1. Consider the following technology for production in the widget industry. Specialized capital is the only input. The capital requirement to produce output $q$ is

$$c(q) = \phi + f(q), \quad q > 0$$

$$= 0, \quad q = 0.$$ 

Assume that $f$ is strictly increasing and strictly convex, and that $\lim_{q \to \infty} f'(q) = \infty$.

New capital can be obtained at a cost of $\gamma$ per unit. Once the capital is obtained it is specific to the industry. Capital depreciates at rate $\delta$. Thus if $x_t$ is the level of new investment in the industry and $k_t$ is the beginning of period capital stock, then

$$k_{t+1} = (1 - \delta)(k_t + x_t).$$

The demand curve in the industry is $D(p)$ and it is strictly downward sloping and $\lim_{p \to \infty} D(p) = 0$, $\lim_{p \to 0} D(p) = \infty$.

There are no other costs in this industry.

(a) Solve for the stationary competitive equilibrium in this industry. Determine firm size in stationary competitive equilibrium.

(b) Suppose the demand is initially in stationary competitive equilibrium as in (a). Then there is an unanticipated permanent shock to demand so that the new demand curve is $D_2(p) = \lambda D_1(p)$, where $D_1$ is the original demand and $\lambda < 1$. Determine the transition path to the new stationary equilibrium.

(c) In what respect is this a theory of mergers in demand downturns? What is missing from this theory? Any suggestions for filling the holes?
2. Recall the production technology from my merger paper with Gautam. The labor cost to invest at a rate of $i$ per unit of capital is $c(i)$ per unit of capital, where $c$ is strictly convex and increasing and $c'(0) = 0$. Suppose that in addition to this technology for producing the joint product of current output and future capital, there exists an alternative technology for producing this joint product that does not employ industry specific capital. With this second technology, $\gamma$ units of labor produces 1 unit of current output and $1 - \delta$ units of next period industry-specific capital.

Suppose that $\beta = 0$ so we can consider a static problem. The period begins with $k^{d_0}$ units of capital held by the dominant firm and $k^{f_0}$ held by the fringe sector. Let $k^o = k^{d_0} + k^{f_0}$ and let $m^o = k^{d_0}/k^o$ be the initial share held by the dominant firm.

There are two stages.

In stage 1, the dominant firm makes a take-it-or-leave it offer to fringe firms to either buy or sell capital. Assume that fringe firms, even though they behave competitively, are not anonymous at this stage. The offer is such that if any fringe turns down the offer, the status quo is maintained and there is no merger (Note this is quite different from the setup in my paper with Gautam. Here the dominant firm get more surplus than in the other paper.) Let $m$ the share of capital held by the dominant firm at the end of this stage.

In stage 2, the dominant firm and the fringe simultaneously choose $q^d$ and $q^f$ (through the original investment technology). In addition, there is the possibility of outside entry $e \geq 0$ by the second technology (at cost $\gamma$ per unit).

(a) Solve for the stationary equilibrium in this industry. Suppose $\gamma$ is low. Do mergers occur in the stationary equilibrium?

(b) Suppose the industry is initially in stationary equilibrium as in (a). Then there is an unanticipated permanent shock to demand so that the new demand curve is $D_2(p) = \lambda D_1(p)$,
where $D_1$ is the original demand and $\lambda < 1$. Determine the transition path to the new stationary equilibrium.

(c) In what respect is this a theory of mergers in demand downturns?

3. Compare the welfare consequence of banning mergers in these two cases. (You don’t need to drive expressions for welfare here, just make the obvious points).

4. (For this question, use a word processing program and include references).

Both models predict an increase in the merger rate in response to an unanticipated downturn. How is this related to Andrade and Stafford’s (1999) findings? Do the models share any other common predictions? Do the two theories have implications that are different? Discuss a potential research project that might make some headway at answering the following question: What is the relative quantitative importance of the two alternative theories in understanding why and when mergers occur?