1. Recall the classic Hotelling spatial model from the previous homework. There are two firms located at the end points of the unit interval [0,1]. Marginal cost is zero for both firms. Firm 1 is at point 0 and firm 2 is at point 1. Consumers are uniformly distributed on the interval. Let \( x \in [0,1] \) denote the location of a particular consumer. A consumer at \( x \) has a transportation cost of \( xt \) to travel to firm 1 and a transportation cost \( (1-x)t \) to travel to firm 2. Thus the transportation cost is \( t \) per unit distance. Assume each consumer has an infinite reservation price for a single unit of the industry product. Thus a consumer at \( x \) buys a single unit for either firm 1 or firm 2 depending on whether the total cost of buying from 1 \( p_1 + xt \) is less than or greater than the total cost of buying from 2, \( p_2 + (1-x)t \).

Consider the following strategic trade model. There is a governent at each endpoint along with the firm. Government 1 maximizes location 1 domestic surplus. This equals firm 1 profit plus government 1 revenue. Analogously, government 2 maximizes location 2 domestic surplus. There is a two stage game like the Brander and Spencer model considered in class. In the first stage, the two governments simultaneously select the subsidies, \((s_1, s_2)\). In the second stage, the two firms simultaneously choose prices \((p_1, p_2)\). Solve for the subgame perfect Nash equilibrium subsidies and prices.

Suppose instead it was a single-stage game where the governments and firms moved at the same time, simultaneously selecting, respectively, subsidies and quantities. What is the Nash equilibrium of this game?

2. Consider the 2-period Kreps and Wilson chain store game discussed in class. Recall the payoff in the stage game as as follows (the first payoff is the entrant’s, the second is the incumbent’s): Entrant out: (0,4). Entrant in and fight: (-3,-1). Entrant in and accommodate: (2,1).

Now suppose there are three periods, \( t \in \{0,1,2\} \). Let \( \alpha_0 \) be the initial beliefs that incumbent \( I \) is weak at period 0. Determine the sequential equilibrium of this model for any \( \alpha_0 \in [0,1] \).