Homework 1 (due Thursday, February 22)

Question 1

Recall the social planner’s problem discussed in Lecture 2. Farmers are uniformly distributed on the line with density \( m \) per mile. There is a fixed cost of setting up a store of \( f \). The marginal cost per widget is \( c \). Farmers have a transportation cost of \( t \) per mile to travel to a store.

In the model discussed in class, each farmer made one trip to the store and purchased one item. Suppose instead that the farmer has to take \( h \) different trips (and that the farmer purchases one widget each trip). For example, if the unit of time is a year and the farmer makes a trip to town once a week then \( h = 52 \). Solve the social planner’s problem of choosing the optimal distance \( D \) between stores and the store size \( S \) to minimize average total cost.

Question 2

Suppose in the monocentric model of the city there are \( H \) individuals with a utility function \( U(x, L) \) over material goods \( x \) and \( L \). The transportation cost per mile is \( t \) for all individuals. If an individual lives a distance \( u \) from the central business district (CBD), the individual’s net wage after transportation cost is \( w - tu \). Let \( p \) be the price of \( x \) and let \( R(u) \) be the rent on land that is \( u \) miles from the CBD. Let \( R \) be the price of farm land. Let \( L(u) \) be the equilibrium land demanded by individuals who live \( u \) miles from the CBD. The population density is \( D(u) = 1/L(u) \). Write down the equilibrium conditions.
Question 3

Below there is a list of changes in exogenous variables. I want you to determine how these changes lead to changes in the following endogenous variables: (1) the equilibrium boundary of the city \( \hat{u} \), (2) the equilibrium bid-rent curve \( R(u) \), and equilibrium population density \( D(u) \). I want you to work through the model to show how you get your results. Use figures to illustrate your results.

(a) An increase in the wage \( w \) (holding transportation cost fixed)
(b) An increase in the price of farm land \( \bar{R} \).
(c) An increase in the transportation cost \( t \).
(d) An increase in the number of individuals \( H \).

Question 4.

Let’s modify the monocentric model a little. Suppose that the location space is the line segment \( u \in [0, 1] \). There is a CBD at each endpoint. There are \( H \) individuals who work at each CBD. The price of farmland is \( \bar{R} \) as before. If \( H \) is low and if \( t \) is high, then the analysis will be the same as before. People will live close to the central business districts at both ends and in the middle of the interval there will be farmland. However, if \( H \) is high enough and \( t \) low enough, there will be no farmland. The urban populations of the two cities will intersect at the halfway mark between the cities. What are the conditions for equilibrium in this modification of the monocentric model? Redo the comparative statics analysis above for this alternative model.