We do not attempt to match our statistics of business wealth to survey data used by the previous literature because of documented measurement issues. See Bhandari et al. (forthcoming).

and not selling, we break the tie with probability π^f. Sellers exchange κ for cash in the amount $v_\kappa(s)$. Brokers own a technology to produce new businesses of size $\bar{\kappa}$. They offer homogeneous price-quantity bundles $(p_{\bar{\kappa}}, \bar{\kappa})$ to all potential buyers. There is free entry in the broker market, and the zero profit condition for the broker is

$$p_{\bar{\kappa}} \int \mathbf{1}_{\text{buy}}(s)\, \mu(s)\; ds = \pi^f \int v_\kappa(s)\, \mathbf{1}_{\text{sell}}(s)\, \mu(s)\; ds,$$

where $\mathbf{1}_{\text{buy}}$ and $\mathbf{1}_{\text{sell}}$ are indicator functions for buying and selling and depend on the state s. Our definition of a competitive equilibrium must be amended to include $(v_\kappa(s), p_{\bar{\kappa}})$ and the condition that the sale price is such that sellers are indifferent between continuing to run their businesses and entering employment with additional cash, as in (4.2).

To discipline π^f and $\bar{\kappa}$, we use data from the 2007 SBO microsample on business acquisitions and sales. The parameter π^f is set such that the fraction of exiting owners who sold their businesses is roughly 9 percent. The size $\bar{\kappa}$ is set such that we match the fact that 8 percent of SBO businesses with at least one owner reported that they acquired their share of the business over the period 2000–2007 through a purchase. We also recalibrate the deterioration rate λ to 0.85 and C-corporate share η to 0.47 in order to match the acquisition profile and sweat income share. The share of income to sweat capital, ϕ, is 15 percent, as in the baseline. The implied intangible intensity in this case is 61 percent, which is in line with the Pratt's estimate of 58 percent used in our baseline calibration.

The main results for this alternative economy are reported in Table 8. As in the baseline, we find an average sweat equity value close to 1.2 times per capita GDP, with the transferable value equal to 26 percent of the total. Lowering tax rates on private businesses produces the same output response as in the baseline, although more of the change is due to increased hours and less to increased sweat capital. This difference is not surprising given owners have access to a broker market in this extension. Overall, given that we parameterize the alternative model to match the small number of sales observed in U.S. data, we find little change in the main results.

8. Conclusions

In this paper, we developed a theory of sweat equity in private business and used it to reassess central questions in public finance. We estimated a value for sweat equity that is large—roughly the same as the value of fixed assets in use in these businesses—and partly transferable through inheritance or sale. We found that abstracting from sweat activity leads to a significant understatement of the effects of lowering tax rates on private businesses. We view these findings as a proof of concept for future analyses of specific tax reforms for private business.

The fact that private business owners make substantial investments in sweat capital is also relevant outside of public finance. There are implications for theories of the firm as well as measurements of business activity. Testable implications of theories of organizational choice and corporate finance crucially rely on the determination of characteristics of business assets, particularly their alienability, specificity, and tangibility. Most of the applied work focuses on one attribute at a time and imposes a strict dichotomy on the nature of the asset: alienable or inalienable, specific or general, tangible or intangible. One of the key messages from our work is that sweat capital does not fit neatly in these dichotomies. Furthermore, the fact that these assets are a substantial part of private business value will likely necessitate a review of some of the classic results concerning the boundary of the firm and its capital structure.

Our findings also warrant improvements in measuring firm-level productivities and monopoly rents. As we demonstrated, standard measures of total factor productivity that rely exclusively on observable factor inputs will be confounded by the presence of unmeasured sweat capital. Similarly, standard measures of markups will be overstated to the extent that returns on sweat capital are interpreted as monopoly rents. With better access to firm-level data and improved econometric methods, much more can be done to unravel sources of income and productivity growth. We expect that the theory developed here will require some updating—especially with regard to the accumulation and transference of sweat capital—but hope such efforts would yield better insight for future policy analyses.

Appendix A. Nomenclature

s	Individual state vector $(a, \kappa, z, \epsilon, j)$
a	Financial assets of individuals held as deposits with intermediaries
κ	Sweat capital stock
z, ϵ	Productivity as a business owner and employee
j	Index for age (y=young,o=old)
V_p, V_w	Value functions indexed by occupation (private owner, worker)
V	Continuation value function of individuals
U_p, U_w	Utility functions of private business owners and workers
c	Consumption composite of private and C-corporate consumption
c_p, c_c	Consumption of private and C-corporate goods and services
ℓ	Leisure time
p	Relative price of private to C-corporate goods
r	Interest rate
w	Wage rate
y_p, y_c	Output of private business, C corporations, and financial intermediaries
k_p, k_c	Fixed assets in production of private and C-corporate goods
x_p, x_c	Investment of private and C-corporate fixed assets
n_p, n_c, n	Employee hours in private business, C corporations, and total
h	Composite labor input of owner and employee hours in private business
h_p, h_κ	Owner hours in production and building sweat capital
e	Intermediate expenses in building sweat capital
f_p, f_κ	Production functions of private businesses for goods and sweat capital
T^b, T^w	Tax schedules for business and wage income
τ_c, τ_p, τ_d	Tax rates on consumption, corporate profits, and corporate dividends
F	Production function of C corporations
v_c, v_I	Value functions of C corporations and financial intermediaries
d_c, d_I	Dividends of C corporations and financial intermediaries
ς	Equity shares in C corporations held by financial intermediaries
q	Price per share of corporate equities
b	Government debt
g	Government spending
$\bar{x}_{nb}, \bar{y}_{nb}$	Investment and income of the nonbusiness sector
v_b, d_b, r_b	Value, dividend, and holding return of hypothetical private mutual fund
v_κ	Value of sweat capital if sold separately from business
$p_{\bar{\kappa}}, \bar{\kappa}$	Price and quantity offers in brokered sales
ii	Intangible intensity
tfp	Measured total factor productivity
m	Measured markup

Appendix B. Model Equilibrium

Before formally defining the concept of equilibrium, we specify the problem of financial intermediaries and the government.

At the beginning of each period, the net worth of an intermediary is the value of its equity shares ς, bonds b, and fixed assets k_p, less the value of deposits owed to households a. During the period, the intermediary receives dividend income from C corporations, interest income from bonds, and rental income on fixed assets and pays interest on deposits. At the end of the period, the intermediary receives new deposits and invests in new equity shares, bonds, and fixed assets. The dynamic program in this case is

$$v_I(x) = \max_{x'}\{d_I + \left(\frac{1+\gamma}{1+r}\right) v_I(x')\}, \tag{B1}$$

where the state vector is $x = [\varsigma, b, k, a]$ and the intermediary dividends are

$$\begin{aligned} d_I = {} & q\varsigma + \int k_p(s)\,\mu(s)\,ds + b - \int a(s)\,\mu(s)\,ds \\ & + (1-\tau_d)\,d_c\varsigma + rb + (r+\delta_k)\int k_p(s)\,\mu(s)\,ds - r\int a(s)\,\mu(s)\,ds \\ & - (1+\gamma)\left[q\varsigma' + b' - \int a'(s)\,\mu(s)\,ds\right] - \int x_p(s)\,\mu(s)\,ds \end{aligned}$$

and q is the per-share price of corporate equities. There is free entry in this sector.

The government spends g; borrows b; and collects taxes on consumption at rate τ_c, labor earnings with schedule T_j^w, private business income with schedule T_j^b, C corporation dividends at rate τ_d, and C corporation profits at rate τ_p. The government budget constraint is given by

$$\begin{aligned} g + (r-\gamma)\,b = {} & \tau_c\left(\int c_c(s)\,\mu(s)\,ds + \int pc_p(s)\,\mu(s)\,ds\right) + \tau_p\,(y_c - wn_c - \delta_k k_c) \\ & + \tau_d\,(y_c - wn_c - (\gamma+\delta_k)\,k_c - \tau_p\,(y_c - wn_c - \delta_k k_c)) \\ & + \sum_{j=y,o}\int T_j^b\,(py_p(s) - (r+\delta_k)\,k_p(s) - wn_p(s) - e(s))\,\mu(s)\,ds \\ & + \sum_{j=y,o}\int T_j^w\,(w\epsilon(s)\,\zeta_j n(s))\,\mu(s)\,ds, \end{aligned} \tag{B2}$$

with all variables divided by the technological trend growth.

A *stationary recursive competitive equilibrium* is value functions V_p, V_w, V; policy functions a', κ', c_c, c_p, ℓ, n, k_p, n_p, h_p, h_κ, and e; C corporation choices n_c, x_c; financial intermediary choices ς, b, k, a; prices r, w, p; and a measure over types indexed by the state s and age j such that

- given prices, the policy functions for private business owners—namely, a', κ', c_c, c_p, ℓ, k_p, n_p, h_p, h_κ, e—solve the dynamic programming problem (3.1) associated with value functions V_p;

- given prices, the policy functions for employees—namely, a', κ', c_c, c_p, ℓ, and n—solve the dynamic programming problem (3.7) associated with value functions V_w;
- given prices, the policy functions for C corporations—namely, n_c and x_c—solve the dynamic programming problem associated with value functions v_c;
- given prices, the policy functions for financial intermediaries—namely, ς', b', k_p', and a'—solve the dynamic programming problem (B1) associated with v_I;
- the labor market clears: $n_c = \int (n(s)\epsilon(s) - n_p(s))\mu(s)\,ds$;
- the asset market clears: $\int a(s)\mu(s)\,ds = b + (1-\tau_d)k_c + \int k_p(s)\mu(s)\,ds$;
- the private business goods market clears: $\int y_p(s)\mu(s)\,ds = \int c_p(s)\mu(s)\,ds$;
- the C-corporate goods market clears:

$$y_c = \int c_c(s)\,\mu(s)\,ds + \int e(s)\,\mu(s)\,ds + (\gamma + \delta_k)\left(k_c + \int k_p(s)\,\mu(s)\,ds\right) + g;$$

- the government budget constraint in equation (B2) is satisfied;
- the measure of types over states $s = (a, \kappa, \epsilon, z, j)$ is invariant.

Appendix C. Model National Accounts

Let y denote GDP, which is the sum of C-corporate output, y_c; private output less intermediate expenses, $\int (py_p(s) - e(s))\mu(s)\,ds$; and nonbusiness income, $\bar{y}_{nb}$. The nonbusiness sector includes households, nonprofits, and government and is included to ensure that the model accounts can be directly compared with U.S. accounts. Let $\bar{x}_{nb}$ be investments of the nonbusiness sector. The nonbusiness income less investments is included with transfers in the tax schedules T_j^b and T_j^w.[32] (See Section 4.8.) With these definitions, we can summarize the accounts as follows:

Income shares:

Business incomes	
Sweat income	$\int (py_p(s) - (r+\delta_k)k_p(s) - wn_p(s) - e(s))\mu(s)\,ds)/y$
Employee compensation	$w(n_c + \int n_p(s)\mu(s)\,ds)/y$
C corporations	wn_c/y
Private business	$w\int n_p(s)\mu(s)\,ds/y$
Capital income	$(y_c - wn_c - \delta_k k_c + (r+\delta_k)\int k_p(s)\mu(s)\,ds)/y$
C corporations	$(y_c - wn_c - \delta_k k_c)/y$
Private business	$(r+\delta_k)\int k_p(s)\mu(s)\,ds/y$
Nonbusiness incomes	$\bar{y}_{nb}/y$

[32] If we were to directly compare *total* fixed assets of the model and data, we would also have to add nonbusiness fixed assets to our measure of capital.

Product shares:

Private consumption	$(\int c_c(s) + pc_p(s))\mu(s)\,ds)/y$
Government consumption	g/y
Investment	$(x_c + \int x_p(s)\mu(s)\,ds + \bar{x}_{nb})/y$
C corporations	x_c/y
Private business	$\int x_p(s)\mu(s)\,ds/y$
Nonbusiness	$\bar{x}_{nb}/y$

The data analogue of *sweat income* is BEA proprietors' income—which includes incomes of sole proprietors and partners—plus IRS S-corporate compensation and business income from trade (including estimates of misreported incomes from audited returns). From this, we subtract payments to capital owned by the businesses, using information on rents and interest payments in IRS income statements. *Business employee compensation* is BEA business compensation less S-corporate compensation. *Business capital income* is BEA rental income, net interest, consumption of fixed capital, and corporate profits less IRS S-corporate business income from trade. *Nonbusiness income* is BEA labor and capital income attributed to factors in the household, nonprofit, and government sectors. On the product side of the accounts, the first category is *private consumption*, which is BEA personal consumption expenditures on nondurable goods and services.[33] *Public consumption* is government consumption of goods and services. Finally, data on investments and nonsweat capital stocks by legal entity are available from the BEA fixed asset tables. In Bhandari and McGrattan (2020), we show how we map all NIPA categories to these theoretically consistent categories.

[33] BEA personal consumption expenditures and capital incomes must be adjusted by adding imputed services for durables and subtracting sales tax.

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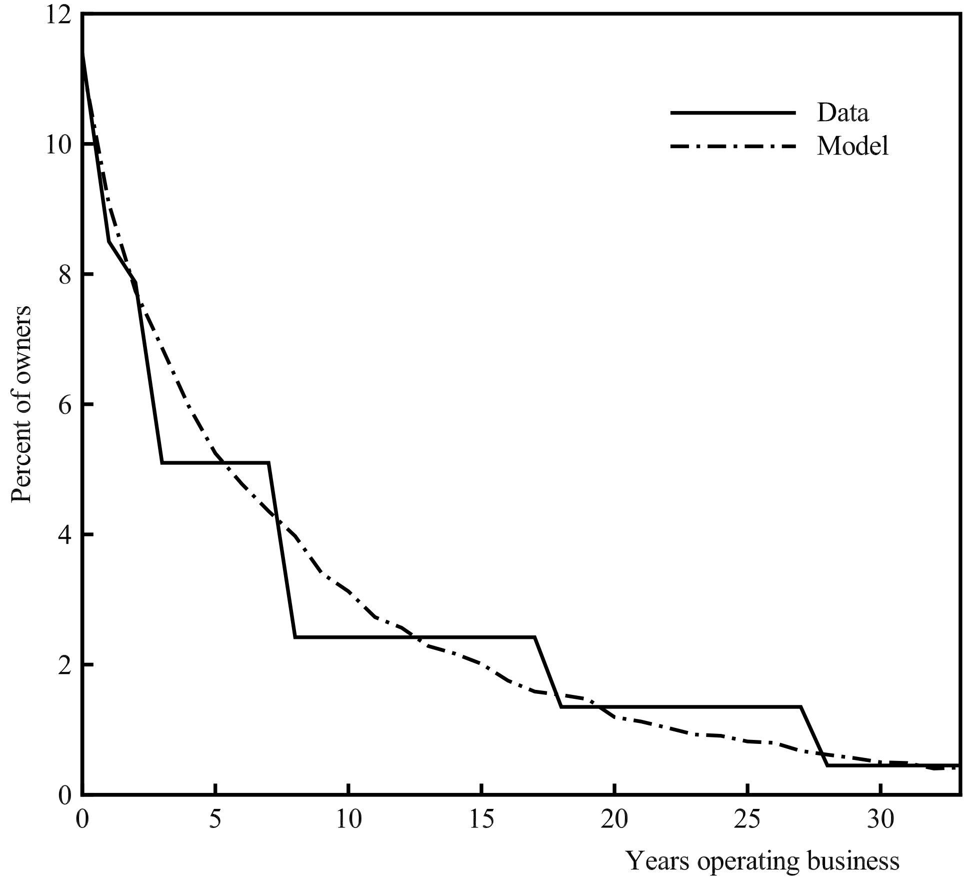
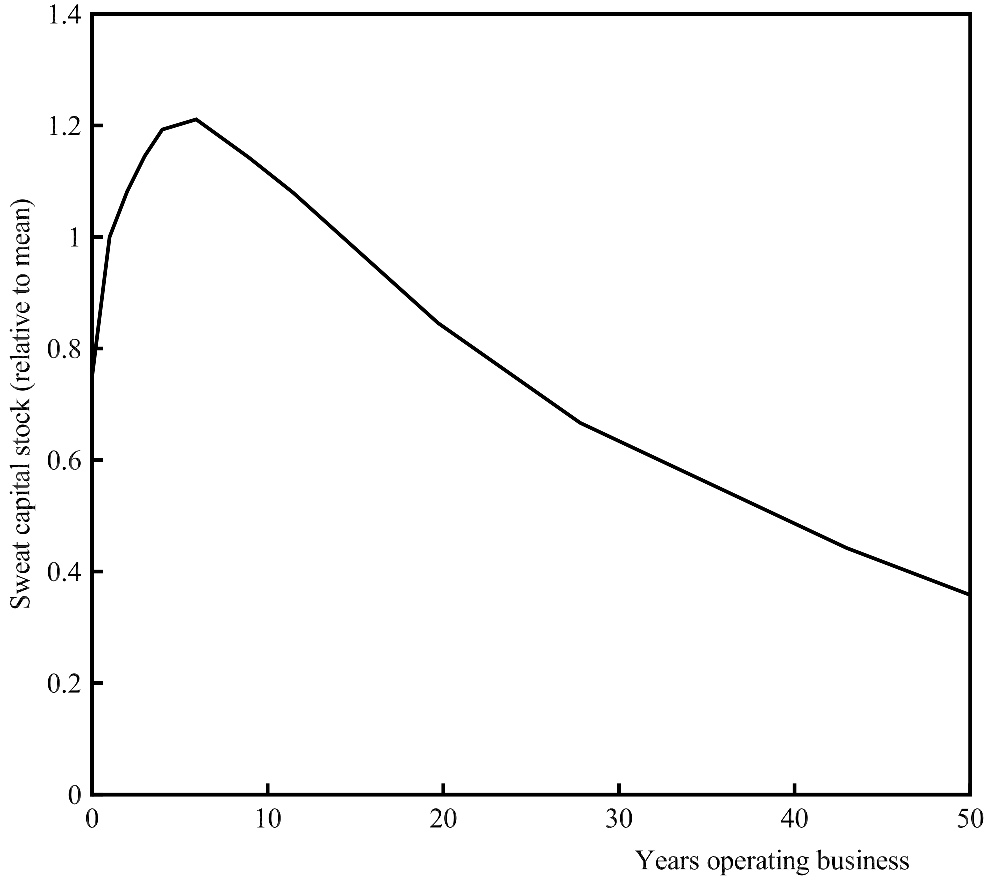
Figure 1. Business Age Profile



Notes. The U.S. estimates are based on the Survey of Business Owners microsample in 2007. The series is the fraction of owners reporting that they began operating their business in the current year, one year ago, two years ago, and so on. Individual information about how and when they acquired the businesses is included in the SBO for up to four owners. For acquisitions prior to 2004, the SBO uses intervals when reporting the data and we plot per-year averages. For the model, we compute the number of years in operation for all owners and record fractions separately for each year.

Figure 2. Sweat Capital by Age



Notes. The figure plots the average sweat capital stock after sorting owners by years since starting their business. The series is then normalized by the mean for all businesses.

Table 1. Baseline Model Parameters

A. Preferences, Technologies, Life Cycle and Financing

Parameter	Expression	Value
Preferences		
C-corporate good share (%)	η	44.9
Love of business parameter	ξ	0
Leisure weight	ψ	0.58
Discount factor	β	0.98
Intertemporal elasticity inverse	σ	1.5
Technologies		
Private business sweat capital share (%)	ϕ	15.0
Sweat capital deterioration (%)	λ	60.0
Sweat capital depreciation (%)	δ_κ	5.8
Sweat capital investment curvature	φ	1.0
Sweat capital owner hour share (%)	ϑ	40.8
Private business owner hour share (%)	ω	42.5
Private business hours substitution parameter	ρ	0.5
Private business fixed asset share (%)	α	30.0
C-corporate fixed asset share (%)	θ	50.2
Fixed asset depreciation (%)	δ_k	4.1
Technology growth (%)	γ	2.0
Life cycle		
Probability of remaining young (%)	π_y	97.8
Probability of remaining old (%)	π_o	93.3
Altruism weight	ι	1
Old-age productivity (%)	ζ_o	50.0
Sweat deterioration at death (%)	λ_d	90.0
Financing		
Working-capital constraint	χ	0

Note: See Section 4 for functional form assumptions, data sources, and details on parameter estimation.

Table 1. Baseline Model Parameters (cont.)

B. Productivity Transition Probabilities

Productivity in t	Productivity in $t+1$				
Business, z_t	.432	.657	1	1.52	2.32
.432	.612	.170	.098	.065	.055
.657	.172	.551	.187	.064	.025
1	.099	.191	.475	.190	.045
1.52	.060	.055	.164	.558	.164
2.32	.046	.009	.034	.135	.776
Employment, ϵ_t	.509	.713	1	1.40	1.97
.509	.424	.549	.027	0	0
.713	.046	.621	.327	.005	0
1	.001	.145	.709	.145	.001
1.40	0	.005	.327	.621	.046
1.97	0	0	.027	.549	.424

Note: See Section 4.7 for details on estimating productivity processes.

Table 1. Baseline Model Parameters (cont.)

C. Government Taxes

Parameter	Expression	Value
Marginal rates, wage schedule	$T_j^{w\prime}(y)$	%
$y/\bar{y} \in [-\infty, \;\; 0.173]$		29.3
$[0.173, 0.262]$		32.4
$[0.262, 0.404]$		34.3
$[0.404, 0.732]$		39.0
$[0.732, 1.409]$		40.0
$[1.409, 3.138]$		40.8
$[3.138, \;\; \infty]$		41.9
Marginal rates, private business schedule	$T_j^{b\prime}(y)$	%
$y/\bar{y} \in [-\infty, \;\; 0.153]$		14.0
$[0.153, 0.304]$		18.3
$[0.304, 0.912]$		20.1
$[0.912, 2.667]$		23.5
$[2.667, 5.727]$		26.2
$[5.727, 9.104]$		26.9
$[9.104, \;\; \infty]$		28.0
Other tax rates		%
Consumption	τ_c	6.5
Profits	τ_p	36.0
Dividends	τ_d	13.3

Note: See Section 4.8 and Bhandari and McGrattan (2020) for details on estimating taxes and transfers.

Table 2. National Account Shares, Data and Baseline Model

	Data	Model
Income Shares		
Business incomes	0.735	0.733
Sweat income	0.090	0.090
Employee compensation	0.331	0.327
C corporations	0.220	0.222
Private business	0.111	0.105
Capital income	0.314	0.317
Nonbusiness incomes	0.265	0.267
Product Shares		
Private nondurable consumption	0.575	0.565
Government consumption	0.133	0.133
Investment	0.292	0.301
C corporations	0.134	0.124
Private business	0.048	0.067
Nonbusiness	0.110	0.110

Note: Three adjustments are made to NIPA income and product: sales taxes are subtracted, consumer durables are classified as investment, and additional IPP categories that are not currently included in NIPA investment are included with investment. See full details in Bhandari and McGrattan (2020).

Table 3. Cross-Sectional Statistics for All Private Businesses

Statistics	Sweat equity	Sale value	Intangible intensity	Gross return	Dividend yield
Mean	1.22	0.32	0.57	7.5	1.6
Std. dev.	0.62	0.45	0.41	21.4	11.0
Percentiles:					
10th	0.64	0.01	0.11	−11.4	−0.1
25th	0.72	0.02	0.12	1.5	0.0
50th	0.95	0.04	0.67	2.0	0.0
75th	1.80	0.59	1.00	15.6	4.3
95th	2.34	1.22	1.00	52.3	15.6
99th	2.43	1.34	1.00	86.5	16.8

Note: The sweat equity for an individual with state s is the present discounted value of net dividends, $\phi p y_p(s) - e(s)$ and is given by $v_b(s)$ in (5.1). The sale value for a business owner with state s is the amount of cash needed for the owner to be indifferent between selling the sweat capital and keeping it and is given by $v_\kappa(s)$ in (4.2). Sweat equity and sale value estimates reported in the table are constructed only for business owners and are divided by per capita GDP. The intangible intensity of a business with state s is the ratio $v_\kappa(s)/(v_\kappa(s)+k_p(s))$. Intensity statistics in the table are constructed only for businesses that are at least one-year old. The gross return $r_b(s, s')$ on the business is the sum of the capital gain to sweat equity $(1+\gamma)v_b(s')/v_b(s) - 1$ plus the dividend yield $d_b(s)/v_b(s)$. Returns and yields are reported in percentage terms.

Table 4. Characteristics of All Private Businesses, Businesses Sorted by Sweat Capital Stocks

Quintile	Business income	Fixed assets	Measured markup	Measured TFP	Financial assets
1	0.00	0.00	−0.41	1.06	4.10
2	0.00	0.01	−0.35	1.07	4.30
3	0.06	0.82	0.07	1.03	5.33
4	0.25	3.37	0.22	0.96	4.87
5	0.61	7.14	0.35	0.87	4.19

Note: The statistics reported are quintile averages. The business income for an owner with state s is $p y_p(s) - (r+\delta_k)k_p(s) - w n_p(s) - e(s)$. Assets in the business are fixed $k_p(s)$ or financial $a(s)$. In the table, we report business income and both types of assets after dividing by per capita GDP. The measured markup is the ratio of business income to cost of goods sold and given by $m(s)$. Measured TFP is the ratio of business output to fixed assets raised to the power 1/3 times employee hours raised to power 2/3 and is given by $\text{tfp}(s)$. The TFP estimates are normalized by the mean value.

Table 5. Effects of Lower Business Tax Rates in Models with and without Sweat Activity

	% Changes from lower tax rates on:			
	Private businesses		All businesses	
	Baseline	No sweat	Baseline	No sweat
Private business activity				
Output	2.2	−0.3	4.6	1.9
Relative price	−2.7	−0.5	2.6	6.3
Sales	−0.5	−0.8	7.4	8.3
Self-employment rate	6.6	2.5	4.7	0.7
Owner hours, production	15.5	–	15.3	–
Owner hours, sweat	8.4	–	7.8	–
Sweat capital	6.0	–	8.7	–
Sweat equity	5.0	–	10.5	–
Total hours	4.2	−1.0	3.5	−1.9
Fixed asset investment	−0.1	−0.5	7.7	8.5
Measured TFP	5.6	0.6	6.5	0.5
Measured markup	5.6	0.0	6.0	0.0
C-corporate activity				
Output	−0.1	−0.7	12.3	13.3
Consumption	−0.5	−0.8	7.4	8.3
Employee hours	−0.7	−0.9	2.4	2.6
Fixed asset investment	0.4	−0.4	23.1	24.0
Aggregates				
Wage rate	0.6	0.3	9.7	10.4
Interest rate	−0.9	−0.4	−14.0	−13.8
GDP	−0.3	−0.5	7.6	8.5
Sweat equity	1.9	–	8.2	–
Consumption	−0.6	−0.8	7.4	8.3
Hours	1.6	−1.0	2.9	1.1
Fixed asset investment	0.3	−0.5	17.7	19.1
Tax revenues	−4.7	−4.1	−1.2	0.3
Debt	−43.0	−23.4	−19.3	−2.6

Note: Measured TFP in the private sector is computed by first aggregating outputs and inputs. Measured markup is computed as the ratio of aggregate business income to aggregate variable costs.

Table 6. Distributional Effects of Lower Tax Rates on Private Business Sorting by Productivity, Sweat Capital, and Financial Assets

Characteristic	Sweat capital	Owner hours	Total hours
Sort by productivity (z)			
Group 1	0.6	0.0	0.0
2	0.5	0.0	0.0
3	0.7	0.1	0.0
4	1.2	1.4	0.7
5	3.1	12.5	3.5
Total change (%)	6.0	14.0	4.2
Sort by sweat capital (κ)			
Quintile 1	0.0	0.0	0.0
2	0.2	0.1	0.0
3	2.3	1.2	1.1
4	8.8	16.0	15.5
5	−5.3	−3.3	−12.4
Total change (%)	6.0	14.0	4.2
Sort by financial assets (a)			
Quintile 1	6.7	9.7	6.7
2	3.7	4.7	2.7
3	1.9	2.4	0.8
4	−0.2	1.3	−0.5
5	−6.1	−4.1	−5.5
Total change (%)	6.0	14.0	4.2

Note: The statistics reported are changes contributed by each group of owners, with the sums equal to column totals. These totals are also reported in column 1 of Table 5. There are five productivity (z) levels in the baseline model. For sweat capital (κ) and financial assets (a), we group by quintiles.

Table 7. Effects of Lower Tax Rates on Labor Inputs of Business Owners and Employees

	% Changes from lower tax rates on:	
Statistics	Owners	Employees
Self-employment rate	6.6	−18.1
Total employee hours	−2.1	14.0
Private business	−4.8	17.6
C-corporate	−0.7	12.2
Total owner hours	14.0	−11.1
Production	15.5	−12.7
Sweat building	8.4	−5.3
Wage rate	0.6	0.5
Relative price	−2.7	4.4

Note: Effects of lowering tax rates on owners are percentage changes in response to lowering T^b. More details for this experiment are shown in Table 5. Effects of lowering tax rates on owners are percentage changes in response to lowering T^n.

Table 8. Sensitivity of Main Results in Extensions of Baseline Model
All Private Businesses

		Extended to include:		
Statistics	Baseline model	Financial frictions	Superstar owners	Brokered sales
Sweat equity	1.22	1.23	1.64	1.17
Sale value	0.32	0.32	0.50	0.30
Gross return	7.5	7.4	10.6	6.7
Effects of lower taxes:				
Output	2.2	2.1	0.6	2.1
Sweat capital	6.0	5.5	7.3	2.1
Self-employment rate	6.6	6.6	15.6	5.6
Owner hours	14.0	13.9	21.0	17.0
Total hours	4.2	4.2	3.4	4.9

Note: Estimates for sweat equity v_b, sale value v_κ, and gross return r_b are averages across all business owners. Sweat equity and sale values are constructed only for business owners and are divided by per capita GDP. The gross return on the business is the sum of the capital gain to sweat equity plus the dividend yield and is reported in percentage terms. Effects of lowering tax rates are percentage changes for private business activities in response to lowering taxes on private businesses. Details of the baseline case are shown in Tables 3 and 5.