Note: You may choose to work with one other person to complete this assignment. If you work as a two-person team, turn in one assignment for the both of you.

1. Consider a general equilibrium version of the Dixit-Stiglitz model mentioned in class. There are two final goods in the economy, an agricultural good and a manufacturing good. Let $A$ denote the quantity of the agricultural good and $Q$ the quantity of the manufactured good. The preferences of the representative consumer are Cobb-Douglas,

$$U(A, Q) = A^\alpha Q^{1-\alpha}.$$ 

The manufactured good is a composite if differentiated inputs each indexed by $x$. Let $q(x)$ denote an amount of differentiated good $x$. The production function for the composite is CES,

$$Q = F(q) = \left( \int_0^\infty q(x)^\frac{1}{\mu} dx \right)^\mu$$

with elasticity of substitution

$$\sigma \equiv \frac{\mu}{\mu - 1}.$$ 

Assume $\mu > 1$.

The consumer is endowed with $L$ units of labor. Agriculture is constant returns to scale, one unit of labor produces one unit of output. The production of a differentiated good requires fixed cost of $\phi$ units of labor and a marginal cost of one unit of labor.

(a) Define an equilibrium of monopolistic competition and solve for the equilibrium.

(b) Consider the problem of a social planner picking the number of differentiated products $n$ and the amount of labor resources allocated to each sector to maximize consumer utility, subject to the resource constraint. How does product variety here compare with the equilibrium variety in monopolistic competition.

Make the following changes in the model. Suppose a type A person can customize a factory for two different B people (but only one can use it), say B₁ and B₂. One of these two B people will turn out to be the efficient producer with productivity \( f(i) + qS \) from operating the factor. The other will be the inefficient producer with productivity \( f(i) + qS - \mu \) for \( \mu > 0 \). Suppose the probability is .5 that B₁ is the efficient producer and B₂ is the inefficient producer. The probability things are reversed is .5. Suppose that rather than Nash Bargaining, the two B people simultaneously submit sealed bids to the type A person for the factory.

(a) Suppose that contracts can be enforced so that the first best can be obtained. Under what assumptions about the technology will the special technology be used in equilibrium?

(b) Suppose ex ante contracting is not feasible so instead we are restricted to subgame perfect equilibria of the game discussed above. Define an equilibrium in this model. Show that if \( \mu \) is close enough to zero, the equilibrium always features specialization.
Question 3

Consider the following variation on the entry and exit model in Pakes, Ostrovsky, and Berry. Suppose there is no variation in the market state variable $z$ (for example the size of the market stays fixed over time). So henceforth ignore this variable. Let $\pi_n$ denote the current profit of an incumbent firm when there are $n$ firms in the industry. Assume $\pi_1 > 0$, $0 \leq \pi_2 < \pi_1$ and $\pi_n = -\infty$, for $n \geq 3$. Obviously there will never be three or more firms in this industry.

As in the paper, incumbent firm draw and exit value $\phi$ each period. Assume it is form the standard exponential distribution ($\lambda = 1$) so the density and c.d.f. is

$$f(\phi) = e^{-\phi}$$
$$F(\phi) = 1 - e^{-\phi}.$$

Suppose there is one possible entrant in each period. It draws an entry cost of $\kappa = 0$ with probability $\gamma$ and cost $\kappa = \infty$ with probability $1 - \gamma$.

(a) Define a Markov-perfect equilibrium in this model. Show how to solve for $VC_1$ and $VC_2$ (the values of continuing) and the cutoffs $\hat{\phi}_1$ and $\hat{\phi}_2$. (Hint: make use of equation (6) in Pakes, Ostrovsky, and Berry and derive four equations in the four unknowns).

(b) Suppose you have data on a market that was generated by the process above. Suppose you know $\delta = .5$. You don’t know $\pi_1$ or $\pi_2$ and you want to estimate these parameters. You have observations for 1000 periods of data. Let $\{(n_t, x_t, e_t), t = 1, 2, \ldots 1000\}$ denote the data where $n_t$ is the number of incumbents in period $t$, and $x_t$ and $e_t$ is the amount of exit and entry decided at time $t$ (to take effect in $t + 1$). Suppose the data is:

$$\#\{n_t = 0\} = 122$$
$$\#\{n_t = 1\} = 532$$
$$\#\{n_t = 2\} = 346$$
$$\#\{e_t = 1\} = 178$$
$$\#\{x_t = 0 \text{ and } n_t = 1\} = 499$$
$$\#\{x_t = 1 \text{ and } n_t = 2\} = 113$$
Use the above data for come up with estimates \( \hat{\gamma}, \hat{\phi}_1 \) and \( \hat{\phi}_2 \) of the entry probability and the exit rules. Then use the method of moments to estimate \( \pi_1 \) and \( \pi_2 \) (note there are two moments and two parameters so the model is exactly identified).

(c) **Extra Credit.** Obtain bootstrap standard errors of your estimates \( \hat{\pi}_1, \hat{\pi}_2, \) and \( \hat{\gamma} \) as follows.

Let \( \hat{F}_n = F(\hat{\phi}_n) \) be the probability of survival given \( n \) firms and \( \hat{\gamma} \) be the entry probability. Simulate 1000 periods of data with these probabilities (use \( n_1 = 1 \) as your initial condition). With this simulated data, re-estimate the parameters. Call these new estimates \( \hat{\pi}_{1,s} \) and \( \hat{\pi}_{2,s} \) and \( \hat{\gamma}_s \) for simulation \( s = 1 \). Repeat these using the original \( \hat{F}_1 \) and \( \hat{F}_2 \) and \( \hat{\gamma} \) for say 100 times. Your bootstrap standard errors are obtained from the variance covariance matrix of \( (\hat{\pi}_{1,s}, \hat{\pi}_{2,s}, \hat{\gamma}_s) \).

(d) **Optional exercise for your own benefit.** Observe that the parameters \( \pi_1, \pi_2 \) and \( \gamma \) could be jointly estimated through maximum likelihood. This would be a nested fixed point approach. Given \( \pi_1, \pi_2 \) and \( \gamma \), use dynamic programing to calculate \( \hat{\phi}_1 \) and \( \hat{\phi}_2 \) and then the likelihood. Pick the vector that maximizes the likelihood.