Background on game theory

- A *game* is the way we describe strategic interaction.
- Strategic interaction means that my *payoff* from what I do depends also on what you do.
  - Before I decide what I should do, I have to think about what you will do,
  - And vice versa.

Some examples

- Some games can be represented by a *payoff matrix* that shows how each player's payoff depends on each player's actions.
- Consider matching coins. R chooses a Row, C chooses a Column. Following table shows player R's payoff:

Player C:

Player R:		Head	Tail
	Head	+1	-1
	Tail	-1	+1

Some examples

- Consider the game of a submarine chasing a merchant ship, each choosing whether to go North or South of an island.
  - If they both choose the same side, the submarine sinks the merchant ship and wins (+1).
  - If they choose different sides, the merchant ship gets away and wins (-1 to the submarine)

Merchant ship

Submarine		North	South
	North	+1	-1
	South	-1	+1

Coin toss & Submarine v. merchant ship

- In abstract form, they're the same game – both:
  - Two-player games.
  - Zero-sum games (what one wins, the other loses).
  - Games of complete information.
  - Same payoff matrix.
- There is no winning *pure* strategy.
- There is a winning *mixed* strategy, though you can't do better than break even:
  - The winning strategy is to toss a coin to decide whether to choose head or tail, or whether to go north or south of the island.

Submarine v. merchant ship, continued

- A change in the payoff will change the winning mixed strategy. E. g., suppose if both go North, the payoff to the submarine is 0.5 (because there is a 50% chance of missing due to rock formations under water).

Captain

Submarine		North	South
	North	+0.5	-1
	South	-1	+1

Submarine v. merchant ship, continued

- Then if the submarine kept flipping coins, the captain should go north.
- But if the captain goes north, the submarine should go North too.
- We won't solve the problem of finding the best strategy for each here, though it is possible.

A new game to consider

Column

		L	M	R
	U	4, 3	5, 1	6, 2
Row	M	2, 1	8, 4	3, 6
	D	3, 0	9, 6	2, 8

- This shows R's payoff followed by C's payoff, in each cell.
- The game is no longer zero-sum. Combined payoffs range from 3 to 15.

A new game to consider

Column

		L	M	R
	U	4, 3	5, 1	6, 2
Row	M	2, 1	8, 4	3, 6
	D	3, 0	9, 6	2, 8

- C's Middle strategy is dominated by Right: No matter what R does, C is better off with R than with M. So he should never choose M.
- If the payoff matrix is *common knowledge*, so R know's C's payoffs, R can reason that C will never choose M. So R should choose U.

A new game to consider

Column

		L	M	R
	U	4, 3	5, 1	6, 2
Row	M	2, 1	8, 4	3, 6
	D	3, 0	9, 6	2, 8

- Then, following the same reasoning, C will know that R will choose U, so C will choose L.
- Each player is using the reasoning of *iterated dominance* to choose a strategy.

But consider this game

C

		L	R
R	U	8, 10	-100, 9
	D	7, 6	6, 5

- What is R's solution based on iterated dominance?
- What strategy would you play as R?

C. Prisoners' dilemma

- Two suspected thieves are caught, put in separate cells, and offered the following deal:
  – Implicate the other thief, and you'll get a reward.
  – Unless he implicates you too, you'll go free.
- Each thief knows that if both stay quiet, there is no evidence to convict, and they both go free.

Prisoners' dilemma

Here's the game (Rat means to implicate your partner, Don't rat means keep quiet):

C

		Don't rat	Rat
R	Don't rat	1, 1	-1, 2
	Rat	2,-1	0, 0

- Clearly no matter what Column does, Row is better off if he rats.
- Similarly no matter what Row does, Column is better off if he rats.

Prisoners' dilemma

		Don't rat	Rat
R	Don't rat	1, 1	-1, 2
	Rat	2,-1	0, 0

- The equilibrium is for both to implicate the other
  - rat, even though they would both be better off if they both kept quiet.

Cooperation game

- The prisoners' dilemma game is an extreme form of a *cooperation game*.
- In a less extreme form, cooperation may turn out to be the equilibrium.
- Consider game on next slide (C means cooperate, D means don't):

Cooperation game

		Column C	Column D
Row	C	2, 2	.5, 1.5
	D	1.5, .5	0, 0

- I calculated those payoffs from the formulas:
  - Row's payoff = a (x + y) – x
  - Column's payoff = a (x + y) – y,
- a = 1.5
- x is Row's contribution (0 or 1)
- y is Column's contribution (0 or 1).
- If 0.5 < a < 1, the game becomes a prisoners' dilemma.

The Tipping Game

- Suppose we use the cooperation game to think about tipping a server in a restaurant I'll never visit again.
- Substitute "server" for Column, "good service" for cooperate, and "poor service" for "don't cooperate".
- Substitute "Ed" for Row. What are the 2 strategies?

Contributing to public radio

- Suppose that instead of a two-player game we have a million-player cooperation game.
- We can use the same formula,
  my payoff = $a(x + y)$
  where x is my contribution and
  y is everyone else's contributions.
- The game matrix doesn't fully capture the issues, but it goes part-way.

Contributing to public radio

- Let C be the other 999,999 players and suppose that they all act in concert.
- This matrix just shows my payoff:

» Other players

		C	D
Ed	C	*1,000,000 a* -1	*a* - 1
	D	999,999 *a*	0, 0

Contributing to public radio

- We really should have 1 million separate columns
  - All 999,999 other players contribute in the 1st column,
  - 999,998 contribute in the 2nd column,
    down to
  - no one contributes in the last column.
- But that table shows the same conclusion:
  - I have no incentive to contribute.

Contributing to public radio

- Since people *do* contribute to public radio, the model of the simple game must be wrong.
- What should be changed?

The Tipping Game

		Server	
		Good svc. (y=1)	Poor svc. (y=0)
Ed	Tip (x=1)	0.5, 0.5	-1, 1.5
	Don't (x=0)	1.5, -1	0, 0

- Ed's payoff: 1.5y - x
- Server's payoff: 1.5x - y
- Is it a prisoner's dilemma? Should I tip?
- Is the game an accurate description of the real-life tipping game?

Prisoners' dilemma and free riders

- "Free rider" is the label we give to those who don't cooperate in a prisoner's dilemma game.
  - That language assumes that other people do cooperate.
- Guilt may make some people contribute, even when it does not seem rational to do so.
- What other motives could cause people to pay when it doesn't seem rational?

Prisoners' dilemma and free riders

- Lake owners' association
- MPR
- Community charity drive-United Way
  - vs. Universityparticipation in charity drive
    - vs. Economics department participation
- The larger and more anonymous the group, the harder it is to make people feel guilty enough to contribute.
- Does that work for other motives too?

Government as a solution

- For a prisoner's dilemma, enforced contribution by everyone can be in everyone's self-interest.
- For large groups, government is a device for overcoming the free-rider problem.
- Collective actions that are in "everyone's" interest are carried out by government, with forced contributions by everyone, through taxes.
- But usually, there is a minority of people who don't value the benefit to be provided.
  - That means that we must balance wishes of majority against those of minority.