Consider a general equilibrium version of the Dixit-Stiglitz model mentioned in class. There are two final goods in the economy, an agricultural good and a manufacturing good. Let $A$ denote the quantity of the agricultural good and $Q$ the quantity of the manufactured good. The preferences of the representative consumer are Cobb-Douglas,

$$U(A, Q) = A^\alpha Q^{1-\alpha}.$$  

The manufactured good is a composite if differentiated inputs each indexed by $x$. Let $q(x)$ denote an amount of differentiated good $x$. The production function for the composite is CES,

$$Q = F(q) = \left( \int_0^\infty q(x)^{\frac{\mu}{\sigma}} \, dx \right)^{\frac{\sigma}{\mu}}$$

with elasticity of substitution

$$\sigma \equiv \frac{\mu}{\mu - 1}.$$  

Assume $\mu > 1$.

The consumer is endowed with $L$ units of labor. Agriculture is constant returns to scale, one unit of labor produces one unit of output. The production of a differentiated good requires fixed cost of $\phi$ units of labor and a marginal cost of one unit of labor.

(a) Solve for the monopolistic competition equilibrium.

(b) Consider the problem of a social planner picking the number of differentiated products $n$ and the amount of labor resources allocated to each sector to maximize consumer utility, subject to the resource constraint. How does product variety here compare with the equilibrium variety in monopolistic competition?

(c) Now add a spatial dimension in this economy. There are two locations 1 and 2. Suppose labor is perfectly mobile. Suppose that (i) it is costless to ship the agricultural good $A$, (ii) it is infeasible to ship the differentiated goods (iii) it is possible to ship the composite manufactured good but there is a cost. The cost for shipping the composite is that a fraction $\tau$ of any good shipped gets lost in transit (this is called an iceberg cost).
Assume that location 1 is better than 2 in that there is a disutility of $\beta > 0$ from locating there. In other words, an individual's utility from consuming $A$ and $Q$ at location 1 is $U_1 = A^\alpha Q^{1-\alpha}$ but the utility from the same bundle at 2 is $U_2 = A^\alpha Q^{1-\alpha} - \beta$. Under what circumstances does there exist an equilibrium where all individuals locate at 2?

**Question 2: Nested-Fixed Point Procedure**

Consider the simple version of the Ericson and Pakes entry model discussed in class. Let $\pi_n$ denote the current profit of an incumbent firm when there are $n$ firms in the industry. Assume $\pi_1 > 0$, $0 \leq \pi_2 < \pi_1$ and $\pi_n = -\infty$, for $n \geq 3$. As noted in class, there will never be three or more firms in this industry.

As in the paper, incumbent firms draw an exit value $\phi$ each period from the exponential density,

$$f(\phi) = \sigma e^{-\frac{\phi}{\sigma}}$$

$$F(\phi) = 1 - e^{-\frac{\phi}{\sigma}}.$$ 

There is one possible entrant in each period. It draws an entry cost of $\kappa = 0$ with probability $\gamma$ and cost $\kappa = \infty$ with probability $1 - \gamma$.

The discount factor is $\beta$.

The data set `entry_exit` (posted on the web site) contains 2000 observations of a simulated history of this industry. It is an ASCII file that contains two variables $(n_t, y_t)$, where row $t$ contains the variables for period $t$. The first variable $n_t$ (column one) is the number of firms in the industry in period $t$. The second variable (column two) $y_t \in \{1, 2, 3, 4, 5, 6\}$ indicates the entry and exit outcome that happens in period $t$ (which takes effect the next period). Let $e_t \in \{0, 1\}$ denote the number of entrants in period $t$ and $x_t \in \{0, 1, 2\}$ denote the number of exiting firms. Then $y_t$ is defined by
Suppose the discount factor and the exit-value distribution parameter are known to be $\beta = .5$ and $\sigma = 1$. The parameters $\pi_1$, $\pi_2$, and $\gamma$ are unknown.

(a) Write a program to solve for the equilibrium of the model for a given value of $\theta = (\pi_1, \pi_2, \gamma)$. Write a procedure for calculating the (log of) the likelihood of the data $\{(n_t, y_t), t = 1, \ldots, 2000\}$ for a given value of $\theta$. Then evaluate the likelihood at $\hat{\theta}$ defined by

$$
\hat{\theta} = \begin{pmatrix}
2.0676 \\
0.1481 \\
0.2876
\end{pmatrix}.
$$

(b) Show that $\hat{\theta}$ maximizes the likelihood. To make things simple, it is sufficient to evaluate the likelihood function at .1 percent above $\hat{\theta}$, and .1 percent below (changing each parameter separately so there are 6 evaluations) and show that the likelihood decreases in each direction. Or you could dig up a maximization procedure and use this to determine the maximum likelihood estimate.

\begin{tabular}{|c|c|c|}
\hline
$y_t$ & $e_t$ & $x_t$ \\
\hline
1 & 0 & 0 \\
2 & 0 & 1 \\
3 & 0 & 2 \\
4 & 1 & 0 \\
5 & 1 & 1 \\
6 & 1 & 2 \\
\hline
\end{tabular}