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Intangible Capital and Measured Productivity

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ABSTRACT

Because firms invest heavily in R&D, software, brands, and other intangible assets—at a rate close to that of tangible assets—changes in measured GDP, which does not include all intangible investments, understate the actual changes in total output. If changes in the labor input are more precisely measured, then it is possible to observe little change in measured total factor productivity (TFP) coincidentally with large changes in hours and investment. This mismeasurement leaves business cycle modelers with large and unexplained labor wedges accounting for most of the fluctuations in aggregate data. In this paper, I incorporate intangible investments into a multi-sector general equilibrium model and parameterize income and cost shares using data from an updated U.S. input and output table, with intangible investments added to final goods and services. I use maximum likelihood methods and quarterly observations on sectoral gross outputs over the period 1985–2014 to estimate processes for latent sectoral TFPs—that have common and idiosyncratic components. Aggregate hours are not used to estimate TFPs, but the model predicts changes in hours that compare well with the actual hours series and account for roughly two-thirds of its standard deviation. I find that sector-specific shocks and industry linkages play an important role in accounting for fluctuations in aggregate U.S. data, and I find that the model's common component of TFP is not correlated at business cycle frequencies with the standard measures of TFP used in the macroeconomic literature.

* The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

1. Introduction

This paper sheds light on a measurement issue that confounds analyses of key macrodata during economic booms and busts. Because firms invest heavily in R&D, software, brands, and other intangible assets—at a rate close to that of tangible assets—changes in GDP, which does not include all intangible investments, understate the actual changes in total output. As a result, it is possible to observe large changes in hours and investment coincidentally with little change in *measured* total factor productivity. In other words, innovation by firms—which is fueled in large part by their intangible investments—may be evident "everywhere but in the productivity statistics."[1] Here, I use theory and recently revised U.S. national accounts to more accurately estimate U.S. total factor productivity at both the aggregate and industry levels.

I develop a dynamic multi-sector general equilibrium model and explicitly incorporate intangible investment. Multiple sectors are needed to account for the vast heterogeneity in intangible investment rates across industries. To parameterize income and cost shares, I start with the 2007 benchmark input-output table and take advantage of the fact that the Bureau of Economic Analysis (BEA) now includes expenditures on intellectual property products—software, R&D, mineral exploration, entertainment, literary, and artistic originals—as part of investment rather than as part of intermediate inputs. I additionally reallocate several categories of intermediate inputs that are under consideration for future inclusion in the BEA fixed assets including computer design services, architectural and engineering services, management consulting services, advertising, and marketing research.

Firms in the model economy have access to two production technologies: one for producing tangible goods and services and another for producing new intangible capital goods and services. Tangible capital is assumed to be a rivalrous input, but intangible capital is

[1] Solow (1987) remarked that the computer age could be seen everywhere but in the economic data.

assumed to be a nonrivalrous input, since knowledge can be used simultaneously in producing consumer goods and services and creating new ideas. I explicitly model industry linkages that occur through purchases of intermediate inputs and through purchases of new tangible investment goods or intangible investment goods. Business cycle fluctuations in the model are assumed to be driven by shocks to industry and aggregate TFP, the impact of which will depend on details of the industry input-use and capital-use linkages.

Because the model includes intangible capital stocks that cannot be accurately measured, it is not possible to use observations on factor inputs and outputs to directly measure the TFP series as has been done in earlier work. (See, for example, Horvath (2000).) Instead, I use maximum likelihood methods to estimate stochastic processes for the latent TFPs—that are assumed to have both sector-specific components and a common component—and derive model predictions for the series via the Kalman smoother. The observations used in the estimation are gross outputs from the BEA by major industry and per capita hours from the Bureau of Labor Statistics (BLS) for three intangible-intensive minor industries over the period 1985:1 to 2014:4. Data on aggregate hours are not used to estimate the TFP processes, but are used as an external check on the model's predictive capability. Previous work has shown that a one-sector, no-intangible version of the model analyzed here has no chance of accounting for fluctuations in aggregate hours.[2]

I find that the model's predicted hours track U.S. hours much better than the simplest one-sector model without intangible investments. The model predicts three sizable booms over the 1985–2014 sample period and then a bust. Moreover, the standard deviation of the model's predicted hours series is 65 percent of the actual series. This implies much less scope for an unexplained labor wedge. If I decompose the predicted changes in hours into components driven by the different shocks, I find that the common TFP shock is

[2] The one-sector, no-intangible version of the model is the prototype model of Chari, Kehoe, and McGrattan (2007, 2016), who use it to show that large labor wedges are needed to account for fluctuations in U.S. hours.

important for the low frequency movements in hours, and the sector-specific TFP shocks are important for higher frequency movements.

I also decompose the variances of the observations used to estimate the latent TFP shocks in order to determine the relative importance of different shocks and to assess the role of input-output linkages. I do this in two ways: by computing the variance decomposition of the ergodic distribution and by decomposing predicted growth rates in specific boom and bust episodes. I find that sector-specific shocks and industry linkages play an important role in accounting for fluctuations in the aggregate and industry gross outputs. With the Kalman smoother, I construct model predictions for all TFP series. Interestingly, I find that the model's common component of TFP is not correlated at business cycle frequencies with the standard measures of TFP used in the macroeconomic literature.

Previous theoretical work related to this paper has either abstracted from intangible capital or been more limited in scope. Long and Plosser (1983) analyzed a relatively simple multi-sector model, arguing that firm- and industry-level shocks could generate aggregate fluctuations. Horvath (1998, 2000) and Dupor (1998) extended their model and studied the nature of industry linkages to determine if independent productivity shocks could in fact generate much variation for aggregate variables. Parameterizing the model to match the input-output and capital-use tables for the 1977 BEA benchmark, Horvath (2000) concludes that sectoral shocks *may* have significant aggregate effects, but he does not compute the model's variance decomposition. More recently, Foerster, Sarte, and Watson (2011) do a full structural factor analysis of the errors from the same multi-sector model, but only use data for sectors within manufacturing and mining. Neither Horvath (2000) nor Foerster et al. (2011) distinguish tangible and intangible investments. McGrattan and Prescott (2010) do distinguish the different investments, but focus only on aggregate data for a specific episode, namely the technology boom of the 1990s. Furthermore, they did their analysis well before the BEA completed the comprehensive revision introducing the category of intellectual property products.

Previous empirical work has documented that intangible investments are large and vary with tangible investments over the business cycle. For example, Corrado, Hulten, and Sichel (2005, 2006) estimate that intangible investments made by businesses are about as large as their tangible investments.[3] McGrattan and Prescott (2014) use firm-level data and show that intangible investments are highly correlated with tangible investments like plant and equipment.

The model is described in Section 2. Parameters of the model are described in Section 3. Section 4 summarizes the results. Section 5 concludes.

2. Model

There is a stand-in household that supplies labor to competitive firms and, as owners of the firms, receives the dividends. There is a government with certain spending obligations that are financed by various taxes on households and firms. Firms produce final goods for households and the government and intermediate inputs for other firms. The sources of fluctuations in the economy are stochastic shocks to firm productivities.[4]

There are J sectors in the economy. Firms in sector j maximize the present value of dividends $\{D_{jt}\}$ paid to their shareholders. I assume that firms in each sector j produce both *tangible* goods and services, Y_j, and *intangible* intangible investment goods and services, X_{Ij}. The technologies available are as follows:

$$Y_{jt} = \left(K^1_{Tjt}\right)^{\theta_j} \left(K_{Ijt}\right)^{\phi_j} \left(\prod_l \left(M^1_{ljt}\right)^{\gamma_{lj}}\right) \left(Z^1_{jt} H^1_{jt}\right)^{1-\theta_j-\phi_j-\gamma_j} \tag{2.1}$$

[3] For more details on measurement of intangible investments in the national accounts, see recent surveys in the BEA's *Survey of Current Business* (U.S. Department of Commerce, 1929–2013). For more details on measurement of R&D investments, see National Science Foundation (1953–2013). For details on entertainment, literary, and artistic originals, see Soloveichik and Wasshausen (2013).

[4] Later, I plan to include shocks to government spending and to tax rates.

$$X_{Ijt} = \left(K_{Tjt}^2\right)^{\theta_j} \left(K_{Ijt}\right)^{\phi_j} \left(\prod_l \left(M_{ljt}^2\right)^{\gamma_{lj}}\right) \left(Z_{jt}^2 H_{jt}^2\right)^{1-\theta_j-\phi_j-\gamma_j} \tag{2.2}$$

and depend on inputs of tangible capital K_{Tj}^1, K_{Tj}^2, intangible capital K_{Ij}, intermediate inputs $\{M_{ljt}^1\}$, $\{M_{ljt}^2\}$, and hours H_j^1, H_j^2. These production technologies are hit by stochastic technology shocks, Z_{jt}^1 and Z_{jt}^2, that could have a common component and sector-specific components. The specific choices for the stochastic processes are discussed below.

Firms in sector j maximize the present value of after-tax dividends on behalf of their owners (households) that discount after-tax future earnings at the rate ϱ_t:

$$\max \ E_0 \sum_{t=0}^{\infty} \left(1 - \tau_{dt}\right) \varrho_t D_{jt},$$

where

$$\begin{aligned}
D_{jt} &= P_{jt} Y_{jt} + Q_{jt} X_{Ijt} - W_{jt} H_{jt} - \sum_l P_{lt} M_{ljt} - \sum_l P_{lt} X_{Tljt} - \sum_l Q_{lt} X_{Iljt} \\
&\quad - \tau_{kt} P_{jt} K_{Tjt} - \tau_{xt} \sum_l P_{lt} X_{Tljt} \\
&\quad - \tau_{pt} \{ P_{jt} Y_{jt} + Q_{jt} X_{Ijt} - W_{jt} H_{jt} - \left(\delta_T + \tau_{kt}\right) P_{jt} K_{Tjt} \\
&\qquad - \sum_l P_{lt} M_{ljt} - \sum_l Q_{lt} X_{Iljt} \} \qquad (2.3) \\
K_{Tjt+1} &= \left(1 - \delta_T\right) K_{Tjt} + \prod_l X_{Tljt}^{\zeta_{lj}} \qquad (2.4) \\
K_{Ijt+1} &= \left(1 - \delta_I\right) K_{Ijt} + \prod_l X_{Iljt}^{\nu_{lj}} \qquad (2.5) \\
M_{ljt} &= M_{ljt}^1 + M_{ljt}^2. \qquad (2.6)
\end{aligned}$$

Dividends are equal to gross output $P_j Y_j + Q_j X_{Ij}$ less wage payments to workers $W_j H_j$, purchased intermediate goods $\sum_l P_l M_{lj}$, new tangible investments $\sum_l P_l X_{Tlj}$, new intangible investments $\sum_l Q_l X_{Ilj}$, and taxes. New investment goods and services are purchased from other sectors and used to update capital stocks as in (2.4) and (2.5). Taxes are levied on property at rate τ_{kt}, investment at rate τ_{xt} (which could be negative if it is an investment tax credit), and accounting profits at rate τ_{pt}.

Households choose consumption C_t and leisure L_t to maximize expected utility:

$$\max \; E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left[(C_t/N_t)\,(L_t/N_t)^{\psi} \right]^{1-\alpha} - 1 \right\} / (1-\alpha)\, N_t \tag{2.7}$$

with the population equal to $N_t = N_0(1+g_n)^t$. The maximization is subject to the following per-period budget constraint:

$$\begin{aligned} &(1+\tau_{ct}) \sum_{j=1}^{J} P_{jt} C_{jt} + \sum_{j=1}^{J} V_{jt} \left(S_{jt+1} - S_{jt} \right) \\ &\quad \leq (1-\tau_{ht}) \sum_{j=1}^{J} W_{jt} H_{jt} + (1-\tau_{dt}) \sum_{j=1}^{J} D_{jt} S_{jt} + \Psi_t, \end{aligned} \tag{2.8}$$

where C_{jt} is consumption of goods made by firms in sector j which are purchased at price P_{jt}, H_{jt} is labor supplied to sector j which is paid W_{jt}, and D_{jt} are dividends paid to the owners of firms in sector j who have S_{jt} outstanding shares that sell at price V_{jt}. Taxes are paid on consumption purchases (τ_{ct}), labor earnings (τ_{ht}) and dividends (τ_{dt}). Any revenues in excess of government purchases of goods and services are lump-sum rebated to the household in the amount Ψ_t.

The composite consumption and leisure that enter the utility function are given by

$$C_t = \left[\sum_j \omega_j C_{jt}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{2.9}$$

$$L_t = N_t - \sum_j H_{jt}. \tag{2.10}$$

Notice that here, I assume CES for consumption and linear for hours. As owners of the firm, the household's discount factor is the relevant measure for ϱ_t in (2.3):

$$\varrho_t = \beta^t U_{ct} / \left[P_t \left(1 + \tau_{ct} \right) \right], \tag{2.11}$$

where P_t is the aggregate price index given by $P_t = [\sum_j \omega_j^{\sigma} P_{st}^{1-\sigma}]^{1/(1-\sigma)}$.

The resource constraints for tangible and intangible goods and services are given as follows:

$$Y_{jt} = C_{jt} + \sum_l X_{Tjlt} + \sum_l M_{jlt} + G_{jt} \tag{2.12}$$

$$X_{Ijt} = \sum_l X_{Ijlt}, \tag{2.13}$$

where Y_j and X_{Ij} are defined in (2.1) and (2.2), respectively. The model economy is closed and, therefore, there is no term for net exports.[5]

I assume that the logs of the sectoral TFP processes are equal to the sum of a sector-specific component $\tilde{Z}^i_{jt}$ and a common component Z_t with factor loading φ_j, that is,

$$\log Z^i_{jt} = \log \tilde{Z}^i_{jt} + \varphi_j \log Z_t \tag{2.14}$$

$$\log \tilde{Z}^i_{jt} = \rho_{ij} \log \tilde{Z}^i_{jt-1} + \eta^i_{jt} \tag{2.15}$$

$$\log Z_t = \rho \log Z_{t-1} + \upsilon_t, \tag{2.16}$$

where $E\eta^i_{jt} = 0$, $E\eta^i_{jt}\eta^i_{jt-1} = 0$, $E\eta^i_{jt}\eta^k_{lt} = 0$ for all i, j, k, l except cases with $j = l$, $E\upsilon_t = 0$, $E\upsilon_t\upsilon_{t-1} = 0$, and $E\upsilon_t\eta^i_{jt} = 0$. In other words, the shocks to TFP are correlated within a sector but not across sectors and not with the common TFP component.[6]

An approximate equilibrium for the model economy can be found by applying a version of Vaughan's (1970) method to the log-linearized first-order conditions of the household and firm maximization problems. The solution can be summarized as an equilibrium law of motion for the logged and detrended state vector x, namely:

$$x_{t+1} = Ax_t + B\varepsilon_{t+1}, \quad E\varepsilon_t\varepsilon'_t = I, \tag{2.17}$$

where $x_t = [\vec{k}_{Tt}, \vec{k}_{It}, \vec{z}_{1t}, \vec{z}_{2t}, z_t, 1]'$ is a $(4J{+}2){\times}1$ state vector, $\vec{k}_{Tt}$ is the $J{\times}1$ vector of logged and detrended tangible capital stocks, $\vec{k}_{It}$ is the $J{\times}1$ vector of logged and detrended

[5] In the empirical implementation, net exports will be included with intermediate and final domestic purchases.

[6] One exception is the government sector (NAICS 92). I assume that shocks to production in NAICS 92 are independent of all other shocks. If I assume otherwise, then the common shock parameter estimates depend importantly on fluctuations of gross output in this sector during the Great Recession, the source of which is unlikely to be a boom in TFP.

intangible capital stocks, $\vec{z}_{1t}$ is the $J \times 1$ vector of logged and detrended sectoral TFPs for production of final goods and services, $\vec{z}_{2t}$ is the $J \times 1$ vector of logged and detrended sectoral TFPs for production of new intangible investments, and z_t is the logged and detrended common shock. The last element of x_t is a 1, which is used for constant terms. The vector ε_t is a $2J+1$ vector of normally distributed shocks. I assume that the only nonzero off-diagonal elements of B are correlations between the tangible and intangible production within a sector. I also estimate, in this case, a stochastic process for the common component, z_t.

In the next section, I use BEA data to parameterize this model economy and to estimate the parameters governing the shock processes in (2.17) using maximum likelihood methods.

3. Parameters

Here, I describe how to parameterize income and cost shares using the 2007 benchmark BEA input-output table and how to estimate processes for components of the sectoral TFPs, namely $\{Z^1_{jt}\}$ and $\{Z^2_{jt}\}$, using data from the BEA and BLS. The remaining parameters, which are also described below, are those related to preferences, growth rates, depreciation, and tax rates and are not critical to the main results.

3.1. Income and Cost Shares

The starting point for my analysis are the input-output tables published by the BEA. In Figure 1, I show an example input-output table. The upper left $J \times J$ matrix has intermediate purchases. The rows are commodities (or inputs) and the columns are the industries using them in production. For the analysis below, I set $J = 15$ and the sectors are the following major industries: (1) agriculture, forestry, fishing, and hunting (NAICS 11); (2) mining (NAICS 21); (3) utilities (NAICS 22); (4) construction (NAICS 23); (5)

manufacturing (NAICS 31-33); (6) wholesale trade (NAICS 42); (7) retail trade (NAICS 44-45); (8) transportation and warehousing (NAICS 48-49); (9) information (NAICS 51); (10) finance, insurance, real estate, rental and leasing (NAICS 52-53); (11) professional and business services (NAICS 54-56); (12) educational services, health care, and social assistance (NAICS 61-62); (13) arts, entertainment, recreation, accommodation, and food services (NAICS 71-72); (14) other services except government (81); and (15) public administration (NAICS 92). Before computing intermediate shares, I reallocate intermediate expenses in several categories of professional and business services—categories that national accountants are considering reallocating—to the matrix of intangible investments listed under final uses. Specifically, I move expenses for computer design services, architectural and engineering services, management consulting services, advertising, and marketing research out of the intermediate inputs matrix and into final uses.

In terms of the model, the intermediate purchases that show up in element (l, j) of the matrix are given by $P_l(M^1_{lj} + M^2_{lj})$. I use the relative shares of these purchases to parameterize the intermediate shares, $\{\gamma_{lj}\}$, in (2.1) and (2.2). The actual shares used in the analysis are reported in Table 1. The first panel of the table shows the values of the intermediate shares γ_{lj}. The first row and column headers indicate the commodity and industry NAICS category, respectively, which in turn correspond to the 15 major industries listed above. Notice that most elements are nonzero, indicating that there are many sectoral linkages.

The upper right part of the table in Figure 1 is the final uses of the commodities. The labels on these final uses are not exactly the same as the BEA's because some adjustments need to be made in order for the theory and data to be consistent. Starting with consumption, I include the nondurable goods and services categories from BEA's personal consumption expenditures (PCE). Expenditure shares for these goods and services are governed by the choice of $\{\omega_j\}$ in (2.9), which I set to align the theoretical and empirical shares. These are shown in the final row of Table 1.

The durable goods component of PCE is included with investments. Specifically, durable equipment is assumed to be part of tangible investment, and software and books are assumed to part of intangible investment. Since the tangible and intangible investments, like intermediate purchases, are used by different industries, I need to assign consumer durable purchases to specific elements of the $J \times J$ matrices. In the case of consumer durable equipment, I assume it is a manufactured commodity (commodity 5) used by the real estate industry (industry 10). In the case of software and books, I assume these are information commodities (commodity 9) used by the real estate industry (industry 10). Another adjustment that must me made is to include the durable capital services and depreciation with consumption services. This adjustment also affects incomes, which I describe later.

Detailed investment data are used to fill in elements of the BEA capital flow tables (also referred to as the capital-use tables).[7] The detailed data are broken down by investment category and industries making the investment.[8] I construct two capital flow tables: tangible and intangible. I include fixed investment in equipment and structures—both public and private—and changes in inventories with tangible investment, and I include the new BEA category of *intellectual property* (IP) *products*—both public and private—with intangible investment.[9]

The IP products include expenditures on software, mineral exploration, research and development (R&D), and entertainment, literary, and artistic originals. Some of this spending is done by firms in-house (and is what the BEA calls own-account). For this spending I reassign the commodity source to the own industry, which is more in line with the theory.

[7] The BEA has not yet published an official capital flow table for the 2007 benchmark input-output accounts. I constructed one with detailed investment data available for the BEA fixed asset tables and very useful correspondence with David Wasshausen of the BEA.

[8] Some adjustments need to be made to reallocate from owners to users since these tables record final users of the capital goods.

[9] This category of investment was added in the 2013 comprehensive revision of the accounts.

Once I have the capital flow tables, I can set the parameters ζ_{lj} and ν_{lj} using the spending shares for tangible investment and intangible investment, respectively.

The second panel of Table 1 shows the tangible capital flow shares ζ_{lj}. Notice that many rows of this panel have only zeros because the commodities produced are neither structures nor equipment. Commodities categorized under construction (NAICS 23) and manufacturing (NAICS 31-33) are the main sources of these investment goods. The third panel is the analogous panel for intangible investments. Commodities categorized under information (NAICS 51) and professional and business services (NAICS 54-56) are most important in this case. In the BEA data, scientific R&D is listed under NAICS 5417 but much of this is specific to other commodities (e.g., chemical manufacturing) and has been assigned accordingly. For this reason, there are nonzero shares on the diagonal of the 15×15 matrix ν that would be zeros in the BEA's table.

The next columns in the final-use table has purchases of government and the rest of world. I list government purchases as 'government consumption' in the table since government investment is included with the private investments. For all of the simulations below, I also add the government consumption in with private spending and thus the theory assumes zeros for this column. The economy is closed and does not have a rest-of-world sector. Thus, I reallocate net exports to the domestic categories of intermediates, consumption, and investment. I do so in a pro rata way.

The panel below intermediate purchases in Figure 1 shows the categories of value added. The first has industry compensation, which is W_jH_j for all j in the model. The second has business taxes that include consumption and excise taxes $\tau_c C_j$ and property taxes $\tau_k K_{Tj}$. The third category is operating surplus which is the sum of all capital income and capital depreciation (including depreciation of consumer durables) less property taxes. Shares of capital income $\{\theta_j, \phi_j\}$ are set so that the total spending on tangible and intangible investment is equal to that in the U.S. data. These shares are shown in the fourth and fifth panels of Table 1. Adding up the income categories is another way to compute GDP (in addition to adding up expenditures or taking industry outputs and subtracting intermediate purchases).

3.2. Shock Processes

Estimates of the parameters governing the shock processes are found by applying maximum likelihood to the following state space system:

$$x_{t+1} = Ax_t + B\varepsilon_{t+1} \tag{3.1}$$

$$y_t = Cx_t, \tag{3.2}$$

where the elements of x_t are defined above (see (2.17)) and assumed to be unobserved, and y_t are quarterly U.S. data for the period 1985:1-2014:4. For y_t, I use detrended gross outputs and, in some intangible-intensive industries, per capita hours. A common trend is used for technology growth for all industry gross outputs.

The sectoral gross outputs are the empirical analogue of $P_{jt}Y_{jt} + Q_{jt}X_{Ijt}$ in equation (2.3).[10] I use gross outputs, rather than data on value added, because there are no issues with the classification of spending as intermediate or final.[11] Definitions of value added have changed over the postwar period. For intangible-intensive sectors that may have some own-account investments still missing from gross output, sectoral hours are used. If only a small subset of hours are used in the estimation, aggregate hours can be used as an external check on the model. Given the failure of the standard one-sector model without intangibles to account for large fluctuations in hours, a comparison of hours is a particularly important test of the new theory.

The model time period is quarterly, but time series on gross outputs by industry are only available annually before 2005. Therefore, before estimating parameters for the shock

[10] Both data and model series are deflated before shocks are estimated.

[11] I also worked with IRS business receipts, which are an important source of information for constructing gross outputs and are available back to the 1920s for many major and minor industries.

processes, I use a Kalman filter to compute forecasts of quarterly gross outputs. The idea is to use other available quarterly data by industry and construct quarterly forecasts for the series of interest, namely, gross outputs. Specifically, I use quarterly estimates of BEA's national income by industry, N_{jt}, quarterly estimates of BLS's employment by industry, E_{jt}, and *annual* estimates of BEA's gross outputs, $G_{jt}, t = 4, 8, 12, \ldots$ where $G_{jt} = 0$ for t not divisible by four. Both the national income and gross output data are divided by the GDP deflator. Then all three series are detrended by applying the filter in Hodrick and Prescott (1997) (with a smoothing parameter of 1600 for the quarterly series and 100 for the annual series).[12]

Let $\hat{G}_{jt}$ be the quarterly gross outputs being forecasted. The first step in deriving a forecast is to estimate A_j and B_j of the following state space system via maximum likelihood:

$$x_{jt+1} = A_j x_{jt} + B_j \epsilon_{t+1}$$

$$y_{jt} = C_{jt} x_{jt}$$

where $x_{jt} = [X_{jt}, X_{j,t-1}, X_{j,t-2}, X_{j,t-3}]'$, $X_{jt} = [N_{jt}, E_{jt}, \hat{G}_{jt}]'$, and $y_{jt} = [N_{jt}, E_{jt}, G_{jt}]'$, and

$$A_j = \begin{bmatrix} a_{1j} & a_{2j} & a_{3j} & a_{4j} \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix}, \quad B_j = \begin{bmatrix} b_j \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_{jt} = \begin{cases} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/4 & 0 & 0 & 1/4 & 0 & 0 & 1/4 & 0 & 0 & 1/4 \end{bmatrix} & \text{if } t \text{ is 4th quarter} \\ \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} & \text{otherwise.} \end{cases}$$

[12] The Hodrick-Prescott filter is only used to when constructing quarterly forecasts for the missing observations in G_{jt}; the low frequency HP trend is added back. Then a common linear trend is removed from the logged gross output series before adding them to the vector y_t in (3.2).

Once I have parameter estimates $\hat{A}_j$ and $\hat{B}_j$, I can construct forecasts of gross outputs in all quarters given the full sample of data, namely $\hat{G}_{jt} = E[G_{jt}|y_{j1},...,Y_{jT}]$, by first applying the Kalman filter and then applying the Kalman smoother. (See Harvey (1989) for details.)

Doing this yields fifteen series of gross outputs for the major industries in the input-output table described earlier. Additionally, I include data on hours per capita for computer and electronic products, broadcasting and communications, and advertising, which are 3-digit industries under manufacturing (industry 5), information (industry 9), and professional and business services (industry 11), respectively. Firms in these minor industries make sizable intangible investments, some of which may be done in-house, which is why I use hours rather than output to identify the Z_{jt}^2 for the sectors j = 5,9,11. Because the hours in these industries account for only 10 percent, I can use the model's prediction for aggregate hours as an external check on the model.

One final step before the TFP processes can be estimated is to set the initial state x_0 in (3.1). Here, I do not use the steady-state values because there are differing growth trends in U.S. industry data. For example, relative to an economy-wide trend, manufacturing has been slowing and information has been growing. Thus, I choose x_0 in such a way that initial investments do not jump. This is easy to do in two steps: start by setting x_0 equal to the steady state and then use the model's prediction for the first period state, $\hat{x}_1$, as the new initial condition.

Given the eighteen observable series in the vector, y_t, and initial conditions for the state, x_0, I again apply the methods in Harvey (1989) to estimate the parameters of the eighteen stochastic TFP processes. These parameters appear as coefficients in A and B in (3.1). The estimated stochastic processes are reported in a separate appendix.

3.3. Other parameters

The remaining parameters are those related to preferences, growth in population and technology, depreciation, and taxes.

For preferences, I set $\alpha = 1$, $\psi = 1.2$, and $\beta = 0.995$. Growth in population is 0.25 percent per quarter. Growth in technology is 0.5 percent per quarter. Depreciation is assumed to be the same for all sectors and both types of capital and is set at 0.8 percent per quarter.[13] Tax rates are based on IRS and national account data and are as follows: $\tau_c = 0.065$, $\tau_d = 0.144$, $\tau_h = .382$, $\tau_k = 0.003$, $\tau_p = 0.33$ and $\tau_x = 0$. For the results below, these rates are held constant.

4. Results

In this section, I first provide evidence of the model's fit by comparing the per capita hours prediction of a baseline model with only one sector and no intangibles to those of the extended multi-sector model with intangible investments. I find that the latter is significantly closer to U.S. hours because the implied TFP series for the baseline model and the estimated series for the extended model behave differently at business cycle frequencies. I decompose the extended model's predictions for aggregate data and show that sector-specific shocks and industry linkages both play an important role in accounting for fluctuations over the period 1985-2014. I then look more closely at the Great Recession, highlighting differences in the model's predicted TFP series and a typical measure of TFP used in the macroeconomic literature.

[13] One issue that arises in models with intangible capital is the lack of identification of all parameters. For example, there is insufficient data to estimate both capital shares and depreciation rates, even in the case of R&D assets that are now included in both NIPA and the BEA's fixed asset tables. The BEA uses estimates of intangible depreciation rates to calculate the return to R&D investments and the capital service costs, which are used in capitalizing R&D investments for their fixed asset tables. Unfortunately, as the survey of Li (2012) makes clear, "measuring R&D depreciation rates directly is extremely difficult because both the price and output of R&D capital are generally unobservable." Li discusses different approaches that have been used to estimate industry-specific R&D depreciation rates, finding that there is a wide range of estimates even within narrow categories. She concludes that "the differences in their results cannot be easily reconciled." (See Li, Table 2.)

4.1. Baseline Model

A useful baseline is the nested model with one sector $S = 1$ and without intangible capital $\phi_s = 0$. This version of the model generates similar results to the model of Kydland and Prescott (1982), which has the distinction of being the standard theoretical benchmark.

In this case, I use the Solow residual as an estimate of the model's (one) TFP series. The Solow residual is real GDP divided by real fixed assets raised to a power (in this case, one-third) times aggregate hours raised to a power (in this case, two-thirds).[14] I assume the logarithm of the Solow residual is a first-order autoregressive process which can be estimated using ordinary least squares. Given the estimates and an initial condition for the process, I can simulate a path for TFP and feed it in to the model's equilibrium decision functions.

Figure 2 shows the baseline model's predicted per capita hours along with actual U.S. hours. As the figure shows, the predicted series does not track the U.S. series and varies much less over the business cycle, barely rising during the technology boom and barely falling during the Great Recession. Why does it vary so little? The answer is that measured TFP—which in this case is the Solow residual—does not fluctuate that much over the cycle in my sample period.

This result will serve as a useful baseline when analyzing the extended model with intangible investments and industry input-output linkages explicitly modeled. I turn to this next.

4.2. Extended Model

In the extended model, predictions of the model's state and decision variables conditional on all of the observations, $\{y_t\}$, are derived using a Kalman smoother. Variables of interest include aggregate hours and the latent TFP series.

[14] The NIPA data do include some intangible investments and the fixed assets do include some intangible capital. Stripping them out will not affect the main results for the baseline model.

Figure 3 shows the extended model's predicted per capita hours along with actual U.S. hours. The figure shows that the predicted hours track actual hours much better than the simplest one-sector model (Figure 2). The model predicts three sizable booms and then a bust, and the standard deviation of the model series is 65 percent of the actual series. This leaves much less room for an unexplained labor wedge.

If I decompose the predicted series in Figure 3 into the component due to the sector-specific shocks and the component due to the common shock, I find that both play a role. Figure 4A plots the predicted hours series with all shocks included and then again with only the common shock. The figure shows that the common shock generates significant variation at a low frequency, which is necessary to account for the lack of recovery after the Great Recession. Figure 4B plots the predicted hours series with all shocks included and then again with only the sector-specific shocks. The sector-specific shocks are more important in generating variation at higher frequencies, for example, in the 2001 and 2008 downturns.

Table 2 shows the variance decomposition for model's ergodic distribution. The rows correspond to the observable data in y_t and I have listed the impacted industry.[15] In the case of major industries, the variances of y_t being decomposed are those of gross outputs, and for the minor intangible-intensive industries, the variances being decomposed are those of per capita hours. The columns correspond to the shocks. The first column is the total variance due to sectoral shocks. This variance is split between own-sector shocks and other-sector shocks. Notice that sectoral shocks are important for all of the industry data. Furthermore, with the exception of mining, the variances attributable to shocks in other industries are nontrivial, indicating that sectoral linkages are playing an important role. In fact, in many industries the role of other-sector shocks is greater than that of the common shock.

[15] The government sector is not listed since I imposed restrictions on the shocks in this sector.

One issue with the variance decomposition in Table 2 is the fact that there are significant trends in the 1985–2014 sample, which will bias these estimates. Most likely, the trends imply more weight on sectoral shocks and less weight on common shocks. Thus, as an alternative summary of the variance decomposition, I decompose the growth rates of gross output in two episodes: the 1990s technology boom and the Great Recession.

The results are shown in Figure 5. Here the rows correspond to the source of shocks, with "common" being due to innovations in Z. The x-axis shows the change in growth attributable to shocks from each source. There are two periods and, therefore, two estimates for each period. The figure shows that the common TFP shock accounts for roughly half of the increase in total gross output in the boom and roughly half of the decrease in the bust. In the technology boom, shocks to TFP in FIRE and professional and business services were also important. In the Great Recession, shocks to manufacturing TFP were important.

Finally, I compare the time series of the model's predicted common TFP shock to the standard measure of TFP used in the literature, namely, the Solow residual, which was the TFP series used in the baseline model. I find they are similar at the low frequency but different at the high frequency. More specifically, if I compare the two series after logging and linearly detrending them, the correlation is 73 percent. However, if I instead use a Hodrick-Prescott filter and compare only cyclical fluctuations, the correlation is only 9 percent. Figure 6 shows these two TFP series—logged and linearly detrended—during the Great Recession, where both are standardized by first subtracting the 2008:1 value and then dividing all values by the standard deviation of the series. The Solow residual falls modestly and briefly and is back to trend by mid-2009, exactly when the Great Recession is declared over by the National Bureau of Economic Research. The estimated TFP series, on the other hand, falls by more and never recovers. This pattern is consistent with much of U.S. real activity since 2008.

5. Conclusion

In the recent comprehensive revision of the national accounts, the BEA has greatly expanded its coverage of intellectual property products. In this paper, I expand the coverage further and use a multi-sector general equilibrium model to quantify the impact of including these products (which I refer to as intangible investments) in both the theory and the measures of GDP and TFP. I find that updating both—both the theory and the data—is quantitatively important for analyzing U.S. aggregate fluctuations.

References

Chari, V.V., Patrick J. Kehoe, and Ellen R. McGrattan. 2007. "Business Cycle Accounting." *Econometrica* 75(3): 781–836.

Chari, V.V., Patrick J. Kehoe, and Ellen R. McGrattan. 2016. "Accounting for Business Cycles," in J. Taylor and H. Uhlig (eds.), *Handbook of Macroeconomics*, forthcoming.

Corrado C., Charles R. Hulten, and Daniel E. Sichel. 2005. "Measuring Capital and Technology: An Expanded Framework," in C. Corrado, J. Haltiwanger, and D. Sichel (eds.), *Measuring Capital in the New Economy*, (Chicago, IL: University of Chicago).

Corrado, Carol A., Charles R. Hulten, and Daniel E. Sichel. 2006. "Intangible Capital and Economic Growth," Finance and Economics Discussion Series, 2006–24, Divisions of Research and Statistics and Monetary Affairs, Federal Reserve Board, Washington, DC.

Dupor, Bill. 1998. "Aggregation and Irrelevance in Multi-Sector Models." *Journal of Monetary Economics* 43(2): 391–409.

Federal Reserve Board of Governors. 1945–2013. *Flow of Funds Accounts of the United States*, (Washington, DC: Board of Governors).

Foerster, Andrew T., Pierre-Daniel G. Sarte, and Mark W. Watson. 2011. *Journal of Political Economy* 119(1): 1–38.

Harvey, Andrew. 1989. *Forecasting, Structural Time Series Models and the Kalman Filter.* (Cambridge, UK: Cambridge University Press).

Hodrick, Robert and Edward C. Prescott. 1997. "Postwar U.S. Business Cycles: An Empirical Investigation." *Journal of Money, Credit, and Banking* 29(1): 1–16.

Horvath, Michael. 1998. "Cyclicality and Sectoral Linkages: Aggregate Fluctuations from Independent Sectoral Shocks." *Review of Economic Dynamics* 1(4): 781–808.

Horvath, Michael. 2000. "Sectoral Shocks and Aggregate Fluctuations." *Journal of Monetary Economics* 45(1): 69–106.

Kydland, Finn E.and Edward C. Prescott. 1982. "Time to Build and Aggregate Fluctuations." *Econometrica* 50(6): 1345–1370.

Li, Wendy C.Y. 2012. "Depreciation of Business R&D Capital," Mimeo, Bureau of Economic Analysis.

Long, John B., Jr., and Charles I. Plosser. 1983. “Real Business Cycles.” *Journal of Political Economy* 91(1): 39–69.

McGrattan, Ellen R. 1994. “The Macroeconomic Effects of Distortionary Taxation.” *Journal of Monetary Economics* 33(3): 573–601.

McGrattan, Ellen R., and Edward C. Prescott. 2010. “Unmeasured Investment and the Puzzling U.S. Boom in the 1990s.” *American Economic Journal: Macroeconomics* 2(4): 88–123.

McGrattan, Ellen R., and Edward C. Prescott. 2014. “A Reassessment of Real Business Cycle Theory.” *American Economic Review, Paper and Proceedings*, 104(5): 177–187.

Roll, Richard and Stephen A. Ross. 1980. “An Empirical Investigation of the Arbitrage Pricing Theory.” *Journal of Finance*, 35(5): 1073-1103.

Soloveichik, Rachel, and David Wasshausen. 2013. “Copyright-Protected Assets in the National Accounts,” Mimeo, Bureau of Economic Analysis.

Solow, Robert. 1987. “We’d better watch out,” *New York Times Book Review*, July 12, 1987, p. 36.

National Science Foundation. 1953–2015. *National Patterns of R&D Resources*, (Washington, DC: National Science Foundation).

Vaughan, David R. 1970. “A Nonrecursive Algebraic Solution for the Riccati Equation.” *IEEE Transactions on Automatic Control* AC-15: 597–599.

U.S. Department of Commerce, Bureau of Economic Analysis. 1929–2015. *Survey of Current Business*, (Washington, DC: U.S. Government Printing Office).

Table 1. Parameters Based on 2007 U.S. Input Output Table

Intermediate goods and services shares (γ_{lj})

NAICS	11	21	22	23	31-33	42	44-45	48-49	51	52-53	54-56	61-62	71-72	81	92
11	.205	.000	.000	.001	.033	.001	.001	.000	.000	.000	.000	.000	.007	.000	.001
21	.003	.069	.107	.005	.037	.000	.000	.003	.000	.001	.000	.000	.001	.001	.004
22	.015	.011	.014	.003	.013	.006	.014	.008	.003	.018	.005	.011	.018	.008	.009
23	.007	.017	.019	.000	.002	.001	.003	.005	.002	.025	.001	.001	.003	.006	.019
31-33	.178	.073	.071	.243	.264	.030	.033	.154	.050	.011	.042	.076	.118	.079	.094
42	.071	.015	.016	.044	.047	.029	.017	.030	.012	.003	.008	.021	.022	.016	.014
44-45	.001	.000	.001	.058	.002	.001	.004	.005	.000	.002	.001	.001	.007	.008	.000
48-49	.033	.023	.067	.018	.022	.047	.053	.123	.015	.007	.015	.010	.014	.009	.018
51	.001	.002	.006	.003	.004	.012	.013	.007	.141	.016	.023	.016	.011	.017	.026
52-53	.045	.032	.052	.023	.015	.086	.126	.093	.050	.212	.088	.136	.097	.159	.040
54-56	.010	.040	.045	.011	.042	.085	.059	.046	.040	.068	.088	.068	.082	.039	.038
61-62	.001	.000	.000	.000	.000	.000	.002	.000	.000	.000	.000	.012	.002	.003	.005
71-72	.001	.002	.010	.002	.003	.005	.003	.004	.021	.010	.018	.011	.025	.006	.009
81	.004	.003	.005	.005	.008	.017	.011	.024	.016	.013	.014	.013	.015	.015	.015
92	.000	.000	.002	.000	.001	.011	.006	.022	.003	.002	.003	.003	.007	.003	.003

Tangible capital flow shares (ζ_{lj})

	11	21	22	23	31-33	42	44-45	48-49	51	52-53	54-56	61-62	71-72	81	92
11	.084	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	.002	.763	.003	.003	.002	.001	.001	.020	.001	.000	.002	.001	.001	.001	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	.154	.054	.431	.058	.165	.228	.477	.261	.320	.329	.205	.430	.574	.496	.699
31-33	.510	.123	.379	.629	.593	.468	.350	.470	.454	.558	.531	.381	.285	.337	.247
42	.129	.031	.096	.160	.124	.191	.089	.119	.115	.016	.135	.097	.072	.086	.040
44-45	.037	.009	.027	.045	.035	.034	.025	.034	.033	.007	.038	.027	.020	.024	0
48-49	.029	.007	.022	.036	.028	.027	.020	.045	.026	.004	.030	.022	.016	.019	.006
51	.008	.002	.006	.009	.007	.007	.005	.007	.008	.001	.008	.006	.004	.005	0
52-53	0	0	0	0	0	0	0	0	0	.066	0	0	0	0	0
54-56	.049	.012	.036	.060	.047	.045	.033	.045	.043	.020	.051	.036	.027	.032	.008
61-62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
71-72	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
81	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
92	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Intangible capital flow shares (ν_{lj})

	11	21	22	23	31-33	42	44-45	48-49	51	52-53	54-56	61-62	71-72	81	92
11	.028	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	.187	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	.115	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	.028	0	0	0	0	0	0	0	0	0	0	0
31-33	0	0	0	0	.725	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	.219	0	0	0	0	0	0	0	0	.003
44-45	0	0	0	0	0	0	.091	0	0	0	0	0	0	0	0
48-49	0	0	0	0	0	0	0	.089	0	0	0	0	0	0	.000
51	.112	.149	.107	.024	.028	.047	.086	.094	.614	.391	.044	.048	.197	.065	.015
52-53	0	0	0	0	0	0	0	0	0	.122	0	0	0	0	0
54-56	.860	.664	.778	.948	.247	.734	.824	.817	.386	.487	.956	.619	.793	.674	.381
61-62	0	0	0	0	0	0	0	0	0	0	0	.333	0	0	0
71-72	0	0	0	0	0	0	0	0	0	0	0	0	.010	0	0
81	0	0	0	0	0	0	0	0	0	0	0	0	0	.261	0
92	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.602

Table 1. Parameters Based on 2007 U.S. Input Output Table (Cont.)

	Tangible capital shares (θ_j)														
NAICS	11	21	22	23	31-33	42	44-45	48-49	51	52-53	54-56	61-62	71-72	81	92
	.301	.558	.384	.167	.165	.127	.136	.132	.201	.408	.059	.076	.142	.130	.102
	Intangible capital shares (ϕ_j)														
	11	21	22	23	31-33	42	44-45	48-49	51	52-53	54-56	61-62	71-72	81	92
	.006	.011	.038	.082	.193	.149	.072	.039	.236	.040	.178	.033	.061	.056	.083
	Consumption shares (ω_j)														
	11	21	22	23	31-33	42	44-45	48-49	51	52-53	54-56	61-62	71-72	81	92
	.005	.000	.021	.000	.118	.038	.089	.022	.033	.202	.018	.163	.068	.031	.193

Table 2. Variance Decomposition, 1985:1–2014:4

Observable	Sector-specific			Common
	Total	Own Industry	Other Industry	Shock
Agriculture (11)	96.4	61.8	34.6	3.6
Mining (21)	99.9	98.8	1.2	0.1
Utilities (22)	98.8	61.9	37.0	1.2
Construction (23)	77.9	39.2	38.7	22.1
Manufacturing (31-33)	91.5	75.7	15.8	8.5
Computers & Electrical	90.9	80.3	10.6	9.1
Wholesale Trade (42)	81.5	32.5	16.8	18.6
Retail Trade (44-45)	60.0	27.5	32.5	40.0
Transportation & Warehousing (48-49)	70.6	29.7	40.9	29.4
Information (51)	74.1	49.4	24.7	25.9
Broadcasting & Telecommunications	78.5	49.8	28.8	21.5
Finance, Insurance & Real Estate (52-53)	64.7	9.0	55.7	35.3
Professional & Business Services (54-56)	73.5	57.8	15.7	26.5
Advertising	63.8	42.0	21.8	36.2
Education, Health & Social Services (61-62)	67.4	8.6	58.9	32.6
Leisure and Hospitality (71-72)	65.1	10.2	54.9	34.9
Other Services (81)	62.5	20.4	42.2	37.5

Figure 1
Input Output Table

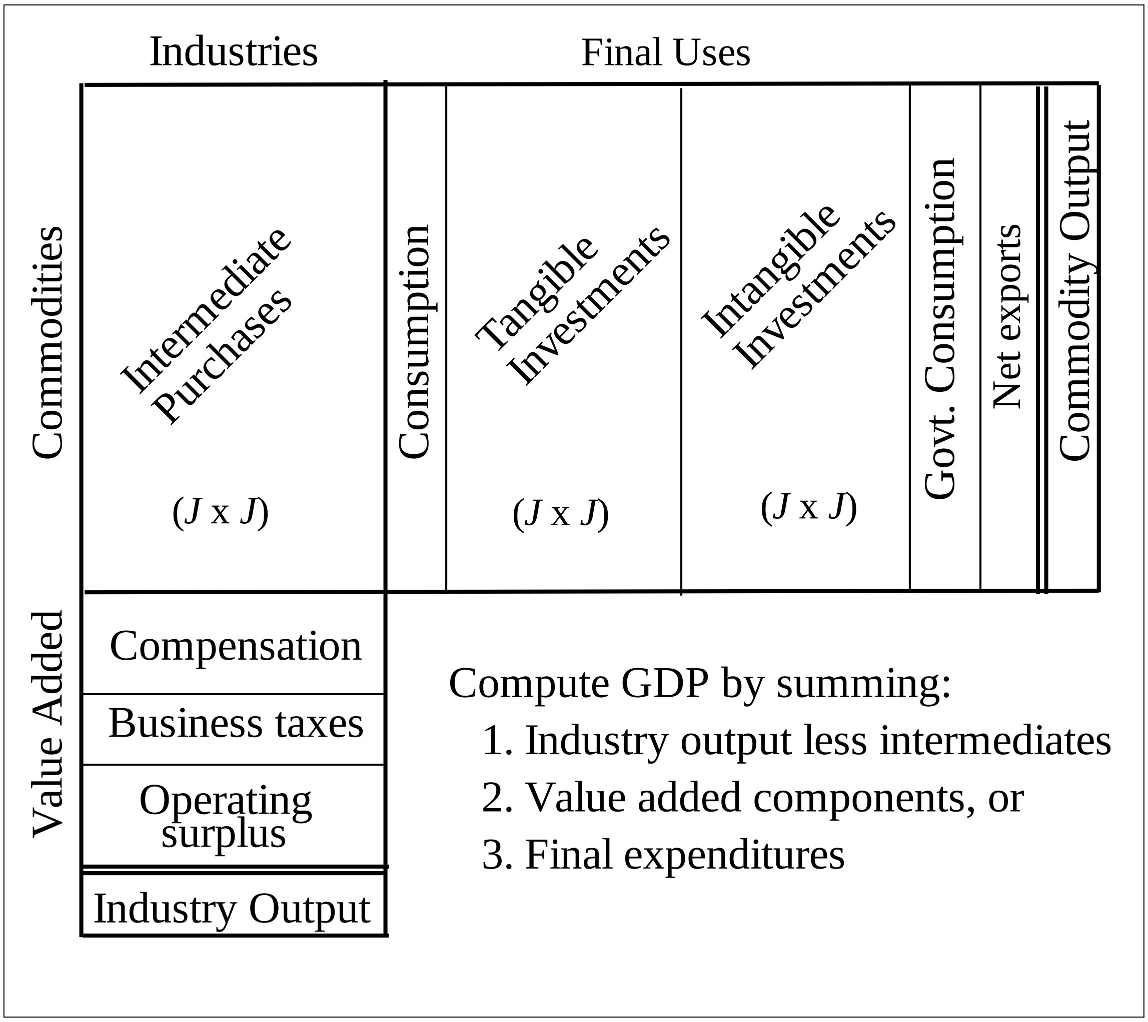


Figure 2

U.S. Hours Per Capita and One-Sector, No-Intangibles Model Prediction

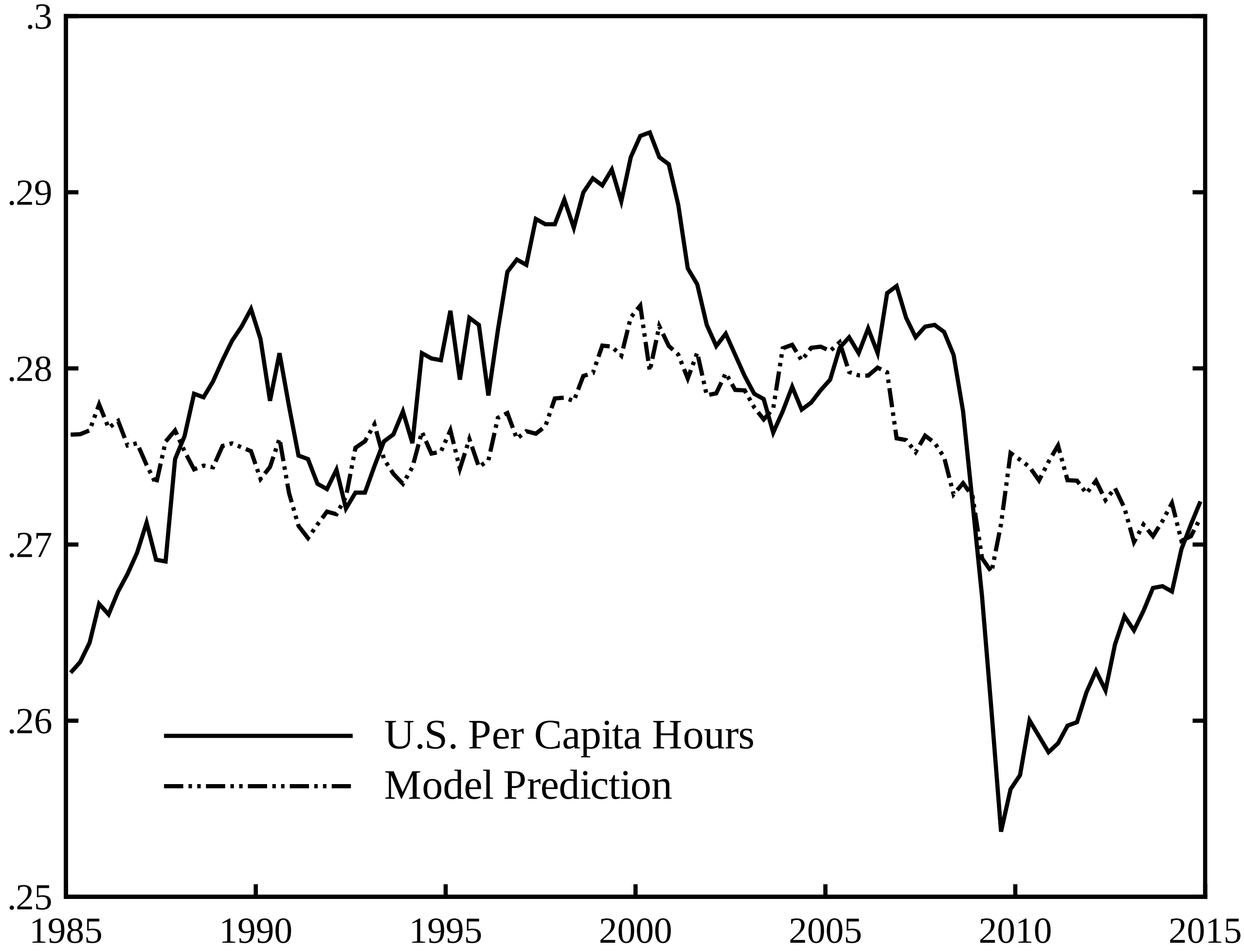


Figure 3

U.S. Hours Per Capita Multi-Sector with Intangibles Model Prediction

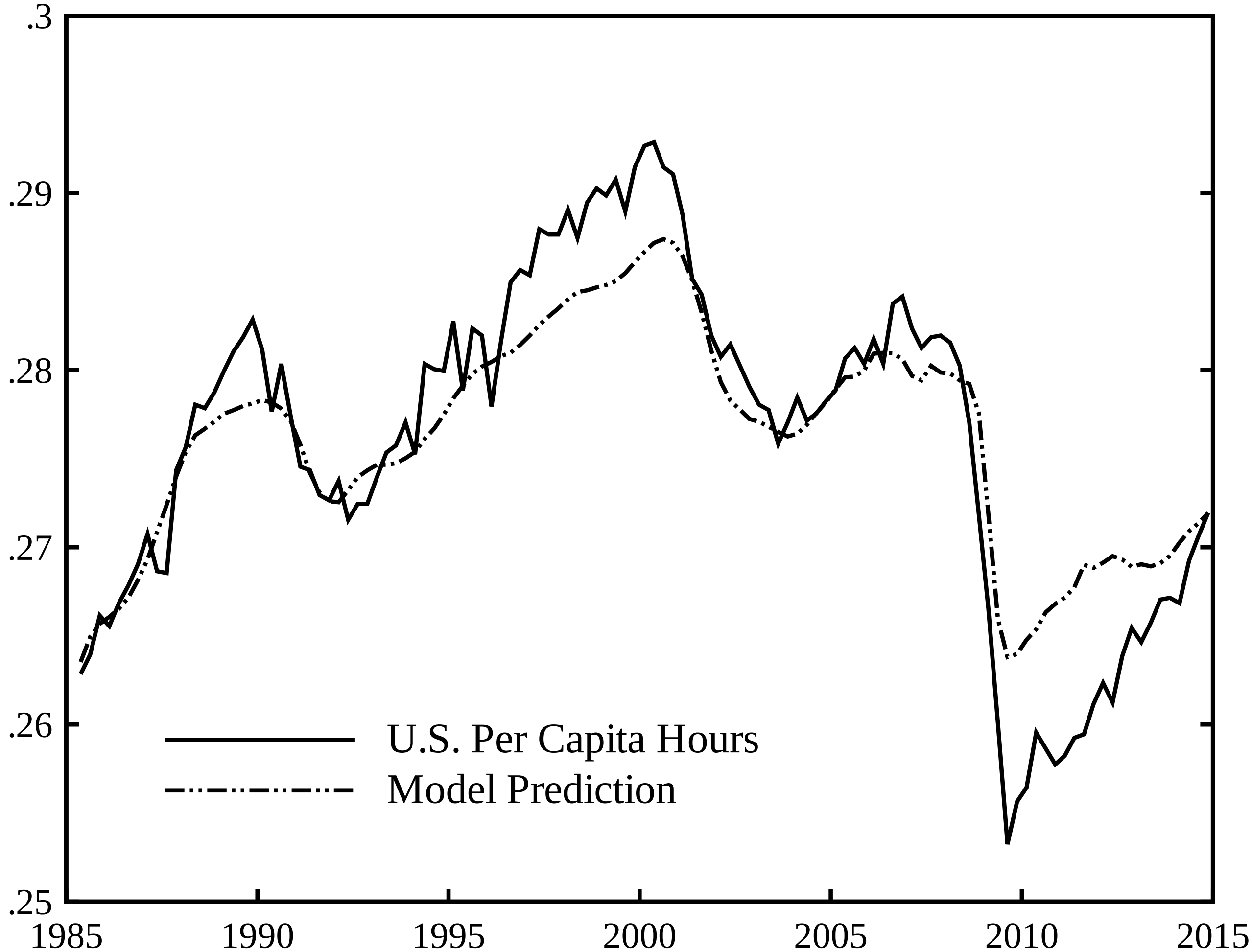
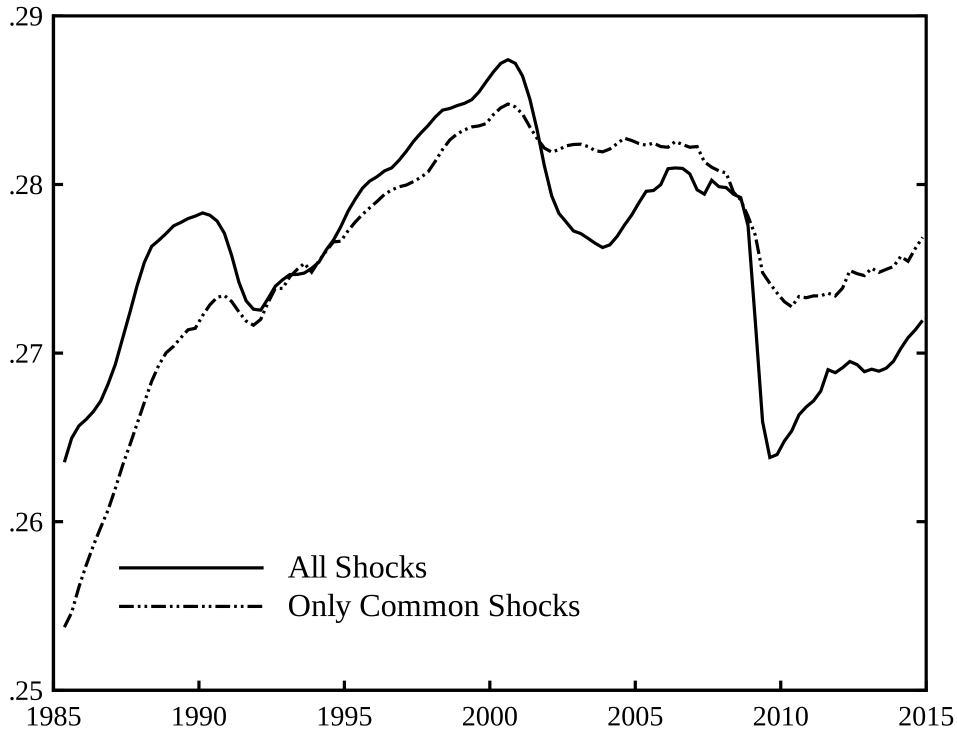


Figure 4

Predicted Hours Per Capita with Subset of Shocks

A. Only Common Shocks



B. Only Sectoral Shocks

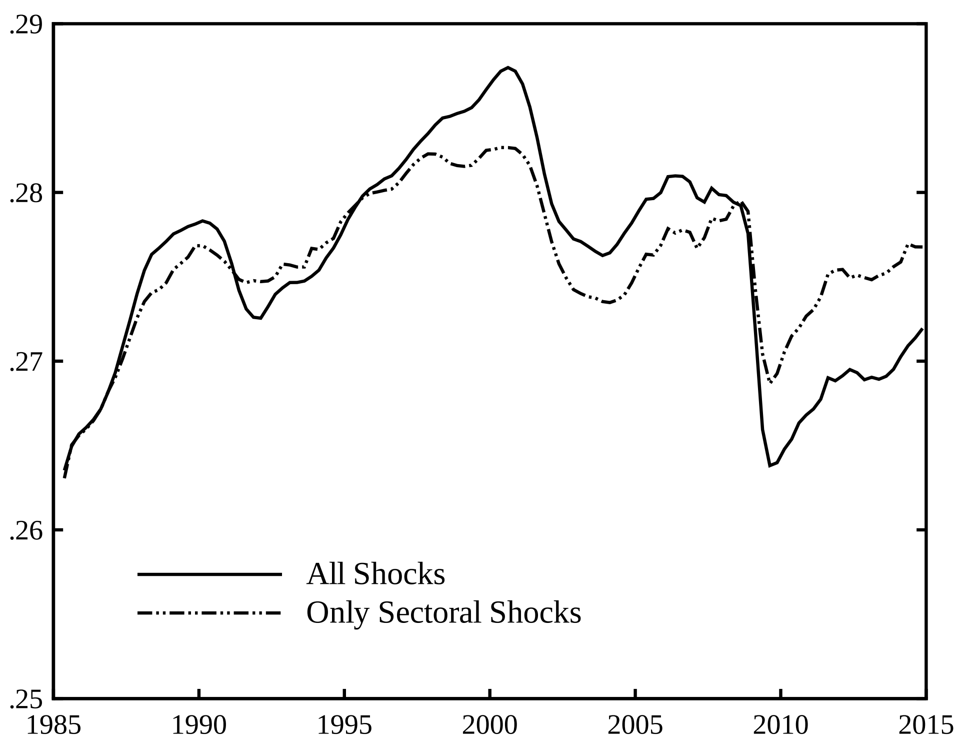


Figure 5
Decomposition of Changes in Gross Output in Two Periods

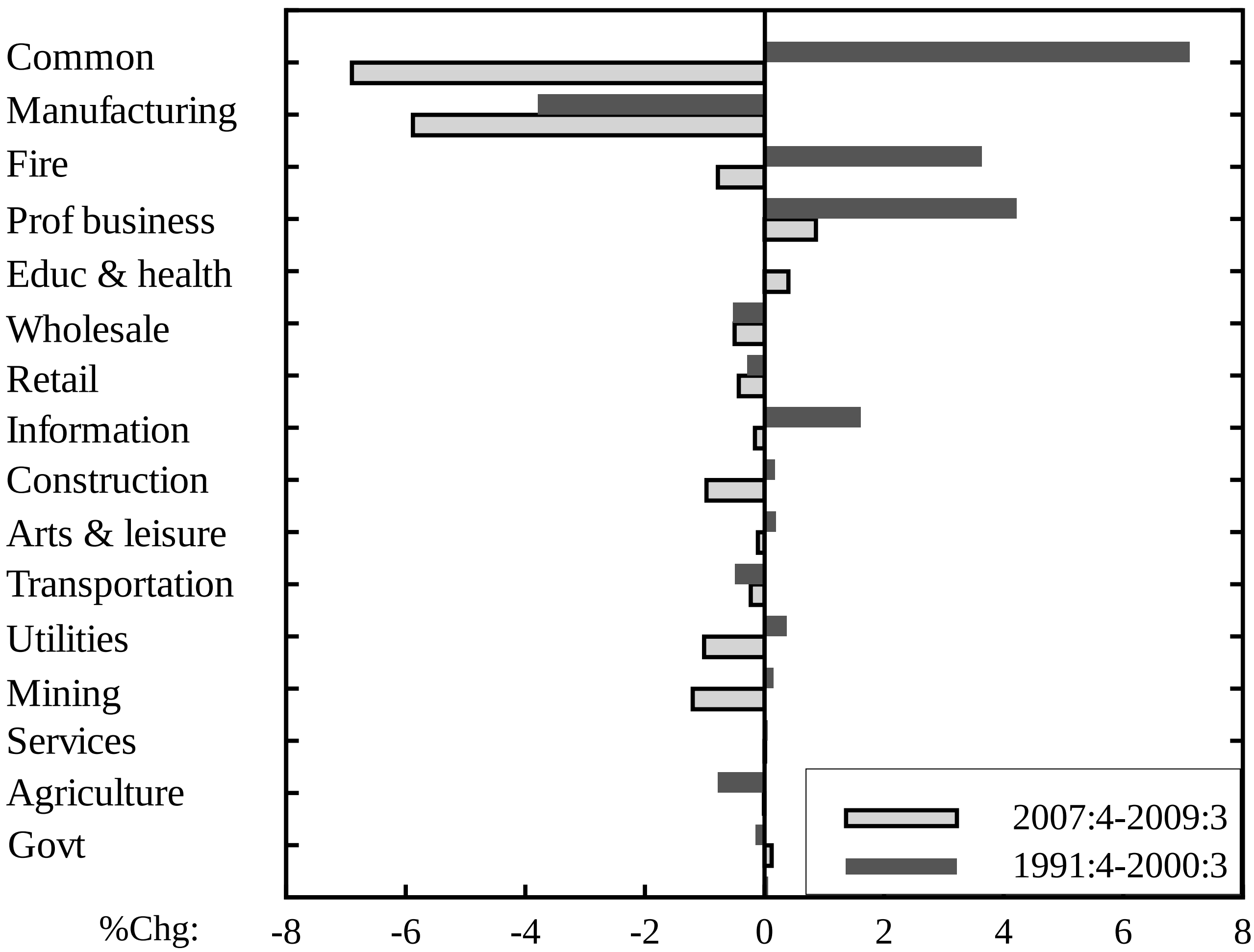


Figure 6

Two Measures of Aggregate TFP