Lecture 3: Jovanovic (1979, JPE)

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McCall (1970) Search Model

- Infinite horizon. Continuous time.
- Jobs drawn from exogenous fixed distribution $F(w)$, with $F(0) = 0$.
- No exogenous separation. Receive $b$ during unemployment.

$$rV = u(b) + \alpha \int \max\{W(w) - V, 0\} \, dF(w),$$

$$\begin{align*}
rW(w) &= u(w).
\end{align*}$$

Some straightforward manipulations yield:

$$\begin{align*}
u(w^*) - u(b) &= \frac{\alpha}{r} \int_{w^*} u(w) - u(w^*) \, dF(w) \\
&\text{marginal benefit of accepting offer} \\
&\text{marginal cost of accepting offer} \\
&\text{PDV of future offers}
\end{align*}$$
Rothschild’s critique:

- Rothschild called search models as "partial-partial equilibrium" models because the wage offer distribution is fixed.

- He argued that if one wanted to endogenize it (with identical firms), it would collapse to a single point because all firms would offer the same wage (same with price dispersion).

- This in turn would invalidate the derivation of the reservation wage policy. So he argued that the framework was internally inconsistent.

- Several models have been developed and shown to overcome this critique (e.g., Burdett and Judd; Burdett and Mortensen).

- Jovanovic generates wage dispersion by adding heterogeneity (in match quality).
Stylized facts that Jovanovic seeks to explain:

1. Wages rise with tenure.
2. Probability of quit falls with tenure.
3. Probability of quit falls with current wage.
Preliminaries

Bayesian Learning and Kalman Filtering:


2. Time Series Econometrics (James Hamilton).

3. Ljungqvist and Sargent, Appendix B has some examples.
Stage 1 (Pre-draw): Every period a worker and firm meet and draw a productivity, \( \theta \), from distribution \( \Pr \{ \theta \leq s \} = F(s) \). First, draw is initially unobservable.

Stage 2:
- Before worker agrees to produce, \( \theta + u \) is observed, where \( u \) is uncorrelated with \( \theta \).
- Firm offers to pay \( E[\theta | \theta + u] \) in period 1 and the expected value of \( \theta \) thereafter based on whatever information becomes available after period 1.
- Then worker makes a decision whether to stay or leave.

Stage 3: If worker produces in period 1, \( \theta \) is revealed to both in period 2. If worker quits he draws a new firm and a new \( \theta' + u' \) in the next period.
Specifics of the Information Structure:

- First, $\theta \sim N(\mu, \sigma_0^2)$, and $u \sim N(0, \sigma_u^2)$
- Worker and firm observe $y = \theta + u$ and use the Kalman filter to "learn" about $\theta$ given $y$.

$$m_0 = E(\theta | y) = E(\theta) + \frac{\text{cov}(\theta, y)}{\text{var}(y)} (y - E(y))$$

$$= \mu + \frac{\sigma_0^2}{\sigma_0^2 + \sigma_u^2} (y - \mu) \equiv \mu + K_0 (y - \mu)$$

$$\sigma_1^2 = E[(\theta - m_0)^2 | y] = \frac{\sigma_0^2}{\sigma_0^2 + \sigma_u^2} \sigma_u^2 = K_0 \sigma_u^2$$

- So the firm offers the worker a wage of $m_0$ in the first period and promises to pay $\theta$ (unknown at the time) in the future.
- Note that from stage 1 perspective, $m_0$ is random with mean $\mu$ and variance $K_0 \sigma_u^2$. So $m_0$ has the same mean and smaller variance than $\theta$. 
Characterizing the Solution (Backwards)

Stage 3: Let $J$ be the value function

- $\theta$ is revealed and worker must decide whether to stay at firm or quit.
  - If he stays, he will never quit in the future.
  - Call the value function of an unemployed worker as $Q$. Therefore, Bellman equation is: 
    $$J(\theta) = \max \{ \theta + \beta J(\theta), \beta Q \}$$  
    (Graph this as $J(\theta)$ versus $\theta$).
  - The solution is
    $$J(\theta) = \begin{cases} 
    \theta + \beta J(\theta) = \frac{\theta}{1-\beta} & \text{for } \theta \geq \bar{\theta} \\
    \beta Q & \text{for } \theta \leq \bar{\theta} 
    \end{cases}$$  
    (1)

- Reservation wage is $\frac{\bar{\theta}}{1-\beta} = \beta Q$, which we can find once we know $Q$. 

Stage 2

Stage 2: Worker has current wage offer $m_0 = E [\theta | \theta + u]$ and conditional probability function $Pr \{ \theta \leq s | \theta + u \} = F \left( s | m_0, \sigma_1^2 \right)$.

- Let $V (m_0)$ be the value function at this stage with Bellman equation:

$$V (m_0) = \max \left\{ \beta Q, m_0 + \beta \int J (\theta') dF (\theta' | m_0, \sigma_1^2) \right\}$$

- Note that the second term is increasing in $m_0$ whereas the first one is constant, so a reservation wage policy is optimal again:

$$V (m_0) = \begin{cases} m_0 + \beta \int J (\theta') dF (\theta' | m_0, \sigma_1^2) & \text{for } m_0 \geq \overline{m_0} \\ \beta Q & \text{for } m_0 \leq \overline{m_0} \end{cases}$$

(2)

- The reservation wage is implicitly defined as:

$$V (\overline{m_0}) = \overline{m_0} + \beta \int J (\theta') dF (\theta' | \overline{m_0}, \sigma_1^2) = \beta Q.$$

- Result 1: Can show that $\overline{m_0} < \overline{\theta}$. So the reservation wage rises over time as the option value of employment goes down.
Stage 1: Worker about to draw a match has value function:

\[ Q = \int V(m_0) \, dG(m_0|\mu, K_0\sigma_0^2) \] (3)

- Does a solution exist? Is it unique?

Combine (1)-(3) to write:

\[ V(m_0) = \max \left\{ m_0 + \beta \int \max \left[ \frac{\theta}{1-\beta}, V(m_1') \, dG(m_1'|\mu, K_0\sigma_0^2) \right] \, dF(\theta|m_0, \sigma_1^2), \right. \]
\[ \left. \beta \int V(m_1') \, dG(m_1'|\mu, K_0\sigma_0^2) \right\} \]

- Defines an operator mapping continuous \( V \) into continuous \( TV \). One can show that this operator satisfies Blackwell’s sufficiency conditions (so is a contraction)

- \( \therefore \) it has a unique fixed point. (HW: Show this.)
Three Observations

1. Because $m_0 < \bar{\theta}$ wages rise with tenure. If we extend the model to multi-period learning we can easily create a monotonically rising average wage with tenure.
   - This is consistent with stylized fact 1 above.
   - Ben-Porath was also consistent with this. How can you distinguish between the two theories?

2. Quits occur between periods 1 and 2 and never after.
   - So quits fall with tenure.

3. In stage 2, the probability of quit is:  
   \[
   \Pr \{ \theta' < \bar{\theta} | m_0 \} = F (\bar{\theta} | m_0, \sigma_1^2) .
   \]
   This is obviously falling with $m_0$.
   - So quits fall with current wage.