Instructions

Choose 3 out of the 5 questions.

This is a take-home file. You have 24 hours to complete this exam after you begin working on it. Please take the exam sometime this week or over the weekend. Pick a time period that is convenient for you. When you are finished, put the completed exam in my mailbox, or email it to me as a pdf file. Note you are on your own to monitor the 24 time period. And I do not expect that the end of the 24 hour period would coincide with a time that it is feasible to turn in the exam. (So if you start at noon on Saturday, then be finished at noon on Sunday, and turn it in on Monday.)

Since these questions are supposed to be something like prelim questions (and questions 1 and 2 are directly from prelims), I include below the standard instructions one finds on a prelim:

Analytical solutions can be derived for some of the problems this examination contains. If the algebra involved proves too cumbersome, however, essentially full credit will be given for careful explanations of the steps that could be followed to derive analytical solutions. If you feel that additional assumptions are required before a unique solution to the problem can be found, specify what they are and why you need them.

Please indicate clearly which questions and which part of the question you are answering. Note also that irrelevant material, even if correct, will receive no credit.
Question 1

Consider the following duopoly model. There are two firms, 1 and 2, and time is discrete, $t \in \{0, 1, 2, \ldots\}$. Let $q_{1,t}$ and $q_{2,t}$ be the output of each firm in period $t$ and let $Q_t = q_{1,t} + q_{2,t}$ be total output. Suppose inverse demand is constant over time and linear, $P = A - Q$.

In each period $t$, the cost per unit of output is a constant $c_t$ that is the same for both firms. The constant average cost $c_t$ can take one of two values, high or low, $c_t \in \{c_L, c_H\}$, where $0 < c_L < c_H < A$. The cost realization in a period is determined by the following stochastic process. The probability of the low cost outcome $c_t = c_L$ depends on the output levels of the two firms in the previous period, i.e. the probability can be written as a function $f(q_{1,t-1}, q_{2,t-1})$ where $0 < f < 1$. Suppose the probability takes the following functional form

$$f(q_{1,t-1}, q_{2,t-1}) = 1 - [1 - g(q_{1,t-1})][1 - g(q_{2,t-1})],$$

where $g(q)$ satisfies $0 < g(q) < 1$, $g'(q) > 0$, $g''(q) < 0$.

Suppose the discount factor is $\beta < 1$.

Assume in each period $t$ the two firms simultaneously choose output levels $q_{1,t}$ and $q_{2,t}$ in a Cournot fashion, after observing the cost realization $c_t$ for the period.

(a) Provide an economic interpretation for specification of the link between $(q_{1,t-1}, q_{2,t-1})$ and $c_t$.

(b) Define a Markov-perfect equilibrium in this model.

(c) Suppose $\beta = 0$. Calculate the Markov-perfect equilibrium.

(d) Consider a two-period version of the model, $t \in \{1, 2\}$ with $\beta > 0$ and let $c_1 = c_H$ be the initial state. Show by contradiction that equilibrium outputs in period 1 when $\beta > 0$ are necessarily different from the equilibrium outputs when $\beta = 0$. 


Question 2. Consider the following model of differentiated products industry. Suppose differentiated goods are indexed by \( x \) and let \( q(x) \) be a quantity of good \( x \). There is a composite industry good that is CES in the differentiated products,

\[
Q = \left[ \int_0^\infty q(x) \frac{1}{2} dx \right]^2,
\]
i.e., the elasticity of substitution is 2. Suppose consumers have an inelastic demand for \( \bar{Q} \) units of the composite good.

There is a fixed cost \( \phi > 0 \) dollars of entering the industry to produce a particular differentiated product. The marginal cost is one dollar.

(a) Define a monopolistic competition equilibrium for this industry. Solve for the equilibrium.

(b) Suppose the government sets a price floor \( \lambda > 2 \), such that the price of any differentiated product \( x \) in this industry is required to satisfy \( p(x) \geq \lambda \). Determine the equilibrium under this policy. How does this policy affect the welfare of consumers and firms?

(c) In the previous two sections we assume that the fixed cost \( \phi \) is constant for all products. Now assume that the fixed cost \( \phi(x) \) depends upon \( x \) and that \( \phi(x) \) is strictly increasing in \( x \). Redo parts (a) and (b) under this alternative assumption.
Question 3

Labor Unions and the Incentive to Organize (A Positive Feedback Model)

Suppose there are a continuum of firms in an industry. The measure of firms is $N$. Let $i$ index firms, $i \in [0, N]$.

Firms have an identical concave production function $q = f(x)$ where $x$ is labor input, and $f' > 0$, $f'' < 0$, and $\lim_{x \to 0} f'(x) = \infty$.

It is a partial equilibrium model of an industry where there is a perfectly elastic supply of labor to the industry at wage $w$.

Let $q(i)$ denote the output of firm $i$ and let

$$Q = \int_{0}^{N} q(i) di$$

by total output of the industry. Suppose there is a demand curve $Q^D = D(p)$ for the industry that is strictly downward sloping. Assume $\lim_{p \to 0} D(p) = \infty$.

(a) Let $\tilde{x}(p, w)$ be the profit-maximizing demand for labor for a representative firm facing output price $p$ and wage $w$. Show how this is determined and show how the competitive equilibrium in this industry is determined.

Now embed the possibility of unions in this environment by considering the following three stage game. It is a model of a unionization decision at the firm level. In stage 1, at each firm workers decide whether or not to form a local union at the firm. When making this decision, workers take as given the decisions of all the other firms. If a union is formed, an organizing cost $\lambda > 0$ is paid by the workers doing the organizing. In stage 2, if a local union was formed at the firm, the local sets a wage $w$ that will apply at this firm. In stage 3, the firm hires an amount of labor $x$ to maximize profits, given the competitive price $p$ it faces in output markets and given the wage $w$. Thus at this stage the firm hires $\tilde{x}(p, w)$ units of labor. If there is no union at this stage the that wage at the firm will be $w$. 

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Assume that in stage 2, the local union takes the competitive $p$ as given and sets $w$ to maximize its total rent $(w - w) \tilde{x}(p, w)$, i.e. its payments in excess of the competitive wage. Note this competitive price will depend upon the fraction of firms in the industry that are organized. In stage 1, when the workers, decide whether or not to organize a particular firm, they take as given the measure $u \in [0, N]$ of the plants in the industry that will be organized. Assume the workers a plant in the organizing decision compare the gain of the rents $(w - w) \tilde{x}(p, w)$ with the cost of organizing $\lambda$.

(b) Define an equilibrium in this model.

(c) Show that if $\lambda$ is sufficiently high then the unique equilibrium is $u = 0$ and we obtain the same outcome as in part (a). Show that if $\lambda = 0$, the unique equilibrium is for $u = N$.

(d) Show there there exists a range of $\lambda$ in which there are three equilibria, one with $u = 0$, another with $u = N$ and an intermediate one where $u \in (0, N)$. 
Question 4

Consider a Cournot version of the model in Syverson. Suppose the demand function of a single consumer is given by \( q = a - p \). Demand a location with population \( S \) is then \( Q = S(a - p) \).

Firms play a game with three stages. In stage 1, \( N \) firms simultaneously enter and each firm pays an entry cost of \( \phi \). In this stage each entrant \( i \) draws a marginal cost \( c_i \in \{c_L, c_H\} \), where \( 0 \leq c_L < c_H < a \). The probability of drawing \( c_i = c_L \) is \( \omega \) and it is i.i.d. across firms. In stage 2 all the entrants observe the vector of cost draws \( (c_1, c_2, c_3, ... c_N) \) of all the entrants. The firms simultaneously decide whether or not to remain in the industry or to exit. Any firm that remains pays a cost \( \gamma \) at this point. By exiting, the cost \( \gamma \) is avoided. In stage 3, the remaining entrants observe which firms are still in the industry. The remaining firms play a Cournot game.

(a) Restrict attention to equilibria where in the exit stage (stage 2), if there is any exit by a firm with low cost \( c_L \), then any entrants with \( c_H \) also exit. Define a subgame perfect equilibrium of this model subject to this restriction.

(b) Observe that there exists a \( \hat{S} \) such that if \( S < \hat{S} \), there is no entry at stage 1, but if \( S > \hat{S} \), there is entry. Calculate \( \hat{S} \).

(c) Fix a market size \( S \). Let \( C_3(S) \) be the expected value of the mean cost in period 3, given equilibrium entry and exit behavior, and conditioned upon at least one firm surviving (so the mean can be calculated). Consider \( S \) in the range just above \( \hat{S} \), so there is one entrant. Discuss how \( C_3(S) \) is determined for \( S \) just above \( \hat{S} \).

(d) Syverson presented simulation results for his model in which the equivalent of \( C_3(S) \) strictly decreased in \( S \). Is that necessarily true here?
Question 5.

In the solution to question 2 of homework set 2 there is a mistake. In the solution, when analyzing the investment choice of the type A agent, I did not take into account the kink in the bidding function at zero. Fix this mistake. Determine the cutoff $\hat{\mu}$ such that for $\mu < \hat{\mu}$, there is specialization and the efficient investment is obtained, while if $\hat{\mu} > 0$ both things do not happen.

In your analysis, assume that $f(0) = 0$. Note I expect that there are two cases here. In case 1,

$$q_S - q_B \geq 0.$$ 

(so a factory even with zero investment is used). In case 2,

$$q_S - q_B < 0$$

(so a factory with no investment is never used).