Incentives and the Theory of the Firm

• Tradeoff between incentives and insurance (Holmstrom)

• Multi-Tasking: Get What You Pay For (Holmstrom and Milgrom)

• Holdup Problem (Grossman and Hart, Williamson)
Classic Moral Hazard

Adam Smith “Division of Labor Limited by the Extent of the Market

- $a$: agent effort, $c(a)$ cost of effort
- $\varepsilon$: events beyond agent’s control
- $y = a + \varepsilon$ total output, publicly observed
- $w(y)$ compensation scheme
— In classic case cost of effort additively separable (cost in utils) so agent solves

\[ \max_a E[u(w(y)) - c(y)] \]

— Here consider effort cost in dollars,

\[ \max_a E[u(w(y)) - c(y)] \]
Suppose

- 1. Restrict attention to linear compensation, \( w(y) = s + by \)

2. Assume CARA, \( u(x) = -\exp(-rx) \)

3. Suppose \( \varepsilon \) is \( N(0, \sigma^2) \)

Agent’s problem

\[
\max_a -e^{-r(s+ba-c(a))} \int_{\varepsilon} e^{-rb\varepsilon} \phi(\varepsilon) d\varepsilon
\]

So \( a^*(b) \) solves \( c'(a) = b \).

- Agent’s certainty equivalent

\[
CE(s, b) = s + ba^*(b) - c(a^*(b)) - \frac{1}{2}rb^2\sigma^2
\]
• Principal expected profit

\[ E\Pi(s, b) = (1 - b)a^*(b) - s \]

• Total Surplus

\[ CE(s, b) + E\Pi(s, b) = a^*(b) - c(a^*(b)) - \frac{1}{2}rb^2\sigma^2 \]

• Optimal slope \( b^* \)

\[ b^* = \frac{1}{1 + r\sigma^2c''} \]
Linearity?

- In problem described above can do better with some step function contract

\[
\begin{align*}
    w_H, & \text{ if } y \geq y_0, \\
    w_L, & \text{ if } y < y_0
\end{align*}
\]

for some \( w_L < w_H \) and some \( y_0 \)

- In general optimal incentive contracts not even monotonic

- Holmstrom and Milgrom rescue linear contracts in reinterpretation.

- In more recent thinking goes beyond tradeoff between incentives and insurance...
You Get What You Pay For

• Suppose

\[ y = a + \varepsilon \]
\[ p = a + \phi \]
\[ w = s + bp \]
\[ a = a_1 + a_2 \]

• Ex 1. \( y = a_1 + a_2, \ p = a_1 \).

• Ex 2. \( y = a_1, \ p = a_1 + a_2 \)

• Ex 3. \( y = a_1, \ p = a_2 \)
Ex. 4

- \( y = f_1 a_1 + f_2 a_2 + \varepsilon \)

- \( p = g_1 a_1 + g_2 a_2 + \phi \)

- \( w = s + bp \)

- \( y - w \)

- Payoff to risk neutral agent \( w - c(a_1, a_2) \) where
  \[
c(a_1, a_2) = \frac{1}{2} a_1^2 + \frac{1}{2} a_2^2
  \]
• Timing

1—Contract \((w = s + bp)\) determined

2—Agent picks \(a_1\) and \(a_2\)

3—\(\epsilon\) and \(\phi\) occur

4—Agent paid

• Agent solves

\[
\max_{(a_1,a_2)} s + E [bg_1a_1 + bg_2a_2 + b\phi] - \frac{1}{2}a_1^2 - \frac{1}{2}a_2^2
\]

So \(a_1^*(b) = g_1b\) and \(a_2^*(b) = g_2(b)\)
- Principal expected payoff

\[ E(y - w) = f_1 a_1^*(b) + f_2 a_2^*(b) - s - b [g_1 a_1^*(b) + g_2 a_2^*(b)] \]

- Agent expected payoff

\[ E(w) - c(a_1, a_2) = s + b [g_1 a_1^*(b) + g_2 a_2^*(b)] - \frac{1}{2} a_1^*(b)^2 - \frac{1}{2} a_2^*(b)^2 \]

- Total Surplus

\[ E[y] - c(a_1, a_2) = f_1 a_1^*(b) + f_2 a_2^*(b) - \frac{1}{2} a_1^*(b)^2 - \frac{1}{2} a_2^*(b)^2 \]

- FONC

\[ f_1 a_1^{*'}(b) + f_2 a_2^{*'}(b) - a_1^*(b)a_1^{*'}(b) - a_2^*(b)a_2^{*'}(b) = 0 \]
\[ f_1 g_1 + f_2 g_2 - g_1^2 b - g_2^2 b = 0 \]
So

\[ b^* = \frac{f_1 g_1 + f_2 g_2}{g_1^2 + g_2^2} = \frac{\|f\|}{\|g\|} \cos(\theta) \]

• Scaling and alignment
Holmstrom and Milgrom

- Multi-tasking and multidimensional contracts

- Observe $p_1, p_2, ..., p_n$

- Payment is $s + b_1 p_2 + b_2 p_2 + ... + b_n p_n$

- Cases where go to corner.

— Extreme case 1: $b_1 = 0, b_2 = 0, ..., b_n = 0$ (employee)

— Extreme case 2: $b_1 = 1, b_2 = 1, ..., b_n = 1$ (separate firm)
The Holdup Problem

Property Rights and the Nature of the Firm

• Two agents, supplier and buyer and one asset.

• Supplier makes investment $x$ in the asset.

—$f(x)$ is benefit if continue with relationship, $f' > 0$ $f'' < 0$, $f(0) = 0$.

—$f_r(x)$ if walk away and can reuse it with another party $f_r(0) = 0$, $f'_r(x) < f'(x)$, $x > 0$.

—0 if buyer takes asset away.
• Buyer makes investment $y$

$-g(y)$ if continue with the relationship

$-g_r(y)$ is walk away and find it new supplier, $g_r(0) = 0$, $g_r'(y) < g'(y), y > 0$

$-0$ if supplier take asset away.

• Incomplete contracts

1—Sign contract (assign residual rights to control). Agree to lump sum transfer and who gets to walk away with the asset.

2—Supplier and buyer pick $x$ and $y$

3—Whoever is assigned residual rights of control can exercise this right. Nash Bargaining with parameter $\alpha$ on the supplier.
Case 1—assign supplier residual rights of control

- Stage 3: Outside option

  — Supplier has value \( f_r(x) \)

  — Buyer has value 0

  — Total value when agree is \( f(x) + g(y) \)

  — Distribution is

\[
\begin{align*}
v_S &= f_r(x) + \alpha [f(x) + g(y) - f_r(x)] \\
v_B &= 0 + (1 - \alpha) [f(x) + g(y) - f_r(x)]
\end{align*}
\]

- Stage 2:
—Supplier problem

\[ \max_x -x + f_r(x) + \alpha [f(x) + g(y) - f_r(x)] \]

\[ \text{FONC} : \quad \alpha f'(x) + (1 - \alpha) f'_r(x) - 1 = 0 \]

Let \( x^{**}_S \) solve above. Let \( x^* \) solve \( f'(x) = 1 \). Note \( x^{**}_S < x^* \) if \( \alpha < 1 \).

—Buyer problem

\[ \max_y -y + 0 + (1 - \alpha) [f(x) + g(y) - f_r(x)] \]

\[ \text{FONC} : \quad (1 - \alpha) g'(y) - 1 = 0 \]

Let \( y^{**}_S \) solve above. Let \( y^* \) solve \( g'(y) = 1 \), \( y^{**}_S < y^* \).
Case 2—assign buyer residual rights of control

- Stage 3: Outside option
  - Supplier has value 0
  - Buyer has value $g_r(y)$
  - Total value when agree is $f(x) + g(y)$
  - Distribution is
    \[ v_S = 0 + \alpha [f(x) + g(y) - f_r(x)] \]
    \[ v_B = g_r(y) + (1 - \alpha) [f(x) + g(y) - f_r(x)] \]

- Stage 2:
—Supplier problem

\[
\max_x -x + \alpha [f(x) + g(y) - f_r(x)]
\]

\[FONC : \ \alpha f'(x) - 1 = 0\]

Let \(x_B^{**}\) solve above. Note \(x_B^{**} < x_S^{**} < x^*\).

—Buyer problem

\[
\max_y -y + g_r(y) + (1 - \alpha) [f(x) + g(y) - f_r(x)]
\]

\[FONC : \ \alpha g_r'(y) + (1 - \alpha)g'(y) - 1 = 0\]

Let \(y_B^{**}\) solve above. \(y_S^{**} < y_B^{**} \leq y^*\) (\(y_B^{**} < y^*\) if \(\alpha > 0\))
Stage 1

- Supplier ownership. Total surplus is

$$TS_S = f(x_S^*) + g(y_S^*) - x_S^* - y_S^*$$
$$< f(x^*) + g(y^*) - x^* - y^*$$

- Buyer ownership

$$TS_B = f(x_B^*) + g(y_B^*) - x_B^* - y_B^*$$
$$< f(x^*) + g(y^*) - x^* - y^*$$

- Pick ownership structure to solve

$$\max \{TS_S, TS_B\}$$

- Divide ex ante surplus somehow.
Generalization to Multiple Assets

- Suppose $A_1, A_2 \ldots A_n$

- Can have general functions $f(x, y), g(x, y)$ where $x = (x_1, \ldots, x_n)$.

- Can specify walkaway returns for various partitions of the assets

- Have vertical integration if one party has all residual rights of control.

- Williamson vs. Hart and Moore
Williamson Hold-up Model

- Two kinds of individuals, type $A$ and type $B$.

$- N_i$ measure of type $i$, $N_A < .5N_B$.

- $t = \{0, 1\}$. $\beta = 1$.

- Each individual has a single labor unit in each period

- Technology 1: Regular

$- type \ j$ produces $q_j$ per unit of time, $q_A > q_B$
• Technology 2: Special

—period 0, type A builds a factory of quality $i$ with $i$ labor units

—period 0 factor has no output

—period 1: output is $f(i) + q_S$, where $f(0) = 0$, $f'(0) > 0$, and $f''(0) < 0$ when managed with one unit of time (of any type)

—factory must be customized in period 0. If another person manages it, output is $q_S$ instead of $f(i)$