Romer’s Model of Expanding Varieties (Part 3)

3. R&D Sector

- A research firm hires researchers to invent new designs (or variety of intermediate good).
- The R&D firm gets a perpetual patent for the invention.
- Sells the patent at price $P_\alpha$ to a firm in sector 2.
- How is $P_\alpha$ determined?
- Using no-arbitrage condition—which is a powerful method, widely used by practitioners (by traders, etc.)
- **No-arbitrage**: Assuming that two assets have the same level of risk and investors can invest in both then both choices should yield the same return.
- The “law of one price” in exchange rate analysis is also an application of the no-arbitrage principle.
3. R&D Sector (cont’d)

So, how much is a patent worth ($P_A$)?
Suppose that a risk-free asset is also available with interest rate $r$.
An investor decides whether (1) to invest $P_A$ dollars in the risk-free asset or (2) to buy a patent, produce an intermediate good for one period, and then sell the patent after a year.
Equating the return from two options yields:

$$rP_A = \pi + \dot{P}_A$$

3. R&D Sector (cont’d)

Rearranging yields: 

$$r = \frac{\pi}{P_A} + \frac{\dot{P}_A}{P_A}$$

Begin by assuming that the solution to the model will display BGP. In this case, $r$ will be constant.
For $r$ to be constant we need both terms on the RHS to be constant. So $\pi$ and $P_A$ must grow at the same rate.
We found before:

$$\pi = \alpha(1-\alpha)\frac{Y}{A} \rightarrow \alpha(1-\alpha)\frac{\dot{Y}}{A}$$

Along BGP, $\frac{y}{A}$ is constant $\rightarrow \pi$ grows at rate $n$.
Therefore, $P_A$ grows at rate: 

$$\frac{\dot{A}}{A} = n$$

So we have:

$$r = \frac{\pi}{P_A} + n \rightarrow P_A = \frac{\pi}{r-n}$$
3. Closing the model

The only thing that is left to solve is the fraction of labor that works as researchers (sector 3) as opposed to workers (sector 1).

We can pin down this fraction again applying the no-arbitrage principle.

That is, an individual must be indifferent between the two options. This will pin down what fraction works in which sector.

We will not do this here (see the textbook if interested).

Efficient Level of R&D

Romer (1990) model is different from other models we have seen so far: the equilibrium allocation is not Pareto optimal.

In other words, a social planner who pools the resources of all individuals and make all decisions will attain better outcomes for everyone compared to the equilibrium.

Notice that this means Adam Smith’s invisible hand is not working perfectly in this economy.

In particular, this social planner would choose more resources to be devoted to R&D, which would imply faster growth.
Why R&D Level is not Optimal? $\dot{A} = \delta L^A A^\phi$

There are three distortions in this economy:

- **Knowledge spillover** (standing on giant’s shoulders $\phi > 0$): researchers contribution to future innovations not compensated. Under-provision of research.

- **Crowding out** (Stepping on others’ toes): Replication of research is not penalized. Overprovision of research.

- **Monopoly distortion** (consumer surplus effect): To the extent that monopolists cannot price-differentiate, they are only able to get a fraction of consumer surplus.
  - In other words, the social return to innovation is higher than the private return to monopolist. So monopolist does not invest sufficiently in R&D.