Econ 8601—9/22/03

Vertical Integration

• Make or Buy Decision

—Significant issue, outsourcing, international division of labor....

—Fundamental for theory of the firm

• Big ideas

—Technology

—Externalties, Coase, Williamson, Grossman and Hart
Technology

Adam Smith “Division of Labor Limited by the Extent of the Market

• Single final good: \( f(L, M) = L^{1-\alpha}M^\alpha \)

\[
M = \left[ \int_0^1 x(t) \frac{1}{\mu} \right]^\mu
\]

\[
\sigma = \frac{\mu}{\mu - 1} \text{ elasticity of substitution}
\]

• How make \( x \)

—Internal technology \( \gamma \) units of labor per unit of output, \( \gamma > \mu \)
—Specialist technology. Marginal cost one labor unit, fixed cost $\theta$ labor units

—Why call one internal and specialist?

• $N$ individuals in economy each with one labor unit
Problem of Final Good Producer

- Suppose \([0, z]\) interval of goods with a specialist producer. Let \(p(t)\) be price of good \(t\) by a specialist producer

\[
\max_{L,x(\cdot)} L^{1-\alpha} \left[ \int_0^z x(t) \frac{1}{\mu} dt \right]^{\mu\alpha} - \int_0^z p(t)x(t) dt - \int_z^1 \gamma wx(t) dt \quad (1)
\]

We need ratio of ratio of value of the marginal product equal to ratio of prices

\[
\frac{\mu\alpha [\mu\alpha (1/\mu)]^{1/\mu} x^{1/\mu - 1}}{\mu\alpha [\mu\alpha (1/\mu)]^{1/\mu} x^{1/\mu - 1}} = \frac{p_1}{p_0}
\]
\[
\left( \frac{x_1}{x_0} \right)^{\frac{-1}{\sigma}} = \frac{p_1}{p_0} \\
\]
\[
x_1 = p_1^{-\sigma} (p_0^\sigma x_0)
\]
Specialist firm is measure zero so takes \( p_0 \) and \( x_0 \) as given. So constant elasticity demand. Set \( p_1 \) to maximize

\[
(p_1 - w) p_1^{-\sigma} k
\]

The FONC

\[
p_1^{-\sigma} k - \sigma (p_1 - w) p_1^{-\sigma-1} k = 0
\]

\[
p_1 = \sigma (p_1 - w)
\]

\[
\frac{p_1 - w}{p_1} = \frac{1}{\sigma}
\]

\[
\frac{p_1}{p_1} = \mu w
\]
There is free entry into intermediate goods production

a zero-profit condition

\[ \mu wx_M - wx_M - w\theta = 0, \]  

(2)

Problem of final goods producers implies:

\[ \frac{x_M^{1-\mu}}{x_I^{1-\mu}} = \frac{\mu w}{\gamma w} \]  

(3)

solving for \( x_I \) in terms of \( x_M \) yields

\[ x_I = \left( \frac{\mu}{\gamma} \right)^{\frac{\mu}{\mu-1}} \cdot x_M \]  

(4)
• Demand equal supply in the labor market.

• Given the Cobb-Douglas, \( L = (1 - \alpha)N \).

• Residual

\[
z (x_M + \theta) + (1 - z)x_I \gamma = \alpha N.
\]

(5)

• Solve for \( z^e \). Two critical levels of \( N \), defined by

\[
N' = \frac{\gamma \theta}{\alpha (\mu - 1)} \left( \frac{\mu}{\mu - 1} \right)
\]

(6)

and

\[
N'' = \frac{\gamma \theta}{\alpha (\mu - 1)} \left( \frac{\mu}{\gamma} \right).
\]

(7)
Assume that $N \leq N''$. The equilibrium number of market equilibrium goods is given by

$$
\begin{align*}
    z^e &= 0, \quad N \leq N' \\
    z^e &= \frac{N - N'}{N'' - N'}, \quad N' < N \leq N''
\end{align*}
$$

(8)