Why Study Wage Income?

- Labor income is 2/3 of GDP.
- Most individuals outside of top 10% have very little capital income. Labor income is all they have.
- So it’s crucial to understand:
  - Why do wages grow over the life cycle?
  - Why does labor income varies so much across individuals?
  - Why has wage inequality increased so much since 1970s in the US?
  - What is the nature of income risk faced by workers?
The Human Capital Model

- Developed with seminar contributions by Becker, Schultz, Ben-Porath, Mincer, and Rosen.

Key Assumptions:

- Skills are general (No firm-specificity. Can you have industry- or occupation-specific skills? Will discuss this later.)

- No slavery (i.e, no commitment by worker).

- Labor markets are competitive.

- Lack of commitment can be solved by making workers pay for their own training (Becker).
Ben-Porath (1967) Model

- Individuals can borrow and lend at a constant interest rate (denoted by $r$), which implies that markets are complete. Let $\delta = 1/(1 + r)$.

- Individuals solve

$$\max_{\{i_t\}_{t=1}^T} \left[ \sum_{t=1}^T \delta^{t-1} R h_t (1 - i_t) \right]$$

s.t

$$h_{t+1} = h_t + A (h_t i_t)^{\alpha}, \quad \text{with} \quad h_0 > 0 \text{ given}$$

$$i_t \in [0, 1]$$

- This formulation does not rest on the assumption of risk-neutrality, but instead requires markets to be complete. [Homework: Prove this statement.]
Ben-Porath (cont’d)

- $\alpha$: measures the returns to scale in investment. When it is small (large), you want to spread (bunch) investment over time.

- Technology could be generalized: $Ah^\beta i^\alpha_s$:

  1. Estimates of $\alpha$ and $\beta$ are typically close.
  2. Present technology gives closed form solutions.
  3. Can also introduce pecuniary costs of investment.
Analyzing the Individual’s Problem

- Define:
  \[ C(Q_s) \equiv h_s i_s = \left( \frac{Q_s}{A} \right)^{1/\alpha}. \]

- With this transformation, the problem of an individual can be written as
  \[
  \max \left\{ Q_s \right\}_{s=1}^S \left\langle \sum_{s=1}^S \delta^{s-1} R(h_s - C(Q_s)) \right\rangle
  \]
  subject to
  \[ h_{s+1} = h_s + Q_s, \quad \text{with} \quad h_0 \text{ given} \]

- Now write it as a Bellman equation:
  \[
  V_s(h_s) = \max_{Q_s \in [0,h_s]} \left[ R(h_s - C(Q_s)) + \delta V_{s+1}(h_s + Q_s) \right]
  \]
Assuming interior solution, the FOC:

\[ RC'(Q_s) = \delta V'_{s+1} (h_s + Q_s) \]

- Envelope:
  \[ V'_s (h_s) = R + \delta V'_{s+1} (h_s + Q_s) \]

- Lead by one period:
  \[ V'_{s+1} (h_{s+1}) = R + \delta \left[ R + \delta V'_{s+2} (h_{s+2}) \right] \]

- ..repeatedly substitute for the RHS to get

\[
RC'(Q_s) = \delta \left\{ R + \delta R + \ldots + \delta^{S-s-1} R \right\} \\
C'(Q_s) = \frac{1}{r} \left[ 1 - \left( \frac{1}{1+r} \right)^{S-s} \right] \\
\to Q_s^* = A^{1-\alpha} \left[ \alpha MB (r, S - s) \right]^{\frac{\alpha}{1-\alpha}}
\]
Solve the model:
- RHS does not depend on $Q$.
- LHS is proportional to $Q_s^{\frac{1-\alpha}{\alpha}}$. If $\alpha > 0.5$, this is a concave function. Alternatively convex.
- How curvature depends on $\alpha$:

![Figure 2: Technology and gains from training.](image-url)
Notice that $Q_s^*$ does NOT depend on current human capital level. This is a consequence of the neutrality assumption.

Is this realistic? ($i_s$ still depends on $h_s$ of course).

It would imply for example that somebody who has high initial human capital will spend less time investing in human capital. seems a bit unrealistic.

It also implies a convergence in the wage distribution ($\text{var}(\log(w))$) if the only heterogeneity is in initial human capital.
# Key Observations

1. Ben-Porath gets several things right:

1. **Investment declines** over the lifecycle in a convex fashion. For example, interpreting $i = 1$ as college enrollment, we have college attendance early on.

2. Wages grow in a **concave** fashion.

3. Can generate a **hump shaped wage** profile by assuming positive depreciation.

4. Investment and wage growth are higher for high ability people.

5. Can generate a rise in wage inequality over the lifecycle without shocks if $A$ differs across people. More on this later.
1 Shortcoming: No well-defined concept of return to skill as mentioned before (eq. (3)).

2 Why do wages grow over the lifecycle?

\[ \begin{align*}
    w_s &= h_s^* - C(Q_s^*) \quad \text{and} \quad w_{s+1} = h_{s+1}^* - C(Q_{s+1}^*) \\
    w_{s+1} - w_s &= h_{s+1} - h_s + C(Q_s) - C(Q_{s+1}) \\
    &= Q_s^* + C(Q_s^*) - C(Q_{s+1}^*)
\end{align*} \]

So it is partly because (i) the stock of human capital grows, and also (ii) simply because individuals reduce investment.

Kuruscu (2006, AER): if \( \alpha \) is large the second effect is the main reason for the rise in wages. He estimates \( \alpha \) to be about 0.95.

If true, the benefit of training is rather low, less than 1%.

Some are skeptical. One reason is that an \( \alpha \) that high implies:

\[ Q_s = A^{\frac{1}{1-\alpha}} [\alpha MB(r, S - s)]^{\frac{\alpha}{1-\alpha}} \]

1 extreme sensitivity of investment to ability differences. (Does this make sense?)

2 extreme sensitivity to changes in MB (interest rates, returns, etc.)
Open Questions

1. If you can say something useful about the value of $\alpha$ you can grab many people’s attention.
   - Move beyond the average wage growth over the lifecycle and look at other moments to identify $\alpha$.

2. One difficulty of human capital model is that OJT is very difficult to measure.

Homework:

- Take the Ben-Porath model. Assume two periods.
- Introduce endogenous leisure: $u(c, l) = \log c + \log l$. Allow for savings between periods 1 and 2.
- Solve for the value function in period 2 and plug into period 1 problem. Can you solve for $(h_1, c_1, l_1)$ using the first order conditions? Why or why not?
- Modify the problem to make it well-behaved and easily solved through the FOCs. Play with preferences if necessary.
Discussion

- Missing from Ben-Porath model:

1. **No Returns to skill:** one cannot easily study what happens to human capital accumulation, etc, when the return to education, etc goes up. We will return to this point.

2. **No Jobs, Occupations, etc:** A bit too simple in its treatment of jobs, occupations.. none of them exist here. They do in real life. So how to combine the two?

3. How about start with specific human capital and the notion of occupation? Consider a model with two types of human capital where the wage of each is determined by a CES production technology. What can you say about the relative employment, wages, education in each occupation depending on movements in relative productivity.. what kinds of transitions would we observe? KEY: is to model what happens when you transit from one occupation to another?
Open Questions


   **Policy questions:** labor market frictions could have substantial welfare losses by discouraging experimentation and learning about one’s best skill.

2. What if learning ability is match-specific and you learn about it after the match. Like Jovanovic (1979) but with human capital accumulation.

   Question: These models are tools for studying economic questions. What other type of question can you fit into this framework?

   - Irreversible investment problems with physical capital.
Leisure in Ben-Porath Model

- Second period problem

\[ V_1(k_1, h_1) = \max_{c_1, n_1} [\log(c_1) + \log((1 - n_1))] \]
\[ c_1 = (1 + r)k_1 + h_1n_1 \]

FOC:
\[ \frac{h_1}{(1 + r)k_1 + h_1n_1} = \frac{1}{1 - n_1} \]

→ Solve \( 1 - n_1 \) and then \( c_1 = (1 - n_1)h_1 = \frac{h_1+(1+r)k_1}{2} \).

\[ V_1(k_1, h_1) = \log\left(\frac{h_1 + (1 + r)k_1}{2}\right) + \log\left(\frac{h_1 + (1 + r)k_1}{2h_1}\right) \]
\[ = 2\log(h_1 + (1 + r)k_1) - \log(h_1) - 2\log(2) \]
Is $V_1$ increasing and concave in $h_1$?

\[
\frac{\partial V_1}{\partial h_1} = \frac{h_1 - (1 + r)k_1}{h_1 (h_1 + (1 + r)k_1)} > 0 \quad \text{if} \quad h_1 > (1 + r)k_1
\]

\[
\frac{\partial^2 V_1}{\partial h_1^2} = \frac{2(1 + r)k_1h_1 - h_1^2 + (1 + r)^2k_1^2}{\#}
\]

You can show that if $k_1$ is large compared to $h_1$ (still satisfying $h_1 > (1 + r)k_1$) one can get a convex value function in $h_1$.

So in period 1, one cannot solve this using Euler equation is not sufficient.
Workaround: Modify preferences:

\[ U = \log(c_1) + \log((1 - n_1) h_1) \]
\[ = \log(c_1) + \log((1 - n_1)) + \log(h_1) \]

Now \( V_1 \) will be:

\[ \tilde{V}_1 = \underbrace{2 \log(h_1 + (1 + r) k_1) - \log(h_1) - 2 \log(2)}_{V_1} + \log(h_1) \]
\[ = 2 \log(h_1 + (1 + r) k_1) - 2 \log(2) \]
Next Lecture

- We will have somebody present the Rosen version of Ben-Porath.