Homework 1
Econ 8004 (Spring 2001)
Holmes

1. Suppose Holmland has a single consumer with quasilinear preferences over widgets $x$ and dollars $y$, $U(x, y) = u(x) + y$, where $u' > 0$ and $u'' > 0$. The consumer is initially endowed with $\overline{y}$ dollars. Suppose the resource cost to produce $x$ widgets in this economy is $c(x)$ dollars, where $c(0) = 0$, $c' > 0$, and $c'' > 0$. Assume further that $u'(0) > c'(0)$ and that $u'(x) < c'(x)$ for $x$ defined by $\overline{y} \equiv c(x)$.

(a) Suppose that a tax of $t$ dollars per unit widget is imposed on the consumer. Suppose the proceeds from the tax are put in a government lockbox. Suppose the consumer and the firm producing widgets behaves competitively. Write down the conditions determining the competitive equilibrium of the economy.

(b) Let $CS(t)$ and $PS(t)$ be consumer and producer surplus as a function of the tax rate $t$. Let $GS(t)$ be government surplus which is total tax revenue in the lockbox. Total surplus is

$$TS(t) = CS(t) + PS(t) + GS(t).$$

Set up the problem of maximizing total surplus. What tax rate $t$ maximizes total surplus? What is the first-order effect of a small tax rate (i.e. what is $TS'(0)$)?

2. Suppose in Holmland above,

$$u(x) = \frac{(10 - x)^2}{2},$$
$$c(x) = \frac{x^2}{2}.$$

(a) What is the competitive equilibrium widget quantity if there are no taxes?

(b) Now suppose that Holmland is opened up to trade with world markets. Suppose the world widget price is $p_w = 2$. Suppose that Holmland is small relative to the
rest of the world so its trade in widgets has no effect on the world price. What is the equilibrium widget production and consumption in Holmland?

(c) Suppose there is a tariff $t = 1$ is imposed on imports of widgets into Holmland. Let $CS_H, PS_H, GS_H$ be the consumer, producer and government surplus accruing to the Holmland parties. Calculate the effect of the tariff relative to no tariff on each surplus component as well as the effect on total Holmland domestic surplus, defined by

$$DS_H = CS_H + PS_H + GS_H.$$ 

Illustrate this on a graph.

(d) Suppose instead of a tariff, there is an import quota of 4 units. Calculate the effect of the import quota on each component of domestic surplus and the total domestic surplus compared to the case of no government intervention in part (b). Compare the effects of a tariff and of a quota.

3. Go back to the case in part 2(a) above where Holmland is closed to trade. Suppose that a monopolist controls production of widgets. Suppose the monopolist is constrained to set proportional prices, i.e., at a price $p$, the consumer picks a quantity level $x \geq 0$ and pays $px$. Calculate the monopoly price and quantity. Compare the difference in consumer and producer surplus with the competitive case and illustrate the changes on a graph.
4. A monopolist has a cost function with constant average cost, \( C(y) = cy \). If faces a demand function with constant elasticity of demand \( \varepsilon \), \( D(p) = p^{-\varepsilon} \), for \( \varepsilon > 1 \).

(a) Suppose that initial the monopolist pays a tax of \( t \) on each unit of output. Let \( p_D \) be the price the consumer pays and let \( p_S \) revenue net of the tax of the monopolist per unit sold, \( p_S = p_D - t \). Solve for the profit maximizing level of \( p_D \) and \( p_S \).

(b) Now consider a change to an ad valorem tax \( \tau \). This is a percentage tax on the sale price paid by the consumer. Thus \( p_D = (1 + \tau)p_S \). What does the ad valorem tax \( \tau \) have to be so that the consumer price \( p_D \) is the same as in part (a)? How does total tax revenue compare in the two cases. What is the intuition for why one is larger than the other?