I. BACKGROUND: SOME BASIC CONCEPTS FROM GAME THEORY

A. A GAME IS AN ABSTRACT REPRESENTATION OF STRATEGIC INTERACTION

Strategic interaction means that my *payoff* from what I do depends also on what you do.

- Before I decide what I should do, I have to think about what you will do
- And vice versa.

B. SOME EXAMPLES

Some games can be represented by a *payoff matrix* that shows how each player's payoff depends on each player's actions.

Consider matching pennies. Following table show's player R's payoff:

		Player C	
		Head	Tail
Player R	Head	+1	-1
	Tail	-1	+1

And consider the game of a submarine and a ship captain choosing whether to go North or South of an island.

- If they both choose the same side, the submarine wins (+1).
- If they choose different sides, the captain wins (-1 to the submarine)

		Captain	
		North	South
Submarine	North	+1	-1
	South	-1	+1

In abstract form, they're the same game. They are both

- Two-player games
- Zero-sum games (what one wins, the other loses)
- Games of complete information.
- There is no winning *pure* strategy.
- There is a winning *mixed* strategy, though "winning" does not mean that you can do better than break even:
- The winning strategy is to toss a coin to decide whether to choose head or tail, or whether to go north or south of the island.
- A change in the payoff will change the winning mixed strategy. E. g., suppose if both go North, the payoff to the submarine is 0.5 (because there is a 50% chance of missing due to rock formations under water).

Captain

		North	South
Submarine	North	+0.5	-1
	South	-1	+1

- Then if the submarine kept flipping coins, the captain should go north. But if the captain goes north, the submarine should go North too.
- We won't solve the problem of finding the best strategy for each here, though it is possible.

A new game to consider:

		Column L	Column M	Column R
Row	U	4, 3	5, 1	6, 2
Row	M	2, 1	8, 4	3, 6
Row	D	3, 0	9, 6	2, 8

- This shows R's payoff followed by C's payoff, in each cell.
- The game is no longer zero-sum. Combined payoffs range from 3 to 15.
- C's Middle strategy is dominated by Right: No matter what R does, C is better off with R than with M. So he should never choose M.
- If the payoff matrix is *common knowledge*, so R know's C's payoffs, R can reason that C will never choose M. So R should choose U.
- Then, following the same reasoning, C will know that R will choose U, so C will choose L.
- Each player is using the reasoning of *iterated dominance* to choose a strategy.

- But consider this game:

		C: L	C: R
R	U	8, 10	-100, 9
	D	7, 6	6, 5

- What is R's solution based on iterated dominance?
- What strategy would you play as R?

C. PRISONERS' DILEMMA

- Two suspected thieves are caught, put in separate cells, and offered the following deal:
  - Implicate the other thief, and you'll get a reward.

- Unless he implicates you too, you'll go free.
- Each thief knows that if both stay quiet, there is no evidence to convict, and they both go free.
- Here's the game (Rat means to implicate your partner, Don't rat means keep quiet):

		C: Don't rat	C: Rat
R	Don't rat	1, 1	-1, 2
	Rat	2,-1	0, 0

- Clearly no matter what Column does, Row is better off if he rats.
- Similarly no matter what Row does, Column is better off if he rats.
- The equilibrium is for both to implicate the other – rat, even though they would both be better off if they both kept quiet.

- The prisoners' dilemma game is an extreme form of a *cooperation game*. In a less extreme form, cooperation may turn out to be the equilibrium. Consider this game (C means cooperate, D means don't):

		Column: C	Column: D
Row	C	2, 2	.5, 1.5
	D	1.5, .5	0, 0

- I calculated those payoffs from the formulas:

  Row's payoff $= 1.5\,(x + y) - x$

  Column's payoff $= 1.5\,(x + y) - y$,

  Where x is Row's contribution to a common cause and y is Column's contribution, each limited to the value 0 if she doesn't cooperate or 1 if she does.

- If $0.5 < a < 1$, the game becomes a prisoners' dilemma.

D. PRISONERS' DILEMMA IN THE REAL WORLD

- (Thaler) How do you set up each of the following situations as a prisoners' dilemma?
  - Contributions to public TV
  - United Way contributions
  - Tipping the server in a restaurant you don't expect to visit again.
  - Tipping the room service maid in a hotel
- Thaler reports on a game exactly like that of the exercise, with *a* set so that it produces a prisoner's dilemma (p. 10).
  - What is the outcome, for a 1-shot game?
  - For a repeated game?

  - Do you understand the distinction between a game with a finite number of repetitions and one which has no known cut-off ("infinitely repeated games")

- Note: Prisoners' dilemma is also called the "free rider problem" or "the tragedy of the commons".