1. Consider the informed/uninformed duopoly model discussed in class. The reservation price is $r$ for all consumers. Each firm has a monopoly on $U$ uninformed consumers. There are $I$ informed consumers who find the products of the two firms to be perfect substitutes. The firms compete in a Bertrand fashion. Define $\bar{p} = Ur(I + U)$.

Suppose we there exists a symmetric mixed strategy equilibrium to this game in which each firm randomizes its price choice over a support $[p', p'']$ with c.d.f $F(p)$. In class we showed that:

(i) $F(p)$ is continuous on the support. (This is the same thing as saying the distribution has no mass points).

(ii) $F(p)$ is strictly increasing. (This is the same thing as saying there are no wholes in the distribution or that the density $f(p) = F'(p)$ is positive over the support).

(iii) $p'' = r$.

We ran out of time before proving

(iv) $p' = \bar{p}$.

Show that property (iv) holds. Note that (i)-(iv) show that the mixed strategy equilibrium constructed in class is the unique symmetric mixed strategy equilibrium.

2. The problem introduces a variant of the classic Hotelling spatial model. There are two firms located at the end points of the unit interval $[0,1]$. Marginal cost is zero for both firms. Firm 1 is at point 0 and firm 2 is at point 1. There are two kinds of consumers, *mobile* and *immobile*.

The mobile consumers are uniformly distributed on the interval. Let $x \in [0,1]$ denote the location of a particular consumer. This consumer has a transportation cost of $xt$ to travel to firm 1 and a transportation cost $(1 - x)t$ to travel to firm 2. Thus the transportation cost is $t$ per unit distance. Assume the mobile consumers have an infinite reservation price for a single unit of the industry product. Thus a mobile consumer will buy a single unit from either firm 1 or firm 2, and will chose the firm with the lowest price.

There are $U$ immobile consumers who cannot purchase from firm 2, but have a reservation price of $r$ for a single unit of firm 1’s product. Analogously, firm 2 has a monopoly on $U$ immobile consumers who have a reservation price of $r$ for its product.

(a) Note that if $t = 0$, the only difference between this model and the one from class is that here the mobile consumers have an infinite reservation price for the industry product. (Here $I = 1$.) Show that the mixed strategy equilibrium from class remains an equilibrium here.

(b) Assume that $t > 0$ and that $U = 0$, to reduce the model to the classic Hotelling setup. Solve for the best response function $R_2(p_1)$, the profit-maximizing price of firm 2 given a choice $p_1$ of firm 1. Graph the response function. Solve for the unique symmetric equilibrium. Is this the unique equilibrium?

(c) Now allow for $U > 0$. Solve for the reaction function $R_2(p_1)$. Is this continuous? Under what condition on $t$ and $U$ will there exist a symmetric pure strategy equilibrium?

3. Suppose that $U = 0$ in the model from question 2. Consider a Stackelberg variant of this game.

(a) Suppose that firm 1 sets its price first, then firm 2 sets its price after seeing firm 1’s price. Solve for the equilibrium sequence of prices. Would firm 1 prefer playing Stackelberg game or the simultaneous move game from question 2? Who does better in the Stackelberg game, the first mover or the second mover?
(b) Now consider a Cournot duopoly model with linear industry demand \( P = 10 - Q \) and marginal cost equal to 0. Consider the Stackelberg model with firm 1 moves first and picks its quantity \( q_1 \) and then firm 2 chooses its quantity. How does the Stackelberg equilibrium output sequence compare with the Cournot equilibrium where the firms set quantity simultaneously. Who does better, the first mover or the second mover? How does the profit of the firms compare with the levels in the simultaneous move game.